FIELD THEORY IN EXTRA DIMENSIONS WITH THIN DEFECTS*

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We discuss some effects associated to thin defects in extra dimensions. After describing the generation of quantum corrections localized at orbifold fixed points, we show some phenomenological implications of the localized terms and point out to generic problems of field theories in the presence of infinitely thin defects, which signal a breakdown of the low-energy expansion. We discuss possible physical interpretations and ways out of these phenomena. Finally, we examine some of these issues in a deconstructed orbifold, which can be regarded as an ultraviolet completion of these models.

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1. Introduction

Many physical systems can be described by fields propagating in a space with lower dimensional defects, possibly boundaries. These infinitely-thin defects are typically idealizations of localized physical backgrounds with a finite size and a certain structure. In this talk we consider the case of field theoretical models in extra dimensions with branes (see [1] for recent reviews). In this setting the defects (or branes) can have different microscopic origins: string-theory D-branes, stable classical field configurations, orbifold fixed planes, *etc.* Nevertheless, their substructure is usually disregarded when extracting phenomenological implications and doing model building in field-theoretical models of extra dimensions. The basic assumption underlying this simplification is that at low energies all observables are fairly insensitive to these ultraviolet details, so that the Dirac-delta limit is a good approximation.

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As a matter of fact, calculations in field theories in the presence of zero-width defects are often plagued with divergences. These divergences arise in the limit of zero thickness and can appear already at the classical level. They signal a breakdown of the field theory at scales where the finite thickness of the defect cannot be neglected, which filtrates into low-energy observables. One example of this situation is classical electrodynamics with point charges. Another one is the calculation of zero point energies of fluctuating fields with boundary conditions (Casimir effect). Brane models in extra dimensions are no exception. The singularities are more and more severe as the codimension of the branes increases. In codimension 2, for instance, one finds logarithmic classical divergences in the presence of mass terms localized on the branes [2]. As we will see, models with branes of codimension 1 are also singular when certain localized operators are present. One important fact of interacting field theories in extra dimensions is that they are non-renormalizable. Therefore, they must be regarded as effective theories, valid below a certain ultraviolet cutoff Λ. Since the couplings have a power-law running, the theory arrives quickly at a nonperturbative regime, so Λ cannot be much larger than the compactification scale $M_{\rm c} = 1/R$.

For definiteness we shall concentrate on orbifold models. An orbifold is basically a manifold in which some points (related by a non-freely-acting symmetry) are identified. Points identified with themselves constitute singular lower-dimensional hyperplanes (fixed "points"). Hence, these models contain infinitely thin defects. Orbifold compactifications of extra dimensions are nowadays standard in string and field theory model building. The main reason is that they give rise to chiral $4D$ fermions without the complications of smooth manifolds such as Calabi–Yau spaces. Here we will focus on the physics of fixed points from a field-theoretical point of view and show that radiative corrections generate divergences localized on them. These divergences indicate that one should include localized operators ("brane terms") in the bare action. In particular, brane kinetic terms (BKT) are a generic feature of brane theories, and many of their properties can be studied in a model-independent way. One can distinguish two kinds of BKT: those with derivatives in the directions parallel to the brane ("parallel BKT") and those with derivatives in the directions orthogonal to the brane ("orthogonal BKT"). We shall first review the phenomenology associated to parallel BKT, which in codimension 1 do not give rise to thin brane singularities. Then, we describe the singularities which appear in the thin-brane limit in the presence of orthogonal BKT. These singularities signify a breakdown of the low-energy expansion of the effective theory. We discuss different interpretations and ways out of this unpleasant situation, emphasizing the idea of classical renormalization. Finally, in order to gain some insight of what the correct, physical approach should be like, we reanalyze these issues in an ultraviolet completion of these models: deconstructed orbifolds.

2. Localized quantum corrections

In this section we give a simple example which shows that orbifold theories have divergent quantum corrections localized on the fixed points [3]. This is allowed because translation invariance is broken there.

Consider a $5D$ ϕ^4 scalar theory with the fifth dimension compactified on an orbifold R/Z_2. R is the real line, parametrized by y, and Z_2 acts as a parity: $y \leftrightarrow -y$, such that $y = 0$ is a fixed point. The Z_2 projection is carried out by identifying $\phi(-y) \sim \pm\phi(y)$. In this section we will work in Euclidean space. One way of performing the calculations would be to work in the fundamental region $[0, \infty)$ imposing Neumann (Dirichlett) boundary conditions at $y = 0$ for an even (odd) field. We follow instead the formalism in [3] and work in the covering space R, with propagators which take into account the even or odd character of ϕ. The interactions, on the other hand, are local in the coordinate y. Using a mixed momentum–position space representation, the Feynman propagator which incorporates the Z_2 parity reads

$$\Delta_\pm(p; y, y') = \frac{e^{-p|y-y'|}}{2p} \pm \frac{e^{-p|y+y'|}}{2p}\,, \tag{1}$$

where here and in the following the upper (lower) sign applies to even (odd) ϕ, and p is the modulus of the four-dimensional momentum. Observe that the propagator depends on both $y - y'$ and $y + y'$. Thus, the momentum in the fifth dimension is conserved up to a sign at tree level.

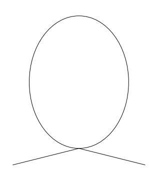

Fig. 1. Tadpole diagram contributing to the two-point function.

The one-loop correction to the two-point function is given by the diagram in Fig. 1. It gives the following contribution to the quadratic part of the quantum effective action:

$$S_2^{(1)} = -\frac{\lambda}{2}\int dy dy' \delta(y-y')\phi(y)\phi(y')\int \frac{d^4p}{(2\pi)^4}\Delta_\pm(p; y, y') \tag{2}$$

$$= -\frac{\lambda}{2}\int dy \phi(y)^2 \int \frac{d^4p}{(2\pi)^4}\left(\frac{1}{2p} \pm \frac{e^{-2p|y|}}{2p}\right) \tag{3}$$

$$= \left[S_2^{(1)}\right]_{\text{bulk}} \pm \left[S_2^{(1)}\right]_{\text{brane}}\,, \tag{4}$$

where the $4D$ dependence of ϕ and the corresponding integration are implicit. The bulk piece is y independent and hence translation invariant. In fact, it coincides with the corresponding correction in the genuine $5D$ theory. It has a cubic $4D$ ultraviolet divergence. On the other hand, $[S_2^{(1)}]_{\rm brane}$ has a profile in the extra dimension:

$$\frac{\lambda}{128\pi^2}\int dy\phi(y)^2\,\frac{1}{y^3}\Big[\;e^{-2\mu y}\,(1+2\mu y+2\mu^2y^2)-e^{-2\Lambda_{4D}y}\\ \times(1+2\Lambda_{4D}y+2\Lambda_{4D}^2y^2)\Big]\,, \tag{5}$$

where Λ_{4D} and μ are, respectively, ultraviolet and infrared $4D$ cutoffs. It is apparent from (4) that the $4D$ UV divergences in $[S_2^{(1)}]_{\rm brane}$ are exactly localized at the fixed point $y=0$. They can be extracted by Taylor expanding $\phi(y)$ about the fixed point and then performing the y integration. Finally, we obtain

$$S_2^{(1)} \;=\; \frac{-\lambda}{16\pi^2}\int dy\,\left[\phi^2(y)\left(\frac{\Lambda_{4D}^3}{6}\pm\delta(y)\frac{\Lambda_{4D}^2}{8}\right)\pm\left(\partial_y^2\phi^2\right)\delta(y)\frac{\log\Lambda_{4D}/\mu}{16}\right]\\ +\,\text{UV finite}\,. \tag{6}$$

We see that the mass and kinetic terms receive, respectively, quadratic and logarithmic $4D$ divergences localized on the fixed point. They should be canceled by appropriate localized counter terms, with finite parts which play the role of renormalized brane couplings. These are free parameters of the theory which in general do not vanish at tree level. In the next sections we study the impact of these new parameters. Note that they have dimensions of length, so that they are expected to be of order of the inverse cutoff of the effective theory, $1/\Lambda$. This is consistent with the fact that the radiative corrections are proportional to the dimensionful coupling λ.

Before finishing this section, let us make two observations. The first is that both bulk and brane ultraviolet divergences are a short-distance effect. Therefore, they are unchanged in the compact case, S^1/Z_2, except for a replication of brane divergences at the second fixed point, $y=\pi R$ (R is the radius of the circle). This can be seen explicitly in an expansion in winding-modes: the divergences in the bulk and at $y=0$ correspond to zero winding [4]. The second observation is that we have regularized the $4D$ integral, while in the fifth dimension we have integrated over arbitrarily short distances. This corresponds to the physical situation in which the distance in the fifth dimension above which the effective theory is valid is much smaller than $1/\Lambda_{4D}$. The opposite scenario corresponds to a lattice in the 5th dimension coarser than $1/\Lambda_{4D}$, and is studied in Sec. 6. If instead we were to use a hard cutoff Λ_5 in the fifth component of the loop momenta (rather than in position space), we would obtain Eq. (6) with $\delta(y)$ replaced by a regularized delta function

$$\delta_{1/\Lambda_5} = \frac{\sin(\Lambda_5 y)}{\pi y} \,. \tag{7}$$

Therefore, we see that although the fixed points in an orbifold are infinitely thin, in a Wilsonian effective theory the brane terms generated by radiative corrections have a finite width, of the order of the inverse cutoff in the fifth dimension.

3. Phenomenology of parallel brane kinetic terms

We have seen that the general action of our orbifold theory should include brane terms. We will concentrate hereafter on the effect of BKT (see [6]) for a general analysis). We can distinguish parallel BKT, with derivatives in the directions parallel to the brane, and orthogonal BKT, with derivatives in the directions orthogonal to the brane (such as the ones we have found in the example above). In this section we consider parallel BKT, for which the thin-brane limit is nonsingular in codimension 1. These are the brane terms that have been most extensively studied. In particular, parallel BKT have been used to construct interesting models of gravity [7], and to improve the phenomenology of Higgsless models in flat space [8] and of models in which the Higgs arises from a higher dimensional gauge boson [9]. Here we review the phenomenology associated to parallel BKT for bulk fermions and gauge bosons [10–12].

For gauge bosons propagating on an S_1/Z_2 orbifold, the free Lagrangian including parallel BKT reads

$$\mathcal{L} = -\frac{1}{2}\left(1 + a_I^A \delta_I\right)\ \mathrm{tr} F_{\mu\nu}F^{\mu\nu} - \mathrm{tr} F_{4\nu}F^{4\nu} \,, \tag{8}$$

where $\delta_I(y) = \delta(y - IR)$ and $I = 0, \pi$ denotes two fixed points. The Lorentz indices μ, ν run over the parallel directions: $\mu, \nu = 0, 1, 2, 3$. We will consider the case in which the gauge symmetry is unbroken in the effective $4D$ theory, which corresponds to even A_μ and odd A_4. Furthermore, we take a_I^A positive to avoid tachions and ghosts in the spectrum. Working in the "axial" gauge $A_4 = 0$ (see [6] for a more rigorous gauge invariant approach), we can perform the Kaluza–Klein (KK) decomposition

$$A_\mu(x, y) = \sum_{n=0}^{\infty} \frac{f_n^A(y)}{\sqrt{2\pi R}} A_{\mu\, n}(x) \,, \tag{9}$$

where $\{f_n^A\}$ is a basis of even functions in S^1. We require that these functions fulfill the differential equations

$$\partial_y^2 f_n^A = -m_n^2 \left(1 + a_I^A \delta_I\right) f_n^A \,. \tag{10}$$

Then, they are orthogonal with respect to the scalar product

$$\langle g, h \rangle = \frac{1}{2\pi R} \int dy \left(1 + a_I^A \delta_I\right) g^*(y) h(y) \tag{11}$$

and the tower of quadratic terms in the reduced $4D$ Lagrangian is diagonal. Imposing the normalization

$$\langle f_n, f_n \rangle = 1 \,, \tag{12}$$

the resulting $4D$ kinetic terms are automatically canonical. The solutions are a massless mode with flat wave function,

$$f_0^A = \frac{1}{\sqrt{1 + \frac{a_0^A + a_\pi^A}{2\pi R}}} \tag{13}$$

and a tower of massive Kaluza–Klein modes

$$f_n^A = N_n \Big[\cos(m_n y) - a_0^A \sin(m_n |y|) \Big] \tag{14}$$

with N_n fixed by (12) and masses determined by the equation

$$\left(4 - a_0^A a_\pi^A m_n^2\right) \tan(\pi m_n R) + 2\left(a_0^A + a_\pi^A\right) m_n = 0 \,. \tag{15}$$

We see that the spectrum and the shape of the wave functions are modified by the BKT. The modifications depend to a great extent on whether the sizes of the BKT at both fixed points are comparable or not. If they are, and $a_{0,\pi}^A \gg R$, the first mode is much lighter than the compactification scale $1/R$, with mass $m_1^2 \sim 2(a_0^A + a_\pi^A)/(a_0^A a_\pi^A \pi R)$ and couplings to brane fields equal in size to the one of the zero mode, while the rest of the massive modes have masses which approach $m_n \sim (n-1)/R$ and suppressed couplings to the branes (Fig. 2). In this scenario (with fermions on the branes) there is the possibility of discovering the first KK mode (a sequential gauge boson) at LHC even for a large compactification scale. When one of the BKT coefficients, say a_π^A, is negligible with respect to the other, all KK modes behave similarly. For $a_0^A \gg 1$ their masses approach $m_n \sim (n - 1/2)/R$ and their couplings to the brane at $y = 0$ are suppressed

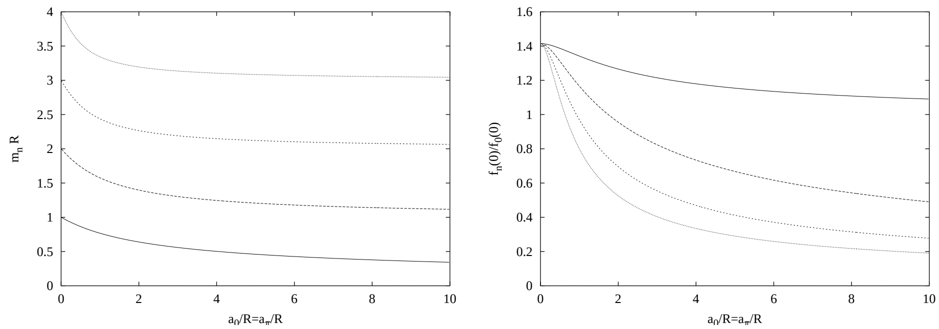

Fig. 2. Masses (left) and couplings to the brane at $y = 0$ normalized to the zero mode coupling (right) for the first few KK modes, $n = 1, 2, 3, 4$ from bottom to top (left) and from top to bottom (right), as a function of $a_0^A = a_\pi^A$.

Let us now turn to bulk (massless) fermions with parallel BKT. Fermions in $5D$ are Dirac spinors, with two chiral components from the four-dimensional point of view: $\Psi = \Psi_{\rm L} + \Psi_{\rm R}$, $\gamma_5 \Psi_{\rm L,R} = \mp \Psi_{\rm R,L}$. Invariance of the bulk kinetic term under Z_2 requires that the left-handed and right-handed components have opposite Z_2 parities. We choose an even left-handed component. The kinetic Lagrangian and gauge couplings read

$$\mathcal{L} = \left(1 + a_I^{\rm L} \delta_I\right) \bar{\Psi}_{\rm L} i \not{D} \Psi_{\rm L} + \bar{\Psi}_{\rm R} i \not{D} \Psi_{\rm R} \,. \tag{16}$$

The term proportional might be naively argued to vanish, based on the odd character of $\Psi_{\rm R}$. However, this is not necessarily so if $\Psi_{\rm R}$ is discontinuous at the fixed points, and this is actually the case for $a_I^{\rm L} \neq 0$. The effect of $a_I^{\rm R}$ has been described in [6]. Here, in order to simplify the analysis, we take $a_I^{\rm R} = 0$. The KK reduction is then very similar to the one for gauge bosons, except for the presence of a chiral zero mode.

The couplings of the four-dimensional effective theory are given by the overlap of wave functions times five-dimensional couplings. In particular, the gauge interactions of the fermions have couplings

$$g_{mnr} = \frac{g_5}{\sqrt{2\pi R}} \int_{-\pi R}^{\pi R} dy \; (1 + a_I^{\rm L} \delta_I) \; \frac{f_m^{\rm L} f_n^{\rm L} f_r^A}{2\pi R} \,. \tag{17}$$

The couplings relevant for phenomenology are the ones with two fermion zero modes and KK excitations of gauge bosons, g_{00r}, which give rise to effective four-fermion operators at low energies. They vanish when $a_0^{\rm L} = a_0^A$ and $a_\pi^{\rm L} = a_\pi^A$, and in particular when BKT are absent. One must also take into account the fact that the profile (and then the couplings) of the Z and

W zero modes is modified by a localized Higgs. A fit to electroweak precision observables including (universal) BKT for bulk fermions and gauge bosons results in bounds on the compactification scale as a function of $a_I^{\rm L}$ and a_I^A, shown in the plots of Fig. 3 [12].

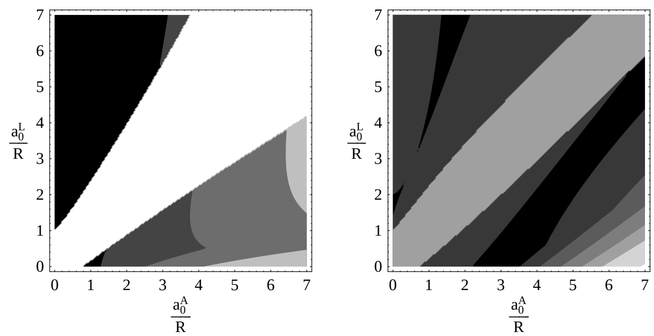

Fig. 3. Bounds at 1 σ on the compactification scale as a function of a_0^A and $a_0^{\rm L}$ for $a_\pi^{{\rm L},A} = a_0^{{\rm L},A}$ (left) and $a_\pi^{{\rm L},A} = 0$ (right). The band along the diagonal corresponds to $U_{\rm eff}$ being too small to give a meaningful lower bound. The other bands correspond, from dark to light, to $M_{\rm c} \geq 2, 3, 3.5, 4$ TeV (left) and $M_{\rm c} \geq 1, 2, 3, 4, 5, 6$ TeV (right).

On the other hand, the localized Higgs also modifies the profile of fermion wave functions. This in turn gives rise to a non-unitary CKM matrix, with departures from the Standard Model proportional to the mass of the quarks involved, *i.e.*, most relevant for top physics [13]. The deviations, which depend on the BKT of fermions as well, give stringent limits coming from the T parameter, which become weaker when $a_0^{\rm L} \gg a_\pi^{\rm L}$ [11] (see also the corresponding plot in the contribution of F. del Aguila and R. Pittau to these proceedings [14]).

4. Orthogonal BKT and thin brane singularities

In this section we study orthogonal BKT and show that they generate singularities in the thin-brane limit [6]. Consider a theory with an even massless complex scalar and a quadratic Lagrangian given by

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \partial_y \phi^\dagger \partial_y \phi + \frac{b}{2} \delta_0 \left[\phi^\dagger \partial_y^2 \phi + (\partial_y^2 \phi^\dagger) \phi \right] . \tag{18}$$

We could have included a BKT $c\delta_0 |\partial_y \phi|^2$ as well, but it has no effect on the exact KK masses and wave functions. We take $b \geq 0$.

The KK reduction is performed in this case by expanding

$$\phi(x, y) = \sum_n \frac{f_n(y)}{\sqrt{2\pi R}} \phi^{(n)}(x) , \tag{19}$$

with f_n given by the eigenvalue equation

$$\left[(1+b\delta_0)\partial_y^2 + b\delta_0'\partial_y + \frac{b}{2}\delta_0''\right] f_n = -m_n^2 f_n\,. \tag{20}$$

In order to make this problem well-posed, we regularize the delta functions: $\delta \to \delta_\varepsilon$, with ε a small length. Then, we solve numerically the differential equation with Neumann boundary conditions (imposed by periodicity and the Z_2 parity). In Fig. 4 we plot the wave function of the first KK mode for a small b and three values of ε. It turns out that the solutions are regularization independent in the limit $\varepsilon \to 0$. They reduce to[1]

$$f_n(y) = \begin{cases} N_n \cos(m_n y) & \text{if } b = 0\,, \\ N_n \sin(m_n |y|) & \text{if } b \neq 0\,, \end{cases} \tag{21}$$

with masses $m_n = n/R$ if $b = 0$, $m_n = (n+1/2)/R$ if $b \neq 0$. The same solutions for $b \neq 0$ would be obtained in the presence of parallel BKT or brane mass terms, *i.e.*, the orthogonal BKT erases the effect of other brane terms. We see that an arbitrarily small b brings about drastic changes in the spectrum and couplings. In particular, the zero mode disappears[2]. This signifies a breaking of perturbation theory in the effective orbifold theory.

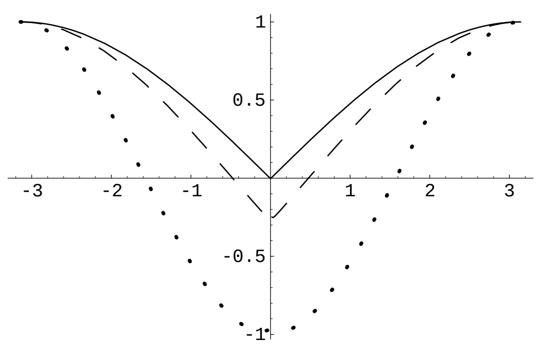

Fig. 4. Wave function of the 1st KK mode for $b = 0.01$ and regularized deltas with $\varepsilon = 10^{-1}$ (dots), 10^{-2} (dashes) and 10^{-3} (solid). The mass is $m_1 = 1.002$, 0.589 and 0.501, respectively. b, ε and m_1 are given in units of R.

[1] In fact, these wave functions do not satisfy (20) when $b \neq 0$! The reason is that the derivatives of f_n do not converge uniformly, due to strong fluctuations near $y = 0$. The safest procedure is to perform the integrals on y before taking the limit $\varepsilon \to 0$. Nevertheless, the wave functions in (21) can be used before integration in the absence of derivative interactions.

[2] More precisely, it is transformed into a tachyon with squared mass approaching $-\infty$ when $\varepsilon \to 0$, and wave function localized at $y = 0$ in that limit.

In fact, if we use perturbation theory and treat the orthogonal BKT as an insertion, we find thin-brane divergences $\delta^n(0)$. The resummation of these divergences yields the singular behavior we have just exhibited.

In the case of fermions the singularities are somewhat milder. Again, the orthogonal BKT generate thin-brane divergences in perturbation theory. But in this case the (chiral) zero mode survives and the masses and wave functions are only discontinuous at $b = 0$ when parallel BKT are present as well. The effect of switching b on is basically to eliminate the dependence on the coefficients of parallel BKT and to make the wave functions discontinuously vanish at $y = 0$. Therefore, for an arbitrarily small nonzero b the bulk fermions decouple from the fields living on the brane. In particular, they cannot get a mass from a Higgs localized on the brane.

Finally, gauge bosons do not have these problems because a b BKT is forbidden by gauge symmetry. The c BKT mentioned above is allowed and it actually produce divergences at higher orders in perturbation theory, but it has no effect when treated exactly.

5. Dealing with thin-brane singularities

We have seen that fields misbehave in the presence of certain orthogonal BKT when the branes are infinitely thin. Taken at its faith value, this result constrains strongly the construction of realistic brane models with bulk fermions — due to the decoupling of the fields from the branes — and/or bulk scalars, because of the absence of zero modes and the appearance of tachyons. Even more problematic is the fact that perturbation theory breaks down, so that possible higher order BKT are out of control. In this section we sketch possible interpretations and ways out of this situation.

5.1. Supersymmetry and critical theories

The most obvious solution is to consider only theories with $b = 0$. Since, as we have seen in Sec. 2, b is in general radiatively induced, we would need a symmetry which protects this BKT against quantum corrections. This is achieved in supersymmetric gauge theories in $5D$ thanks to a combination of supersymmetry and gauge invariance [6]. Indeed, the b BKT for fermions and sfermions in the hypermultiplet is an F term, whereas gauge bosons and gauginos are protected by gauge symmetry. Hence, putting $b = 0$ by hand at tree level we obtain a well-behaved theory. This could be natural in superstring theory. However, in realistic scenarios supersymmetry must be broken and in general one expects finite corrections to b which reintroduce the thin-brane singularities.

It is interesting to note that in supersymmetric theories with BKT (parallel or orthogonal) the behavior of bosons and fermions in the same $4D$ $N=1$ supermultiplet is matched as the result of higher order (singular) BKT in the bosonic sector. This is an example which shows how higher order operators can change drastically low-energy physics in the presence of thin defects. We can take advantage of this effect and consider a class of theories with arbitrary b and a tower of higher-order brane terms such that the low-energy physics be smooth. In such critical theories, the coefficients of the higher-order operators are not arbitrary, but perfectly fine tuned functions of b. At quantum level this corresponds to a reduction of couplings. These relations among operators of different orders may be natural if the effective theory comes from an appropriate fundamental theory with fewer parameters. Since the higher-order terms are fixed by the first order terms, it is plausible that the tower can be resummed. In fact, for a scalar we can consider a theory described by

$$\mathcal{L} = \frac{1 - b\delta_0}{\left(1 - \frac{b}{2}\delta_0\right)^2} \partial_\mu \phi^\dagger \partial^\mu \phi - \partial_y \left(\frac{\phi^\dagger}{1 - \frac{b}{2}\delta_0} \right) \partial_y \left(\frac{\phi}{1 - \frac{b}{2}\delta_0} \right) . \tag{22}$$

This Lagrangian reduces to (18) at first order in b. At higher orders, however, the thin-brane divergences are canceled out, and the KK exact masses and wave functions behave smoothly in the $\varepsilon \to 0$ limit. As a matter of fact, after a singular field redefinition $\phi = (1 - b\delta_0/2)\tilde{\phi}$ the Lagrangian (22) reduces to

$$\mathcal{L} = (1 - b\delta_0)\, \partial_\mu \tilde{\phi}^\dagger \partial^\mu \tilde{\phi} - \partial_y \tilde{\phi}^\dagger \partial_y \tilde{\phi} \, , \tag{23}$$

which has only inoffensive parallel BKT. Note, nevertheless, that this field redefinition may introduce (or cancel) singularities in the interaction Lagrangian.

5.2. Thin coefficients and classical renormalization

Since the singularities appear only in the thin-brane limit, a trivial solution is to use thick branes, regularized by a finite ε. The problem here is that the physics then depends on the shape and size of the branes. Moreover, the radiative corrections suggest that $1/\varepsilon$ is of the order of the UV cutoff of the effective theory, Λ. Then, from the point of view of an infrared observer (at energies of order $1/R$) the brane must be pretty thin, with the consequent abrupt behavior. In order words, organizing the effective theory as a series of powers of $1/\Lambda$ requires expanding about $\varepsilon = 0$.

If we identify $\varepsilon = 1/\Lambda$ and the coefficients of BKT are also $O(1/\Lambda)$, then we have $a = \alpha\varepsilon$, $b = \beta\varepsilon$ with α, β of order one. This means that the coefficients a, b are naturally "thin"[3] and in the expansion in ε we should keep α, β constant. Doing this, the effect of parallel terms vanishes in the

[3] This picture could be changed by geometric factors if Λ was not very large [10].

limit $\varepsilon \to 0$. Note that this limit corresponds to $\Lambda \to \infty$, in which all the bulk couplings vanish as well. So, we must consider at least the first order corrections. At order ε a nontrivial smooth result is obtained. This indicates that parallel BKT give suppressed corrections to bulk physics. On the other hand, the numerical calculations show that the effect of orthogonal terms is still singular in terms of β. This is quite surprising because it means that an operator proportional to a "weak" delta (we mean a function with support $\{0\}$ and vanishing integral) is able to change the physics dramatically. In terms of the $1/\Lambda$ expansion, we find that the orthogonal BKT are stronger than O(1).

On the other hand, preliminary numeric calculations suggest that, writing $\beta = (\varepsilon/R)^{1/2}\xi$, the physics is smooth and nontrivial in the parameter ξ. This is confirmed by the calculations in deconstructed theories below. If this behavior is universal at small ε for any regularization of the brane (although the adequate power of ε might depend on the regularization), it implies that we can consider a, b as bare parameters with a convenient ε dependence, $a_0 = a_{\mathrm{R}}$, $b_0 = (\varepsilon/\mu)^{3/2} b_{\mathrm{R}}$, such that the renormalized parameters a_{R} and b_{R} describe smooth physics in the limit $\varepsilon \to 0$ (see [2] for an application of this method to $6D$ theories with mass brane terms and [15] for an alternative approach). In this way, our ignorance about thin brane physics is absorbed into the bare parameters. Since this procedure must be performed already at tree level, it is often called classical renormalization. Of course, it should be implemented at the quantum level as well. This can be done by adding quantum counter terms with a finite part with an ε dependence as in the bare parameters above. Classical renormalization can be also carried out perturbatively by introducing counter terms with thin brane divergences [6]. These counter terms are higher order operators. In fact, implementing classical renormalization in this manner is equivalent to working with the critical theories described in the previous subsection.

Classical renormalization of thin-brane singularities gives us a smooth parametrization of the physics of brane models. This is clearly an improvement, but one may worry about legitimacy of the procedure: does the renormalized theory describe correctly, at low-energies, the physics of defects? The situation is analogous to the one in Casimir problems. There, the traditional approach is also to perform a "classical" thin-brane renormalization in order to find a well-defined, finite Casimir energy. However, this approach has recently been criticized by Graham *et al.* [16]. These authors argue that classical renormalization arbitrarily eliminates divergences which have a physical significance: they show that the zero-point energies depend on the microscopic details of the interaction of the bulk fields with the branes. In the problem at hand, there seems to be a universal behavior for small ε, which allows to absorb the impact of the brane into bare parameters. However, we should still wonder about the natural size of the renormalized parameters. This depends in turn on the size of bare parameters, which will be ultimately given by the fundamental theory completing our effective theory at high energies. We will come back to this point in the next section, where we study a toy-model completion. At the level of the effective action we can estimate their size using radiative corrections: Assuming that there are no strong cancellations and that $\varepsilon \sim 1/\Lambda$, radiative corrections provide a lower bound of the size of the bare couplings: $a_0, b_0 \sim \varepsilon$. This means that the renormalized coupling $a_{\rm R}$ is naturally small (with $\alpha \sim$O(1)) while $b_{\rm R}$ is naturally very large. But for large $b_{\rm R}$ we recover the solutions displayed above when $b_0 \neq 0$! This effect can be described in a dual manner as the renormalization group flow of the bare BKT: the parallel BKT are irrelevant (just as bulk gauge interactions in $5D$), whereas the orthogonal BKT are relevant operators in the infrared. $b_0 = 0$ is an unstable fixed point of this flow. This contrasts with the case of mass brane terms in $6D$, which are marginal and give rise to interesting logarithmic runnings [2].

6. Deconstructing orbifolds

In order to know better what the physically sensible treatment of thin brane singularities should be like, we would need to match the low energy effective theory to the microscopic theory describing the brane model at short distances. In this section we follow this idea and try to gain some insight on the problem by studying a deconstructed version of a $5D$ orbifold theory, which can be regarded as an ultraviolet completion of the effective theory in the continuum. We will just sketch the procedure. A more complete analysis will be presented in [17].

Deconstructed theories of extra dimensions were introduced in [5]. A deconstructed model is basically a $4D$ theory with a replication of gauge groups with common gauge coupling g and a series of "link" scalars transforming under couples of gauge groups. This structure of sites connected by links is best represented by a *moose* diagram. Fig. 5 describes (before folding) a moose with a linear periodic gauge structure. A discrete extra dimension with spacing $s = 1/(gv)$ is generated when the link fields aquire a vev v. Then, most of the gauge symmetry is Higgsed and the gauge bosons adquire a mass matrix which is nothing but a discrete version of the extra dimensional derivative. Translation invariance in the extra dimension corresponds to having a common gauge coupling for all the groups and all the links getting the same vev. One can also add scalar and fermionic matter at each site to deconstruct bulk scalars and fermions. Alternatively, one can add, for instance, a fermion at one site only. This is the deconstructed version of a model with a fermion localized on a brane. Note that in this case translation invariance is broken. The nice thing about deconstructed theories is that, unlike theories in extra dimensions, they are well-behaved in the ultraviolet. For instance, the links can be linear-sigma-model fields with a potential such that they spontaneously get a common vev.

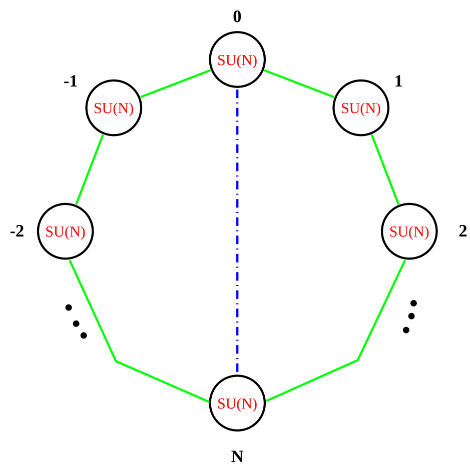

Fig. 5. Moose of a deconstructed circle. A site orbifold is constructed by folding it across the vertical dashed–dotted line.

We can define deconstructed orbifolds by conveniently identifying the fields at different sites and links. For example, a deconstructed S_1/Z_2 orbifold is obtained when the periodic moose of Fig. 5 is folded across the dashed–dotted line. In fact, for an even number of sites, we can define two inequivalent deconstructed orbifolds: a "site orbifold" when we fold across sites, as in the figure, and a "link orbifold" when we fold across links. For odd number of sites there is only one possibility: a "site-link orbifold". The "site orbifolds" are particular cases of aliphatic models, which are non periodic linear mooses with boundaries, and particular interactions at the border sites (mimicking boundary conditions). It is worthwhile to note, however, that the orbifold picture gives in a natural way the boundary interactions which in the aliphatic picture are introduced *ad hoc* to reproduce the usual $5D$ spectra.

The orbifolding breaks translational symmetry at the fixed sites (links) and therefore localized radiative corrections are expected. To see this explicitly, consider a moose with a *global* $\prod_{i=-N+1}^{N} \mathrm{SU}(\mathcal{N})_i$ symmetry, scalars ϕ_i in the adjoint of $\mathrm{SU}(\mathcal{N})_i$ and scalar link fields χ_j transforming under the $(\mathcal{N},\bar{\mathcal{N}})$ of $(\mathrm{SU}(\mathcal{N})_{j-1/2},\mathrm{SU}(\mathcal{N})_{j+1/2})$. It is convenient to use a notation in which, for a site (link) orbifold, the indices for the sites are integer (semi-odd) and the indices for the links and semi-odd (integer). Then, the orbifold projection is performed in both cases by identifying

$$\phi_{-i} \sim \pm\phi_i\,, \tag{24}$$

$$\chi_{-i} \sim \chi_i^\dagger\,. \tag{25}$$

We should note that this toy model without gauge fields will have a bunch of Goldstone bosons. This is unrealistic, but it does not interfere with the problems we are studying here.

For a finite spacing (finite v), the quantum divergences are the same in the unbroken and broken phases. Therefore, if we are only interested in extracting the divergent parts we can work in the unbroken theory, which has a diagonal free action, and put $\chi = v$ at the end. This is equivalent to treating the discrete derivatives in the fifth dimension as insertions. In this formalism, the propagator of the scalar field is simply the standard $4D$ propagator times $\delta_{ii} \pm \delta_{i-i}$. To illustrate what sort of corrections one finds at one loop, let us consider again the diagram in 1. Its contribution to the effective action is proportional to

$$\sum_i \int d^4p\, \phi_i^2(\delta_{ii} \pm \delta_{i-i})\tfrac{1}{p^2} \tag{26}$$

$$\sim \Lambda_{4D}^2 \sum_i (1 \pm \delta_{i0})\phi_i^2\,. \tag{27}$$

Therefore, there is again a divergence localized at the brane $i = 0$ for a site orbifold. On the other hand, this diagram gives no localized divergence in a link orbifold, since i is in this case semi-odd. This makes sense, since in the link orbifold the field ϕ "jumps" the fixed point. The opposite result is found for contributions to the discrete kinetic term, which come from the same diagram with an insertion of the operator $\phi^2|\chi|^2$. This effect has no counterpart in the continuum, where the positions of sites and links collapse. On the other hand, we should also take into account diagrams with virtual links. These go beyond a simple discretization of the $5D$ theory. The brane terms generated by them can be understood as brane terms included at tree level in the $5D$ effective theory.

In general, the bulk and brane one-loop contributions have the same size. They are proportional to the adimensional coupling constants of the deconstructed theory and to a factor of v for every insertion of χ. The factors of v are precisely the ones needed to build the discrete derivatives. At energies lower than $\Lambda = 1/s$ the deconstructed theory can be described by an effective $5D$ theory with cutoff Λ. Taking into account that in the continuum limit $\delta_{i0} \to s\delta(x)$, we see that our expectations above are confirmed in the deconstructed scenario, namely, the coefficients of the brane terms have a $1/\Lambda$ suppression.

In [18] we have analyzed the effect on masses and wave functions of general BKT in a deconstructed (discretized) theory. We have shown that if we keep the continuum parameters a, b and the compactification radius R constant, the results of the previous sections are reproduced in the continuum limit. We can also show that, as we anticipated, physics is still singular in the continuum when we keep the adimensional parameters $\alpha = a/s, \beta = b/s$ fixed (which implies vanishing b). Finally, writing $\beta = \xi s^{1/2}$ one finds smooth results in terms of ξ, a. Indeed, the wave functions and KK masses in the continuum are given by (14) and (15) with $a_I \to a_I - \xi_I^2/m^2$ (for $m \neq 0$). We shall give more details in [17].

At any rate, we stress again that the one loop corrections generate α, β independent of s. The infrared of the deconstructed theory corresponds to KK modes with mass $\ll 1/s$ [4]. Therefore, the calculations in deconstructed orbifolds support the interpretation at the end of Sec. 5. It seems then that the correct low-energy physics of bulk fermions and scalars in noncritical theories with branes is given by a large $b_{\rm R}$. However, it is not clear whether this picture is stable under higher order quantum corrections. In particular, one may worry about the generation of higher order BKT with thin brane singularities. In this sense, two loop calculations in orbifold models can be crucial to establish or invalidate the loop expansion in extra dimensional models with thin defects.

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