

RESTORATION OF CHIRAL SYMMETRY  
IN EXCITED HADRONS\*

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Physics of the low-lying and high-lying hadrons in the light flavor sector is reviewed. While the low-lying hadrons are strongly affected by the spontaneous breaking of chiral symmetry, in the high-lying hadrons the chiral symmetry is restored. A manifestation of the chiral symmetry restoration in excited hadrons is a persistence of the chiral multiplet structure in both baryon and meson spectra. Meson and baryon chiral multiplets are classified. A relation between the chiral symmetry restoration and the string picture of excited hadrons is discussed.

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**1. Introduction**

It was believed by many people (and is still believed by some) that there should be some universal physical picture (model) for all usual hadrons<sup>1</sup>. If we consider, as example, atoms, there is indeed a universal picture for all excitations: electrons move in the central Coulomb field of the nucleus. Such a system is essentially nonrelativistic and relativistic effects appear only as very small corrections to the nonrelativistic description. However, hadrons in the  $u, d, s$  sector are more complex systems. This complexity comes in particular from the very small masses of  $u$  and  $d$  quarks. These small masses guarantee that the role of relativistic effects, such as creation of pairs from the vacuum, should be important. If so in the  $u, d, s$  quark sector a description should incorporate valence quarks, sea quarks and gluonic degrees of freedom.

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<sup>1</sup> Under “usual” hadrons we assume those ones which are not glueballs and with quantum numbers which are provided by the minimal  $\bar{q}q$  or  $qqq$  quark Fock component.

The main message of these lectures is that physics of the low-lying hadrons in the  $u, d, s$  sector is essentially different from the physics of the highly excited states. In the former case spontaneous breaking of chiral symmetry is crucial for physics implying such effective degrees of freedom as constituent quarks (being essentially quasiparticles [1]), constituent quark — Goldstone boson coupling [1,2], *etc.* In the latter case, on the other hand, spontaneous breaking of chiral symmetry in the QCD vacuum becomes irrelevant, which is referred to as effective chiral symmetry restoration or chiral symmetry restoration of the second kind [3–7]. Hence in this case other degrees of freedom become appropriate and probably the string picture [8] with “bare” quarks of definite chirality at the ends of the string [9] is a relevant description.

It is not a surprise that physics of the high-lying excitations and of the low-lying states is very different in complex systems. Remember that in Landau’s Fermi-liquid theory (QCD is a particular case of such a theory) the quasiparticle degrees of freedom are relevant only to the low-lying excitations while high-lying levels are excitations of bare particles.

These lectures consist of the following sections. In the second one we review chiral symmetry of QCD. The third section is devoted to a description of the low-lying hadrons, which are strongly affected by spontaneous breaking of chiral symmetry. Empirical hadron spectra are reviewed in Section 4. In the fifth section we introduce chiral symmetry restoration in highly excited hadrons. In the next section a toy pedagogical model will be discussed which clearly illustrates that there is no mystery in symmetry restoration in high-lying spectra. Implications of the quark–hadron duality in QCD for spectroscopy are discussed in Section 7. In Sections 8 and 9 we will classify chiral multiplets of excited mesons and baryons respectively. In Section 10 it is shown that a simple potential constituent quark model is incompatible with the chiral symmetry restoration in excited hadrons. A relation between the chiral symmetry restoration and the string picture of excited hadrons is discussed in Section 11. Finally, a short summary will be presented in the conclusion part.

## 2. Chiral symmetry of QCD

Consider the chiral limit where quarks are massless. It is definitely justified for  $u$  and  $d$  quarks since their masses are quite small compared to  $\Lambda_{\text{QCD}}$  and the typical hadronic scale of 1 GeV; in good approximation they can be neglected. Define the right- and left-handed components of quark fields

$$\psi_R = \frac{1}{2}(1 + \gamma_5)\psi, \quad \psi_L = \frac{1}{2}(1 - \gamma_5)\psi. \quad (1)$$

If there is no interaction, then the right- and left-handed components of the quark field get decoupled, as it is well seen from the kinetic energy term

$$\mathcal{L}_0 = i\bar{\Psi}\gamma_\mu\partial^\mu\Psi = i\bar{\Psi}_L\gamma_\mu\partial^\mu\Psi_L + i\bar{\Psi}_R\gamma_\mu\partial^\mu\Psi_R, \quad (2)$$

see Fig. 1.

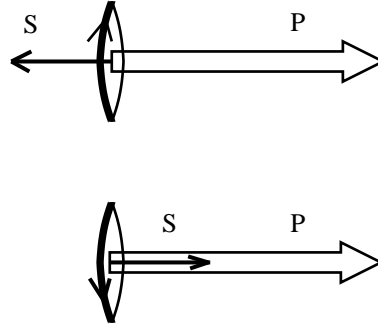


Fig. 1. Left-handed and right-handed massless fermions.

In QCD the quark–gluon interaction Lagrangian is vectorial,  $\bar{\psi}\gamma^\mu\psi A_\mu$ , which does not mix the right- and left-handed components of quark fields. Hence in the chiral limit the left- and right-handed components of quarks are completely decoupled in the QCD Lagrangian. Then, assuming only one flavor of quarks such a Lagrangian is invariant under two independent global variations of phases of the left-handed and right-handed quarks:

$$\psi_R \rightarrow \exp(i\theta_R)\psi_R; \quad \psi_L \rightarrow \exp(i\theta_L)\psi_L. \quad (3)$$

Such a transformation can be identically rewritten in terms of the vectorial and axial transformations:

$$\psi \rightarrow \exp(i\theta_V)\psi; \quad \psi \rightarrow \exp(i\theta_A\gamma_5)\psi. \quad (4)$$

The symmetry group of these phase transformations is

$$U(1)_L \times U(1)_R = U(1)_A \times U(1)_V. \quad (5)$$

Consider now the chiral limit for two flavors,  $u$  and  $d$ . The quark–gluon interaction Lagrangian is insensitive to the specific flavor of quarks. For example, one can substitute the  $u$  and  $d$  quarks by properly normalized orthogonal linear combinations of  $u$  and  $d$  quarks (*i.e.* one can perform a rotation in the isospin space) and nothing will change. Since the left- and right-handed components are completely decoupled, one can perform two independent isospin rotations of the left- and right-handed components:

$$\psi_R \rightarrow \exp\left(i\frac{\theta_R^a \tau^a}{2}\right) \psi_R; \quad \psi_L \rightarrow \exp\left(i\frac{\theta_L^a \tau^a}{2}\right) \psi_L, \quad (6)$$

where  $\tau^a$  are the isospin Pauli matrices and the angles  $\theta_R^a$  and  $\theta_L^a$  parameterize rotations of the right- and left-handed components. These rotations leave the QCD Lagrangian invariant. The symmetry group of these transformations,

$$SU(2)_L \times SU(2)_R, \quad (7)$$

is called chiral symmetry.

Actually in this case the Lagrangian is also invariant under the variation of the common phase of the left-handed  $u_L$  and  $d_L$  quarks, which is the  $U(1)_L$  symmetry and similarly — for the right-handed quarks. Hence the total chiral symmetry group of the QCD Lagrangian is

$$\begin{aligned} U(2)_L \times U(2)_R &= SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \\ &= SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A. \end{aligned} \quad (8)$$

This is a symmetry of the QCD Lagrangian at the classical level. At the quantum level the  $U(1)_A$  symmetry is explicitly broken due to axial anomaly, which is effect of quantum fluctuations. The  $U(1)_V$  symmetry is responsible for the baryon number conservation and will not be discussed any longer.

Now generally if the Hamiltonian of a system is invariant under some transformation group  $G$ , then one can expect that one can find states which are simultaneously eigenstates of the Hamiltonian and of the Casimir operators of the group,  $C_i$ . Now, if the ground state of the theory, the vacuum, is invariant under the same group, *i.e.* if for all  $U \in G$

$$U|0\rangle = |0\rangle, \quad (9)$$

then eigenstates of this Hamiltonian corresponding to excitations above the vacuum can be grouped into degenerate multiplets corresponding to the particular representations of  $G$ . This mode of symmetry is usually referred to as the Wigner–Weyl mode. Conversely, if (9) does not hold, the excitations do not generally form degenerate multiplets in this case. This situation is called spontaneous symmetry breaking.

If chiral symmetry were realized in the Wigner–Weyl mode, then the excitations would be grouped into representations of the chiral group. The representations of the chiral group are discussed in detail in the following sections. The important feature is that the every representation except the trivial one necessarily implies parity doubling. In other words, for every baryon with the given quantum numbers and parity, there must exist another baryon with the same quantum numbers but opposite parity and which

must have the same mass. In the case of mesons the chiral representations combine, *e.g.* the pions with the  $f_0$  mesons, which should be degenerate. This feature is definitely not observed for the low-lying states in hadron spectra. This means that Eq. (9) does not apply; the continuous chiral symmetry of the QCD Lagrangian is spontaneously (dynamically) broken in the vacuum, *i.e.* it is hidden. Such a mode of symmetry realization is referred to as the Nambu–Goldstone one.

The independent left and right rotations (6) can be represented equivalently with independent isospin and axial rotations

$$\psi \rightarrow \exp\left(i\frac{\theta_V^a \tau^a}{2}\right) \psi; \quad \psi \rightarrow \exp\left(i\gamma_5 \frac{\theta_A^a \tau^a}{2}\right) \psi. \quad (10)$$

The existence of approximately degenerate isospin multiplets in hadron spectra suggests that the vacuum is invariant under the isospin transformation. Indeed, from the theoretical side the Vafa–Witten theorem [10] guarantees that in the local gauge theories the vector part of chiral symmetry cannot be spontaneously broken. The axial transformation mixes states with opposite parity. The fact that the low-lying states do not have parity doublets implies that the vacuum is not invariant under the axial transformations. In other words the almost perfect chiral symmetry of the QCD Lagrangian is dynamically broken by the vacuum down to the vectorial (isospin) subgroup

$$\text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(2)_I. \quad (11)$$

The noninvariance of the vacuum with respect to the three axial transformations requires existence of three massless Goldstone bosons, which should be pseudoscalars and form an isospin triplet. These are identified with pions. The nonzero mass of pions is entirely due to the *explicit* chiral symmetry breaking by the small masses of  $u$  and  $d$  quarks. These small masses can be accounted for as a perturbation. As a result the squares of the pion masses are proportional to the  $u$  and  $d$  quark masses [11]

$$m_\pi^2 = -\frac{1}{f_\pi^2} \frac{m_u + m_d}{2} (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) + O(m_{u,d}^2). \quad (12)$$

That the vacuum is not invariant under the axial transformation is directly seen from the nonzero values of the quark condensates, which are order parameters for spontaneous chiral symmetry breaking. These condensates are the vacuum expectation values of the  $\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$  operator and at the renormalization scale of 1 GeV they approximately are

$$\langle \bar{u}u \rangle \simeq \langle \bar{d}d \rangle \simeq -(240 \pm 10 \text{ MeV})^3. \quad (13)$$

The values above are deduced from phenomenological considerations [12]. Lattice gauge calculations also confirm the nonzero and rather large values for quark condensates. However, the quark condensates above are not the only order parameters for chiral symmetry breaking. There exist chiral condensates of higher dimension (vacuum expectation values of more complicated combinations of  $\bar{\psi}$  and  $\psi$  that are not invariant under the axial transformations). Their numerical values are difficult to extract from phenomenological data, however, and they are still unknown.

To summarize this section. There exists overwhelming evidence that the nearly perfect chiral symmetry of the QCD Lagrangian is spontaneously broken in the QCD vacuum. Physically this is because the vacuum state in QCD is highly nontrivial which can be seen by the condensation in the vacuum state of the chiral pairs. These condensates break the symmetry of the vacuum with respect to the axial transformations and as a consequence, there is no parity doubling in the low-lying spectrum. However, as we shall show, the role of the chiral symmetry breaking quark condensates becomes progressively less important once we go up in the spectrum, *i.e.* the chiral symmetry is effectively restored, which should be evidenced by the systematical appearance of the approximate parity doublets in the highly lying spectrum. This is the subject of the following sections.

### 3. A few words about chiral symmetry breaking and low-lying hadrons

A key to understanding of the low-lying hadrons is spontaneous breaking of chiral symmetry (SBCS). Hence it is instructive to overview physics of SBCS. An insight into this phenomenon is best obtained from the pre-QCD Nambu and Jona-Lasinio model [1]. Its application to such questions as formation of constituent quarks as quasiparticles in the Bogoliubov sense, their connection to the quark condensate and appearance of the low-lying collective excitations — Goldstone bosons — was a subject of intensive research for the last two decades and is reviewed *e.g.* in Ref. [13]. Actually all microscopical models of SBCS in QCD, such as based on instantons [14] or other topological configurations, or on nonperturbative resummation of gluon exchanges [15], or on assumption that the Lorentz scalar confining interaction is an origin for SBCS [16], all share the key elements and ideas of the NJL picture. The only essential difference between all these models is a specification of those interactions that are responsible for SBCS.

Any interquark interaction in QCD mediated by the intermediate gluon field, in the local approximation, contains as a part a chiral-invariant 4-fermion interaction

$$(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2. \quad (14)$$

The first term represents Lorentz-scalar interaction. This interaction is an attraction between the left-handed quarks and the right-handed antiquarks and *vice versa*. When it is treated nonperturbatively in the mean-field approximation, which is well justified in the vacuum state, it leads to the condensation of the chiral pairs in the vacuum state

$$\langle 0|\bar{\psi}\psi|0\rangle = \langle 0|\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L|0\rangle \neq 0. \quad (15)$$

Hence it breaks chiral symmetry, which is a nonperturbative phenomenon. This dynamics is described by the famous gap equation which is similar to the one of Bardeen–Cooper–Schrieffer theory of superconductivity. This attractive interaction between bare quarks can be absorbed into a mass of a quasiparticle. This is provided by means of Bogoliubov transformation: Instead of operating with the original bare quarks and antiquarks one introduces quasiparticles. Each quasiparticle is a coherent superposition of bare quarks and antiquarks. Bare particles have both well-defined helicity and chirality, while quasiparticles have only definite helicity and contain a mixture of bare quarks and antiquarks with opposite chirality. This trick allows us to absorb the initial Lorentz-scalar attractive interaction between the bare quarks into a mass of the quasiparticles. These quasiparticles with dynamical mass can be associated with the constituent quarks. An important feature is that this dynamical mass appears only at low momenta, below the ultraviolet cutoff  $\Lambda$  in the NJL model, *i.e.* where the low-momentum attractive interaction between quarks is operative. All quarks with momenta higher than  $\Lambda$  remain undressed. In reality, of course, this step-function behaviour of the dynamical mass should be substituted by some smooth function. Hence in the vacuum a system of massless interacting quarks at low momenta can be effectively substituted by a system of the *noninteracting* quasiparticles with dynamical mass  $M$ . This mechanism of dynamical symmetry breaking and of creation of quasiparticles with dynamical mass is a very general one and persists in different many-fermion systems — from the superconductors to the atomic nuclei.

Once the chiral symmetry is spontaneously broken, then there must appear collective massless Goldstone excitations. Microscopically their zero mass is provided by the second term of Eq. (14). This term represents an attraction between the constituent quark and the antiquark with the pion quantum numbers. Without this term the pion would have a mass of  $2M$ . When this term is nonperturbatively and relativistically iterated, see Fig. 2,

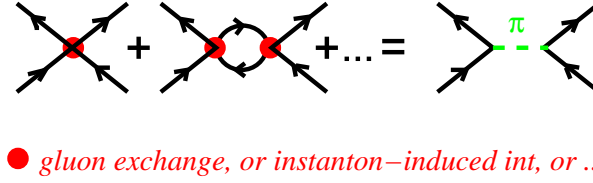


Fig. 2. Pion as a relativistic bound state in the quark–antiquark system.

the attraction between the constituent quarks in the pion exactly compensates the  $2M$  energy and the pion becomes massless. This happens because of the underlying chiral symmetry since it is this symmetry dictates that the strengths of the interactions represented by the first and by the second terms in Eq. (14) are equal. So the pion is a relativistic bound state of two quasiparticles. It contains  $\bar{Q}Q, \bar{Q}Q\bar{Q}Q, \dots$  Fock components. The pion (as any Goldstone boson) is a highly collective excitation in terms of the original (bare) quarks and antiquarks  $q$  and  $\bar{q}$  because the quasiparticles  $Q$  and  $\bar{Q}$  themselves are coherent collective excitations of bare quarks.

Now we will go to the low-lying baryons. A basic ingredient of the chiral quark picture of Manohar and Georgi [2] is that the constituent quarks inside the nucleon are strongly coupled to the pion field and this coupling is regulated by the Goldberger–Treiman relation. Why this should be so can be seen directly from the Nambu and Jona-Lasinio mechanism of chiral symmetry breaking. In terms of the massless bare quarks the axial current,  $A_\mu = \bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi$ , is conserved,  $\partial^\mu A_\mu = 0$ . If one works in terms of free massive quasiparticles, then it is not conserved,  $\partial^\mu A_\mu = 2iM\bar{\psi}\gamma_5\vec{\tau}\psi$ . How to reconcile this? The only solution is that the full axial current in the symmetry broken regime (which must be conserved) contains in addition a term which exactly cancels  $2iM\bar{\psi}\gamma_5\vec{\tau}\psi$ . It is straightforward to see that this additional term must represent a process where the axial current creates from the vacuum a massless pseudoscalar isovector boson and this boson in turn couples to the quasiparticle, see Fig. 3. It is this consideration which forced Nambu to postulate in 1960 an existence of the Nambu–Goldstone boson in the symmetry broken regime, which must be strongly coupled to the quasiparticle.

It was suggested in Ref. [17] that in the low-momentum regime (which is responsible for masses) the low-lying baryons in the  $u, d, s$  sector can be approximated as systems of three confined constituent quarks with the residual interaction mediated by the Goldstone boson field. Such a model was designed to solve a problem of the low-lying baryon spectroscopy. Microscopically this residual interaction appears from the  $t$ -channel iterations of those gluonic interactions in QCD which are responsible for chiral symmetry breaking [18], see Fig. 2. An essential feature of this residual interaction is



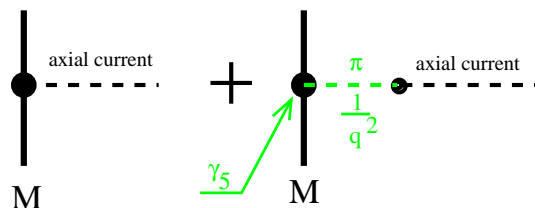


Fig. 3. A full axial current in the symmetry broken regime.

that it is a flavor- and spin-exchange interaction of the form

$$- \text{flavor}(i) \cdot \text{flavor}(j) \text{ spin}(i) \cdot \text{spin}(j).$$

This specific form of the residual interaction between valence constituent quarks in baryons allows us not only to generate octet-decuplet splittings but what is more important to solve at the same time the long-standing puzzle of the ordering of the lowest excitations of positive and negative parity in the  $u, d, s$  sector. This physics is a subject of intensive lattice studies and recent results [19–22] do show that the correct ordering is achieved only close to the chiral limit and hence is related to spontaneous breaking of chiral symmetry. The results [21] also evidence a node in the wave function of the radial excitation of the nucleon (Roper resonance) which is consistent with the  $3Q$  leading Fock component of this state.

#### 4. Low- and high-lying hadron spectra

If one looks carefully at the nucleon excitation spectrum, see Fig. 4, one immediately notices regularities for the high-lying states starting approximately from the  $M \sim 1.7$  GeV region. Namely the nucleon (and delta) high-lying states show obvious patterns of parity doubling: The states of the same spin but opposite parity are approximately degenerate. There are couple of examples where such parity partners have not yet experimentally been seen. Such doublets are definitely absent in the low-lying spectrum. The high-lying hadron spectroscopy is a difficult experimental task and the high-lying spectra have never been systematically explored. However, it is conceptually important to answer a question whether the parity partners exist systematically or not. If yes, and the existing data hint at it, then it would mean that some symmetry should be behind this parity doubling and this symmetry is not operative in the low-lying spectrum. What is this symmetry and why is it active only in the high-lying part of the spectrum? Clearly, if the parity doubling is systematic, then it rules out a description of the highly-excited states in terms of the constituent quarks (it will be discussed in one the following sections). Hence the physics of the low-lying and high-lying states is very different.

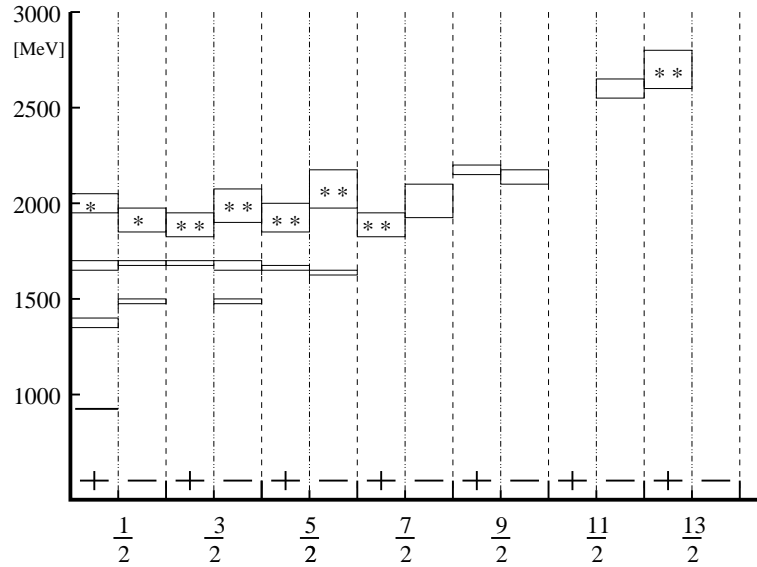


Fig. 4. Nucleon excitation spectrum. Those states which are not yet established are marked by \*\* or \* signs according to PDG classification.

It has been suggested some time ago that this parity doubling reflects restoration of the spontaneously broken chiral symmetry of QCD [3]. We have already discussed in the previous sections that the underlying chiral symmetry of the QCD Lagrangian would imply, if the QCD vacuum was trivial, a systematical parity doubling through the whole spectrum. However, the chiral symmetry of QCD is dynamically broken in the QCD vacuum, which leads to the appearance of the constituent quarks. The constituent (dynamical) mass of quarks results from their coupling to the quark condensates of the vacuum. We have also discussed that a description in terms of the constituent quarks makes sense only at low momenta. Typical momenta of valence quarks in the low-lying hadrons are below the chiral symmetry breaking scale, hence the chiral symmetry is broken in the low-lying states. The idea of Ref. [3] was that the typical momenta of valence quarks in highly excited hadrons are higher than the chiral symmetry breaking scale and hence these valence quarks decouple from the quark condensates of the QCD vacuum. Consequently the chiral symmetry is effectively restored in highly excited hadrons.

Clearly, if the chiral symmetry restoration indeed occurs, then it must be seen also in excited mesons. There are no systematic data on highly excited mesons in PDG. If one uses results of the recent systematic partial wave analysis of the proton-antiproton annihilation at LEAR at 1.8–2.4 GeV,

performed by the London–St.Petersburg group [23, 24], then once a careful chiral classification of the states is done [6, 7] one clearly sees direct signs of chiral symmetry restoration, see, *e.g.*, Fig. 5 where  $\pi$  and  $\bar{n}n = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} f_0$  states are shown (which must be chiral partners in the chiral symmetry restored regime). These facts force us to take seriously the possibility of chiral symmetry restoration in excited hadrons and also to concentrate experimental efforts on the systematical study of highly excited hadrons. Clearly the results on meson spectroscopy from the  $\bar{p}p$  annihilation at LEAR as well as on highly excited baryons must be checked and *completed* at the future facilities like PANDA at GSI as well as at JPARC and at the existing accelerators like at JLAB, Bonn, SPRING8, BES, *etc.* This should be one of the priority tasks.

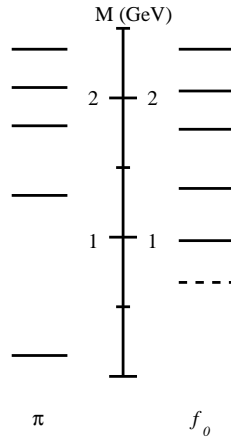


Fig. 5. Pion and  $\bar{n}n = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} f_0$  states.

## 5. Chiral symmetry restoration in excited hadrons by definition

The systematic approach to the symmetry restoration based on QCD has been formulated in Ref. [4, 5]. By definition an effective symmetry restoration means the following. In QCD the hadrons with the quantum numbers  $\alpha$  are created when one applies the local interpolating field (current)  $J_\alpha$  with such quantum numbers on the vacuum  $|0\rangle$ . This interpolating field contains a combination of valence quark creation operators at some point  $x$ . Then all the hadrons that are created by the given interpolator appear as intermediate states in the two-point correlator, see Fig. 6,

$$\Pi = i \int d^4x e^{iqx} \langle 0 | T \{ J_\alpha(x) J_\alpha^\dagger(0) \} | 0 \rangle, \quad (16)$$

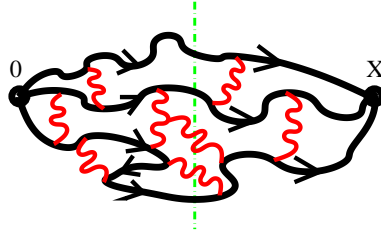


Fig. 6. Two-point correlator.

where all possible Lorentz and Dirac indices (specific for a given interpolating field) have been omitted. Consider two local interpolating fields  $J_1(x)$  and  $J_2(x)$  which are connected by chiral transformation,

$$J_1(x) = U J_2(x) U^\dagger, \quad (17)$$

where  $U$  is an element of the chiral group. Then, if the vacuum was invariant under chiral group,

$$U|0\rangle = |0\rangle,$$

it follows from (16) that the spectra created by the operators  $J_1(x)$  and  $J_2(x)$  would be identical. We know that in QCD one finds

$$U|0\rangle \neq |0\rangle.$$

As a consequence the spectra of two operators must be in general different. However, it may happen that the noninvariance of the vacuum becomes unimportant (irrelevant) high in the spectrum. Then the spectra of both operators become close at large masses (and asymptotically identical). This would mean that chiral symmetry is effectively restored. We stress that this effective chiral symmetry restoration does not mean that chiral symmetry breaking in the vacuum disappears, but only that the role of the quark condensates that break chiral symmetry in the vacuum becomes progressively less important high in the spectrum [4, 5]. One could say, that the valence quarks in high-lying hadrons decouple from the QCD vacuum. In order to avoid a confusion with the chiral symmetry restoration in the vacuum state at high temperature or density one also refers this phenomenon as chiral symmetry restoration of the second kind.

## 6. A simple pedagogical example

It is instructive to consider a very simple quantum mechanical example of symmetry restoration high in the spectrum. Though there are conceptual differences between the field theory with spontaneous symmetry breaking and the one-particle quantum mechanics (where only explicit symmetry

breaking is possible), nevertheless this simple example illustrates how this general phenomenon comes about.

The example we consider is a two-dimensional harmonic oscillator. We choose the harmonic oscillator only for simplicity; the property that will be discussed below is quite general one and can be seen in other systems. The Hamiltonian of the system is invariant under  $U(2) = SU(2) \times U(1)$  transformations. This symmetry has profound consequences on the spectrum of the system. The energy levels of this system are trivially found and are given by

$$E_{N,m} = (N + 1); \quad m = N, N - 2, N - 4, \dots, -(N - 2), -N, \quad (18)$$

where  $N$  is the principal quantum number and  $m$  is the (two dimensional) angular momentum. As a consequence of the symmetry, the levels are  $(N + 1)$ -fold degenerate.

Now suppose we add to the Hamiltonian a  $SU(2)$  symmetry breaking interaction (but which is still  $U(1)$  invariant) of the form

$$V_{SB} = A\theta(r - R), \quad (19)$$

where  $A$  and  $R$  are parameters and  $\theta$  is the step function. Clearly,  $V_{SB}$  is not invariant under the  $SU(2)$  transformation. Thus the  $SU(2)$  symmetry is explicitly broken by this additional interaction, that acts only within a circle of radius  $R$ . As a result one would expect that the eigenenergies will not reflect the degeneracy structure of seen in Eq. (18) if the coefficients  $R, A$  are sufficiently large. Indeed, we have solved numerically for the eigenstates for the case of  $A = 4$  and  $R = 1$  in dimensionless units and one does not see a multiplet structure in the low-lying spectrum as can be seen in Fig. 7.

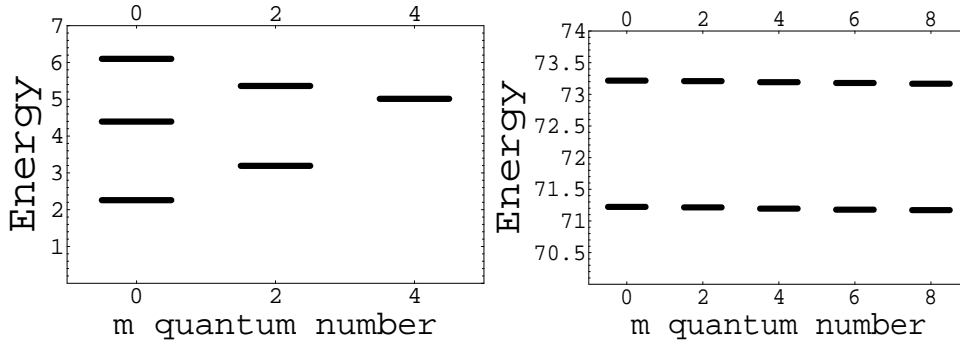


Fig. 7. The low-lying (left panel) and highly-lying (right panel) spectra of two-dimensional harmonic oscillator with the  $SU(2)$ -breaking term.

What is interesting for the present context is the high-lying spectrum. In Fig. 7 we have also plotted the energies between 70 and 74 for a few of the lower  $m$ 's. A multiplet structure is quite evident — to very good approximation the states of different  $m$ 's form degenerate multiplets and, although we have not shown this in the figure these multiplets extend in  $m$  up to  $m = N$ .

How does this happen? The symmetry breaking interaction plays a dominant role in the spectroscopy for small energies. Indeed, at small excitation energies the system is mostly located at distances where the symmetry breaking interaction acts and where it is dominant. Hence the low-lying spectrum to a very large extent is motivated by the symmetry breaking interaction. However, at high excitation energies the system mostly lives at large distances, where physics is dictated by the unperturbed harmonic oscillator. Hence at higher energies the spectroscopy reveals the  $SU(2)$  symmetry of the two-dimensional harmonic oscillator.

### 7. The quark–hadron duality and chiral symmetry restoration

A question arises to which extent the chiral symmetry restoration of the second kind can be theoretically predicted in QCD. There is a heuristic argument that supports this idea [4, 5]. The argument is based on the well controlled behaviour of the two-point function (16) at the large space-like momenta  $Q^2 = -q^2$ , where the operator product expansion (OPE) is valid and where all nonperturbative effects can be absorbed into condensates of different dimensions [25]. The key point is that all nonperturbative effects of spontaneous breaking of chiral symmetry at large  $Q^2$  are absorbed into quark condensate  $\langle \bar{q}q \rangle$  and other quark condensates of higher dimension. However, the contribution of these condensates into correlation function is regulated by the Wilson coefficients. The latter ones are proportional to  $(1/Q^2)^n$ , where the index  $n$  is determined by the quantum numbers of the current  $J$  and by the dimension of the given quark condensate. The higher dimension, the larger  $n$ . It is important that contributions of all possible chiral noninvariant terms of OPE are suppressed by inverse powers of  $Q^2$ , the higher dimension of the condensate, the less important is the given condensate at large  $Q^2$ . Hence, at large enough  $Q^2$  the two-point correlator becomes approximately chirally symmetric. At these high  $Q^2$  a matching with the perturbative QCD (where no SBCS) can be done. In other words, though the chiral symmetry is broken in the vacuum and all chiral noninvariant condensates are not zero, their influence on the correlator at asymptotically high  $Q^2$  vanishes. This is in contrast to the situation of low values of  $Q^2$ , where the role of chiral symmetry breaking in the vacuum is crucial. Hence, at  $Q^2 \rightarrow \infty$  one has

$$\Pi_{J_1}(Q^2) - \Pi_{J_2}(Q^2) \sim \frac{1}{Q^n}, \quad n > 0, \quad (20)$$

where  $J_1$  and  $J_2$  are interpolators which are connected by the chiral transformation according to (17).

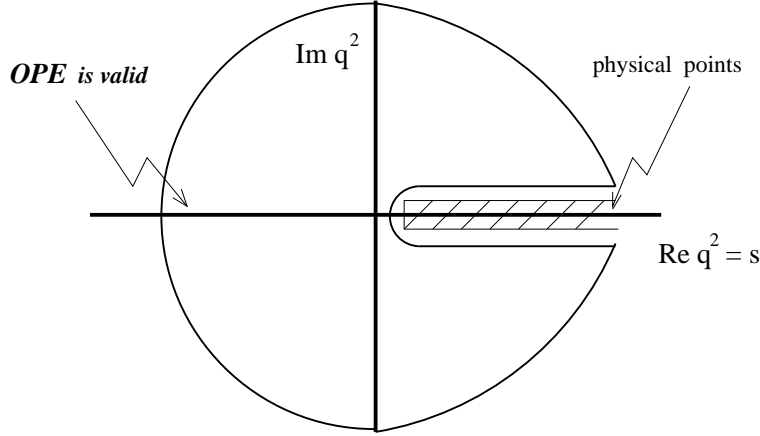


Fig. 8. The two-point correlator in the complex  $q^2$  plain.

Now we can use causality of the local field theory and hence analyticity of the two-point function. Then we can invoke into analysis a dispersion relation,

$$\Pi_J(Q^2) = \int ds \frac{\rho_J(s)}{Q^2 + s - i\epsilon}, \quad (21)$$

where the spectral density  $\rho_J(s)$  is defined as

$$\rho_J(s) \equiv \frac{1}{\pi} \text{Im}(\Pi_J(s)). \quad (22)$$

The integration in this equation is performed along the cut in Fig. 8. Since the large  $Q^2$  asymptotics of the correlator is given by the leading term of the perturbation theory, then the asymptotics of  $\rho(s)$  at  $s \rightarrow \infty$  must also be given by the same term of the perturbation theory if the spectral density approaches a constant value (if it oscillates, then it must oscillate around the perturbation theory value). Hence both spectral densities  $\rho_{J_1}(s)$  and  $\rho_{J_2}(s)$  at  $s \rightarrow \infty$  must approach the same value and the spectral function becomes chirally symmetric. This theoretical expectation, that the high  $s$  asymptotics of the spectral function is well described by the leading term of the perturbation theory has been tested *e.g.* in the process  $e^+e^- \rightarrow \text{hadrons}$ , where the interpolator is given by the usual electromagnetic vector current. This process is described in standard texts on QCD, for the recent data see [26]. Similarly, the vector and the axial vector spectral densities must coincide in the chiral symmetry restored regime. They have been measured

in the  $\tau$  decay by the ALEPH and OPAL collaborations at CERN [27,28]. It is well seen from the results that while the difference between both spectral densities is large at the masses of  $\rho(770)$  and  $a_1(1260)$ , it becomes strongly reduced towards  $m = \sqrt{s} \sim 1.7$  GeV.

While the argument above about chiral symmetry restoration in the spectral density is rather general and can be believed to be experimentally established, strictly speaking it does not necessarily imply that the high lying hadron resonances must form chiral multiplets. The reason is that the approximate equality of two spectral densities would necessarily imply hadron chiral multiplets only if the spectrum was discrete. In reality, however, the high-lying hadrons are rather wide overlapping resonances. In addition, it is only completely continuous non-resonant spectrum that is described by the chiral invariant leading term of perturbation theory. Nevertheless, it is indeed reasonable to assume that the spectrum is still quasidiscrete in the transition region  $\sqrt{s} \geq 1.7$  GeV where one approaches the chiral invariant regime. If so in this region the observed hadrons should fall into approximate chiral multiplets.

The question arises then what is the functional behaviour that determines approaching the chiral-invariant regime at large  $s$ ? Naively one would expect that the operator product expansion of the two-point correlator, which is valid in the deep Euclidean domain, could help us. This is not so, however, for two reasons. First of all, we know phenomenologically only the lowest dimension quark condensate. Even though this condensate dominates as a chiral symmetry breaking measure at the very large space-like  $Q^2$ , at smaller  $Q^2$  the higher dimensional condensates, which are suppressed by inverse powers of  $Q^2$ , are also important. These condensates are not known, unfortunately. But even if we knew all quark condensates up to a rather high dimension, it would not help us. This is because the OPE is only an asymptotic expansion [29]. While such kind of expansion is very useful in the space-like region, it does not define any analytical solution which could be continued to the time-like region at finite  $s$ . While convergence of the OPE can be improved by means of the Borel transform and it makes it useful for SVZ sum rules for the low-lying hadrons, this cannot be done for the higher states. So in order to estimate chiral symmetry restoration effects one indeed needs a microscopic theory that would incorporate *at the same time* chiral symmetry breaking and confinement.

## 8. Chiral multiplets of excited mesons

Here we limit ourselves to the two-flavor version of QCD. There are two reasons for doing this. First of all, the  $u$  and  $d$  quark masses are very small as compared to  $\Lambda_{\text{QCD}}$ . Thus the chiral  $\text{SU}(2)_L \times \text{SU}(2)_R$  and more generally the



$U(2)_L \times U(2)_R$  symmetries of the QCD Lagrangian are nearly perfect. This is not the case if the  $s$  quark is included, and *a priori* it is not clear whether one should regard this quark as light or “heavy”. The second reason is a practical one — there are good new data on highly excited  $u, d$  mesons observed in  $\bar{p}p$  annihilation [23, 24], but such data are still missing for the strange mesons. Certainly it would be very interesting and important to extend the analysis to the  $U(3)_L \times U(3)_R$  case. One hopes that the present results will stimulate the experimental and theoretical activity in this direction.

Mesons reported in Ref. [23, 24] are obtained in  $\bar{p}p$  annihilations, hence according to OZI rule we have to expect them to be  $\bar{q}q$  states with  $u$  and  $d$  valence quark content. Hence we will consider

$$U(2)_L \times U(2)_R = SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A, \quad (23)$$

the full chiral group of the QCD Lagrangian. In the following chiral symmetry will refer to specifically the  $SU(2)_L \times SU(2)_R$  symmetry.

The irreducible representations of this group can be specified by the isospins of the left-handed and right-handed quarks,  $(I_L, I_R)$ . The total isospin of the state can be obtained from the left- and right-handed isospins according to the standard angular momentum addition rules

$$I = |I_L - I_R|, \dots, I_L + I_R. \quad (24)$$

All hadronic states are characterised by a definite parity. However, not all irreducible representations of the chiral group are invariant under parity. Indeed, parity transforms the left-handed quarks into the right-handed ones and *vice versa*. Hence while representations with  $I_L = I_R$  are invariant under parity (*i.e.* under parity operation every state in the representation transforms into the state of opposite parity within the same representation), this is not true for the case  $I_L \neq I_R$ . In the latter case parity transforms every state in the representation  $(I_L, I_R)$  into the state in the representation  $(I_R, I_L)$ . We can construct definite parity states only combining basis vectors from both these irreducible representations. Hence it is only the direct sum of these two representations

$$(I_L, I_R) \oplus (I_R, I_L), \quad I_L \neq I_R, \quad (25)$$

that is invariant under parity. This reducible representation of the chiral group is an irreducible representation of the larger group, the parity-chiral group

$$SU(2)_L \times SU(2)_R \times C_i, \quad (26)$$

where the group  $C_i$  consists of two elements: identity and inversion in 3-dimensional space<sup>2</sup>. This symmetry group is the symmetry of the QCD La-

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<sup>2</sup> In the literature language is sometimes used in a sloppy way and the representation (25) is referred to erroneously as an irreducible representation of the chiral group.

grangian (neglecting quark masses), however only its subgroup  $SU(2)_I \times C_i$  survives in the broken symmetry mode. The dimension of the representation (25) is

$$\dim_{(I_a, I_b) \oplus (I_b, I_a)} = 2(2I_a + 1)(2I_b + 1). \quad (27)$$

When we consider mesons of isospin  $I = 0, 1$ , only three types of irreducible representations of the parity-chiral group exist.

(i) (0,0) Mesons in this representation must have isospin  $I = 0$ . At the same time  $I_R = I_L = 0$ . This can be achieved when either there are no valence quarks in the meson<sup>3</sup>, or both valence quark and antiquark are right or left. If we denote  $R = (u_R, d_R)$  and  $L = (u_L, d_L)$ , then the basis states of both parities can be written as

$$|(0, 0); \pm; J\rangle = \frac{1}{\sqrt{2}}(\bar{R}R \pm \bar{L}L)_J. \quad (28)$$

Note that such a system can have spin  $J \geq 1$ . Indeed, valence quark and antiquark in the state (28) have definite helicities, because generically helicity = +chirality for quarks and helicity = -chirality for antiquarks. Hence the total spin projection of the quark-antiquark system onto the momentum direction of the quark is  $\pm 1$ . The parity transformation property of the quark-antiquark state is then regulated by the total spin of the system [37]

$$\hat{P}|(0, 0); \pm; J\rangle = \pm(-1)^J|(0, 0); \pm; J\rangle. \quad (29)$$

(ii) (1/2, 1/2) In this case the quark must be right and the antiquark must be left, and *vice versa*. These representations combine states with  $I = 0$  and  $I = 1$ , which must be of opposite parity. The basis states within the two distinct representations (denoted as “a” and “b”, respectively) of this type are

$$|(1/2, 1/2)_a; +; I = 0; J\rangle = \frac{1}{\sqrt{2}}(\bar{R}L + \bar{L}R)_J, \quad (30)$$

$$|(1/2, 1/2)_a; -; I = 1; J\rangle = \frac{1}{\sqrt{2}}(\bar{R}\bar{\tau}L - \bar{L}\bar{\tau}R)_J, \quad (31)$$

and

$$|(1/2, 1/2)_b; -; I = 0; J\rangle = \frac{1}{\sqrt{2}}(\bar{R}L - \bar{L}R)_J, \quad (32)$$

$$|(1/2, 1/2)_b; +; I = 1; J\rangle = \frac{1}{\sqrt{2}}(\bar{R}\bar{\tau}L + \bar{L}\bar{\tau}R)_J. \quad (33)$$

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<sup>3</sup> Hence glueballs must be classified according to this representation [30]; with no quark content this representation contains the state of only one parity.

In these expressions  $\vec{\tau}$  are isospin Pauli matrices. The parity of every state in the representation is determined as

$$\hat{P}|(1/2, 1/2); \pm; I; J\rangle = \pm(-1)^J|(1/2, 1/2); \pm; I; J\rangle. \quad (34)$$

The mesons in the representations of this type can have any spin. Note that the two distinct  $(1/2, 1/2)_a$  and  $(1/2, 1/2)_b$  irreducible representations of  $SU(2)_L \times SU(2)_R$  form one irreducible representation of  $U(2)_L \times U(2)_R$ .

(iii)  $(0,1) \oplus (1, 0)$  The total isospin is 1 and the quark and antiquark must both be right or left. This representation is possible only for  $J \geq 1$ . The basis states are

$$|(0,1) + (1, 0); \pm; J\rangle = \frac{1}{\sqrt{2}}(\bar{R}\vec{\tau}R \pm \bar{L}\vec{\tau}L)_J \quad (35)$$

with parities

$$\hat{P}|(0,1) + (1, 0); \pm; J\rangle = \pm(-1)^J|(0,1) + (1, 0); \pm; J\rangle. \quad (36)$$

In the chirally restored regime the physical states must fill out completely some or all of these representations. We have to stress that the usual quantum numbers  $I, J^{PC}$  are not enough to specify the chiral representation for  $J \geq 1$ . It happens that some of the physical particles with the given  $I, J^{PC}$  belong to one chiral representation (multiplet), while the other particles with the same  $I, J^{PC}$  belong to the other multiplet. Classification of the particles according to  $I, J^{PC}$  is simply not complete in the chirally restored regime. This property will have very important implications as far as the amount of the states with the given  $I, J^{PC}$  is concerned.

In order to make this point clear, we will discuss some of the examples. Consider first the mesons of spin  $J = 0$ , which are  $\pi, f_0, a_0$  and  $\eta$  mesons with the  $u, d$  quark content only. The interpolating fields are given as

$$J_\pi(x) = \bar{q}(x)\vec{\tau}\gamma_5 q(x), \quad (37)$$

$$J_{f_0}(x) = \bar{q}(x)q(x), \quad (38)$$

$$J_\eta(x) = \bar{q}(x)\gamma_5 q(x), \quad (39)$$

$$J_{a_0}(x) = \bar{q}(x)\vec{\tau}q(x). \quad (40)$$

These four currents belong to the irreducible representation of the  $U(2)_L \times U(2)_R = SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$  group. It is instructive to see how these currents transform under different subgroups of the group above.

The  $SU(2)_L \times SU(2)_R$  transformations consist of vectorial and axial transformations in the isospin space (10). The axial transformations mix the currents of opposite parity:

$$J_\pi(x) \leftrightarrow J_{f_0}(x) \quad (41)$$

as well as

$$J_{a_0}(x) \leftrightarrow J_\eta(x). \quad (42)$$

The currents (41) form the basis of the  $(1/2, 1/2)_a$  representation of the parity-chiral group, while the interpolators (42) transform as  $(1/2, 1/2)_b$ .

The  $U(1)_A$  transformation (4) mixes the currents of the same isospin but opposite parity:

$$J_\pi(x) \leftrightarrow J_{a_0}(x) \quad (43)$$

as well as

$$J_{f_0}(x) \leftrightarrow J_\eta(x). \quad (44)$$

All four currents together belong to the representation  $(\frac{1}{2}, \frac{1}{2})_a \oplus (\frac{1}{2}, \frac{1}{2})_b$  which is an irreducible representation of the  $U(2)_L \times U(2)_R$  group.

If the vacuum were invariant with respect to  $U(2)_L \times U(2)_R$  transformations, then all four mesons,  $\pi, f_0, a_0$  and  $\eta$  would be degenerate (as well as all their excited states). Once the  $U(1)_A$  symmetry is broken explicitly through the axial anomaly, but the chiral  $SU(2)_L \times SU(2)_R$  symmetry is still intact in the vacuum, then the spectrum would consist of degenerate  $(\pi, f_0)$  and  $(a_0, \eta)$  pairs. If in addition the chiral  $SU(2)_L \times SU(2)_R$  symmetry is spontaneously broken in the vacuum, the degeneracy is also lifted in the pairs above and the pion becomes a (pseudo)Goldstone boson. Indeed, the masses of the lowest mesons are [38]<sup>4</sup>

$$m_\pi \simeq 140\text{MeV}, \quad m_{f_0} \simeq 400 - 1200\text{MeV}, \quad m_{a_0} \simeq 985\text{MeV}, \quad m_\eta \simeq 782\text{MeV}.$$

This immediately shows that both  $SU(2)_L \times SU(2)_R$  and  $U(1)_V \times U(1)_A$  are broken in the QCD vacuum to  $SU(2)_I$  and  $U(1)_V$ , respectively.

If one looks at the upper part of the spectrum, then one notices that the four successive highly excited  $\pi$  mesons and the corresponding  $\bar{n}n f_0$  mesons form approximate chiral pairs [6]. This is well seen from the Fig. 5. This pattern is a clear manifestation of the chiral symmetry restoration. However, given the importance of this statement these highly excited  $\pi$  and  $f_0$  mesons must be reconfirmed in other kind of experiments.

A similar behaviour is observed from a comparison of the  $a_0$  and  $\eta$  masses [6]. However, there are two missing  $a_0$  mesons which must be discovered in order to complete all chiral multiplets. (Technically the identification of the spinless states from the partial wave analysis is a rather difficult task.) There is a little doubt that these missing  $a_0$  mesons do exist. If one puts the four high-lying  $\pi, \bar{n}n f_0, a_0$  and  $\bar{n}n \eta$  mesons on the *radial* Regge trajectories, see Fig. 9, one clearly notices that the two missing  $a_0$  mesons lie on the

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<sup>4</sup> The  $\eta$  meson mass given here was obtained by unmixing the  $SU(3)$  flavor octet and singlet states so it represents the pure  $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$  state, see for details Ref. [6].

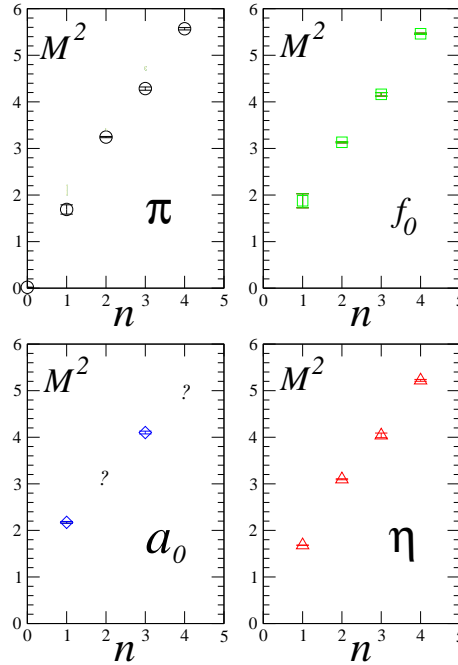


Fig. 9. Radial Regge trajectories for the four successive high-lying  $J = 0$  mesons.

linear trajectory with the same slope as all other mesons [23, 24]. If one reconstructs these missing  $a_0$  mesons according to this slope, then a pattern of the  $a_0 - \eta$  chiral partners appears, similar to the one for the  $\pi$  and  $f_0$  mesons.

For the  $J \geq 1$  mesons the classification is a bit more complicated. Consider  $\rho(1, 1^{--})$  mesons as example. Particles of this kind can be created from the vacuum by the vector current,  $\bar{\psi}\gamma^\mu\vec{\tau}\psi$ . Its chiral partner is the axial vector current,  $\bar{\psi}\gamma^\mu\gamma^5\vec{\tau}\psi$ , which creates from the vacuum the axial vector mesons,  $a_1(1, 1^{++})$ . Both these currents belong to the representation  $(0,1)+(1, 0)$  and have the right-right  $\pm$  left-left quark content. Clearly, in the chirally restored regime the mesons created by these currents must be degenerate level by level and fill out the  $(0,1)+(1, 0)$  representations. Hence, naively the amount of  $\rho$  and  $a_1$  mesons high in the spectrum should be equal. This is not correct, however.  $\rho$ -mesons can be also created from the vacuum by other type(s) of current(s),  $\bar{\psi}\sigma^{0i}\vec{\tau}\psi$  (or by  $\bar{\psi}\partial^\mu\vec{\tau}\psi$ ). These interpolators belong to the  $(1/2, 1/2)$  representation and have the left-right  $\pm$  right-left quark content. In the regime where chiral symmetry is strongly broken (as in the low-lying states) the physical states are mixtures of different representations. Hence these low-lying states are well coupled to both  $(0,1)+(1, 0)$

and  $(1/2, 1/2)$  interpolators. However, when chiral symmetry is (approximately) restored, then each physical state must be strongly dominated by the given representation and hence will couple only to the interpolator which belongs to the same representation. This means that  $\rho$ -mesons created by two distinct currents in the chirally restored regime represent physically different particles. The chiral partner of the  $\bar{\psi}\sigma^{0i}\vec{\tau}\psi$  (or  $\bar{\psi}\partial^\mu\vec{\tau}\psi$ ) current is  $\varepsilon^{ijk}\bar{\psi}\sigma^{jk}\psi$  ( $\bar{\psi}\gamma^5\partial^\mu\psi$ , respectively)<sup>5</sup>. The latter interpolators create from the vacuum  $h_1(0, 1^{+-})$  states. Hence in the chirally restored regime, some of the  $\rho$ -mesons must be degenerate with the  $a_1$  mesons  $((0,1)+(1, 0)$  multiplets), but the others — with the  $h_1$  mesons  $((1/2, 1/2)$  multiplets)<sup>6</sup>. Consequently, high in the spectra the combined amount of  $a_1$  and  $h_1$  mesons must coincide with the amount of  $\rho$ -mesons. This is a highly nontrivial prediction of chiral symmetry.

Actually it is a very typical situation. Consider  $f_2(0, 2^{++})$  mesons as another example. They can be interpolated by the tensor field  $\bar{\psi}\gamma^\mu\partial^\nu\psi$  (properly symmetrised, of course), which belongs to the  $(0,0)$  representation. Their chiral partners are  $\omega_2(0, 2^{--})$  mesons, which are created by the  $\bar{\psi}\gamma^5\gamma^\mu\partial^\nu\psi$  interpolator. On the other hand  $f_2(0, 2^{++})$  mesons can also be created from the vacuum by the  $\bar{\psi}\partial^\mu\partial^\nu\psi$  type of interpolator, which belongs to the  $(1/2, 1/2)$  representation. Its chiral partner is  $\bar{\psi}\gamma^5\partial^\mu\partial^\nu\vec{\tau}\psi$ , which creates  $\pi_2(1, 2^{-+})$  mesons. Hence in the chirally restored regime we have to expect  $\omega_2(0, 2^{--})$  mesons to be degenerate systematically with some of the  $f_2(0, 2^{++})$  mesons  $((0,0)$  representations) while  $\pi_2(1, 2^{-+})$  mesons must be degenerate with *other*  $f_2(0, 2^{++})$  mesons (forming  $(1/2, 1/2)$  multiplets). Hence the total number of  $\omega_2(0, 2^{--})$  and  $\pi_2(1, 2^{-+})$  mesons in the chirally restored regime must coincide with the amount of  $f_2(0, 2^{++})$  mesons.

These examples can be generalized to mesons of any spin  $J \geq 1$ . Those interpolators which contain only derivatives  $\bar{\psi}\partial^\mu\partial^\nu\dots\psi$  ( $\bar{\psi}\vec{\tau}\partial^\mu\partial^\nu\dots\psi$ ) have quantum numbers  $I = 0, P = (-1)^J, C = (-1)^J$  ( $I = 1, P = (-1)^J, C = (-1)^J$ ) and transform as  $(1/2, 1/2)$ . Their chiral partners are  $\bar{\psi}\vec{\tau}\gamma^5\partial^\mu\partial^\nu\dots\psi$  ( $\bar{\psi}\gamma^5\partial^\mu\partial^\nu\dots\psi$ , respectively) with  $I = 1, P = (-1)^{J+1}, C = (-1)^J$  ( $I = 0, P = (-1)^{J+1}, C = (-1)^J$ , respectively). However, interpolators with the same  $I, J^{PC}$  can be also obtained with one  $\gamma^\eta$  matrix instead one of the derivatives,  $\partial^\eta$ :  $\bar{\psi}\partial^\mu\partial^\nu\dots\gamma^\eta\dots\psi$  ( $\bar{\psi}\vec{\tau}\partial^\mu\partial^\nu\dots\gamma^\eta\dots\psi$ ). These latter interpolators belong to  $(0,0)$   $((0,1)+(1, 0))$  representation. Their chiral partners are  $\bar{\psi}\gamma^5\partial^\mu\partial^\nu\dots\gamma^\eta\dots\psi$  ( $\bar{\psi}\vec{\tau}\gamma^5\partial^\mu\partial^\nu\dots\gamma^\eta\dots\psi$ ) which have  $I = 0, P = (-1)^{J+1}, C = (-1)^{J+1}$  ( $I = 1, P = (-1)^{J+1}, C = (-1)^{J+1}$ ). Hence in the chirally restored regime the physical states created by these different types of interpolators will belong to different representations and will be distinct particles while

<sup>5</sup> Chiral transformation properties of some interpolators can be found in Ref. [31].

<sup>6</sup> Those  $\rho(1, 1^{--})$  and  $\omega(0, 1^{--})$  mesons which belong to  $(1/2, 1/2)$  cannot be seen in  $e^+e^- \rightarrow \text{hadrons}$ .

having the same  $I, J^{PC}$ . One needs to indicate chiral representation in addition to usual quantum numbers  $I, J^{PC}$  in order to uniquely specify physical states of the  $J \geq 1$  mesons in the chirally restored regime.

The available data for the  $J = 1, 2, 3$  mesons are systematized in Ref. [7]. Below we show the chiral patterns for the  $J = 2$  mesons, where the data set seems to be complete.

(0,0)	
$\omega_2(0, 2^{--})$	$f_2(0, 2^{++})$
$1975 \pm 20$	$1934 \pm 20$
$2195 \pm 30$	$2240 \pm 15$
(1/2, 1/2)	
$\pi_2(1, 2^{-+})$	$f_2(0, 2^{++})$
$2005 \pm 15$	$2001 \pm 10$
$2245 \pm 60$	$2293 \pm 13$
(1/2, 1/2)	
$a_2(1, 2^{++})$	$\eta_2(0, 2^{-+})$
$2030 \pm 20$	$2030 \pm ?$
$2255 \pm 20$	$2267 \pm 14$
(0,1)+(1, 0)	
$a_2(1, 2^{++})$	$\rho_2(1, 2^{--})$
$1950^{+30}_{-70}$	$1940 \pm 40$
$2175 \pm 40$	$2225 \pm 35$

We see systematic patterns of chiral symmetry restoration. In particular, the amount of  $f_2(0, 2^{++})$  mesons coincides with the combined amount of  $\omega_2(0, 2^{--})$  and  $\pi_2(1, 2^{-+})$  states. Similarly, number of  $a_2(1, 2^{++})$  states is the same as number of  $\eta_2(0, 2^{-+})$  and  $\rho_2(1, 2^{--})$  together. All chiral multiplets are complete. While masses of some of the states can and will be corrected in the future experiments, if new states might be discovered in this energy region in other types of experiments, they should be either  $\bar{s}s$  states or glueballs.

The data sets for the  $J = 1$  and  $J = 3$  mesons are less complete and there are a few missing states to be discovered [7]. Nevertheless, these spectra also offer an impressive patterns of chiral symmetry.

It is important to see whether there are also signatures of the  $U(1)_A$  restoration. This can happen if two conditions are fulfilled [4]: (i) unimportance of the axial anomaly in excited states, (ii) chiral  $SU(2)_L \times SU(2)_R$

restoration (*i.e.* unimportance of the quark condensates which break simultaneously both types of symmetries in the vacuum state). Some evidence for the  $U(1)_A$  restoration has been reported in Ref. [6] on the basis of  $J = 0$  data. Yet missing  $a_0$  states have to be discovered to complete the  $U(1)_A$  multiplets in the  $J = 0$  spectra. In this section we will demonstrate that the data on *e.g.*  $J = 2$  mesons present convincing evidence on  $U(1)_A$  restoration.

First, we have to consider which mesonic states can be expected to be  $U(1)_A$  partners. The  $U(1)_A$  transformation connects interpolators of the same isospin but opposite parity. But not all such interpolators can be connected by the  $U(1)_A$  transformation. For instance, the vector currents  $\bar{\psi}\gamma^\mu\psi$  and  $\bar{\psi}\vec{\tau}\gamma^\mu\psi$  are invariant under  $U(1)_A$ . Similarly, the axial vector interpolators  $\bar{\psi}\gamma^5\gamma^\mu\psi$  and  $\bar{\psi}\vec{\tau}\gamma^5\gamma^\mu\psi$  are also invariant under  $U(1)_A$ . Hence those interpolators (states) that are members of the  $(0, 0)$  and  $(0, 1) + (1, 0)$  representations of  $SU(2)_L \times SU(2)_R$  are invariant with respect to  $U(1)_A$ . However, interpolators (states) from the *distinct*  $(1/2, 1/2)$  representations which have the same isospin but opposite parity transform into each other under  $U(1)_A$ . For example,  $\bar{\psi}\psi \leftrightarrow \bar{\psi}\gamma^5\psi$ ,  $\bar{\psi}\vec{\tau}\psi \leftrightarrow \bar{\psi}\vec{\tau}\gamma^5\psi$ , and those with derivatives:  $\bar{\psi}\partial^\mu\psi \leftrightarrow \bar{\psi}\gamma^5\partial^\mu\psi$ ,  $\bar{\psi}\vec{\tau}\partial^\mu\psi \leftrightarrow \bar{\psi}\vec{\tau}\gamma^5\partial^\mu\psi$ , *etc.* If the corresponding states are systematically degenerate, then it is a signal that  $U(1)_A$  is restored. In what follows we show that it is indeed the case.

$f_2(0, 2^{++})$	$\eta_2(0, 2^{-+})$
$2001 \pm 10$	$2030 \pm ?$
$2293 \pm 13$	$2267 \pm 14$
$\pi_2(1, 2^{-+})$	$a_2(1, 2^{++})$
$2005 \pm 15$	$2030 \pm 20$
$2245 \pm 60$	$2255 \pm 20$

We see clear approximate doublets of  $U(1)_A$  restoration. Hence two distinct  $(1/2, 1/2)$  multiplets of  $SU(2)_L \times SU(2)_R$  can be combined into one multiplet of  $U(2)_L \times U(2)_R$ . So we conclude that the whole chiral symmetry of the QCD Lagrangian  $U(2)_L \times U(2)_R$  gets approximately restored high in the hadron spectrum.

It is useful to quantify the effect of chiral symmetry breaking (restoration). An obvious parameter that characterises effects of chiral symmetry breaking is a relative mass splitting within the chiral multiplet. Let us define the *chiral asymmetry* as

$$\chi = \frac{|M_1 - M_2|}{(M_1 + M_2)}, \quad (45)$$



where  $M_1$  and  $M_2$  are masses of particles within the same multiplet. This parameter gives a quantitative measure of chiral symmetry breaking at the leading (linear) order and has the interpretation of the part of the hadron mass due to chiral symmetry breaking.

For the low-lying states the chiral asymmetry is typically 0.2–0.6 which can be seen *e.g.* from a comparison of the  $\rho(770)$  and  $a_1(1260)$  or the  $\rho(770)$  and  $h_1(1170)$  masses. If the chiral asymmetry is large as above, then it makes no sense to assign a given hadron to the chiral multiplet since its wave function is a strong mixture of different representations and we have to expect also large *nonlinear* symmetry breaking effects. However, at meson masses about 2 GeV the chiral asymmetry is typically within 0.01 and in this case the hadrons can be believed to be members of multiplets with a tiny admixture of other representations. Unfortunately there are no systematic data on mesons below 1.9 GeV and hence it is difficult to estimate the chiral asymmetry as a function of mass ( $\sqrt{s}$ ). Such a function would be crucially important for a further progress of the theory. So a systematic experimental study of hadron spectra is difficult to overestimate. However, thanks to the  $0^{++}$  glueball search for the last 20 years, there are such data for  $\pi$  and  $f_0$  states, as can be seen from Fig. 5 (for details we refer to [6, 30]). According to these data we can reconstruct  $\chi(\sqrt{s} \sim 1.3 \text{ GeV}) \sim 0.03 \div 0.1$ ,  $\chi(\sqrt{s} \sim 1.8 \text{ GeV}) \sim 0.008$ ,  $\chi(\sqrt{s} \sim 2.3 \text{ GeV}) \sim 0.005$ . We have to also stress that there is no reason to expect the chiral asymmetry to be a universal function for all hadron channels. Hadrons with different quantum numbers feel chiral symmetry breaking effects differently, as can be deduced from the operator product expansions of two-point functions for different currents. A task of the theory is to derive these chiral asymmetries microscopically.

## 9. Chiral multiplets of excited baryons

Now we will consider chiral multiplets of excited baryons [4, 5]. The nucleon or delta states have a half integral isospin. Then such a multiplet cannot be an irreducible representation of the chiral group  $(I_L, I_R)$  with  $I_L = I_R$ , because in this case the total isospin can only be integral. Hence the minimal possible representation that is invariant under parity transformation is the one of (25). Empirically, there are no known baryon resonances within the two light flavors sector which have an isospin greater than  $3/2$ . Thus we have a constraint from the data that if chiral symmetry is effectively restored for very highly excited baryons, the only possible representations for the observed baryons have  $I_L + I_R \leq 3/2$ , *i.e.* the only possible representations are  $(1/2, 0) \oplus (0, 1/2)$ ,  $(1/2, 1) \oplus (1, 1/2)$  and  $(3/2, 0) \oplus (0, 3/2)$ . Since chiral symmetry and parity do not constrain the possible spins of the states these multiplets can correspond to states of any fixed spin.

The same classification can actually be obtained assuming that chiral properties of excited baryons are determined by three massless valence quarks which have a definite chirality. Indeed the one quark field transforms as

$$q \sim \left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right). \quad (46)$$

Then all possible representations for the three-quark baryons in the chirally restored phase can be obtained as a direct product of three "fundamental" representations (46). Using the standard isospin coupling rules separately for the left and right quark components, one easily obtains a decomposition of this direct product

$$\begin{aligned} & \left[\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)\right]^3 = \left[\left(\frac{3}{2}, 0\right) \oplus \left(0, \frac{3}{2}\right)\right] \\ & + 3 \left[\left(1, \frac{1}{2}\right) \oplus \left(\frac{1}{2}, 1\right)\right] + 3 \left[\left(0, \frac{1}{2}\right) \oplus \left(\frac{1}{2}, 0\right)\right] + 2 \left[\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)\right]. \end{aligned} \quad (47)$$

The last two representations in the expansion above are identical group-theoretically, so they can be combined with the common multiplicity factor 5. Thus, according to the simple-minded model above, baryons in the chirally restored regime will belong to one of the following representations:

$$\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right); \quad \left(\frac{3}{2}, 0\right) \oplus \left(0, \frac{3}{2}\right); \quad \left(\frac{1}{2}, 1\right) \oplus \left(1, \frac{1}{2}\right). \quad (48)$$

The  $(1/2, 0) \oplus (0, 1/2)$  multiplets contain only isospin 1/2 states and hence correspond to parity doublets of nucleon states (of any fixed spin)<sup>7</sup>. Similarly,  $(3/2, 0) \oplus (0, 3/2)$  multiplets contain only isospin 3/2 states and hence correspond to parity doublets of  $\Delta$  states (of any fixed spin)<sup>8</sup>. However,  $(1/2, 1) \oplus (1, 1/2)$  multiplets contain both isospin 1/2 and isospin 3/2 states and hence correspond to multiplets containing both nucleon and  $\Delta$  states of both parities and any fixed spin<sup>9</sup>.

Summarizing, the phenomenological consequence of the effective restoration of chiral symmetry high in  $N$  and  $\Delta$  spectra is that the baryon states will fill out the irreducible representations of the parity-chiral group (26).

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<sup>7</sup> If one distinguishes nucleon states with different electric charge, *i.e.* different isospin projection, then this "doublet" is actually a quartet.

<sup>8</sup> Again, keeping in mind different charge states of delta resonance it is actually an octet.

<sup>9</sup> This representation is a 12-plet once we distinguish between different charge states.

If  $(1/2, 0) \oplus (0, 1/2)$  and  $(3/2, 0) \oplus (0, 3/2)$  multiplets were realized in nature, then the spectra of highly excited nucleons and deltas would consist of parity doublets. However, the energy of the parity doublet with given spin in the nucleon spectrum *a priori* would not be degenerate with the the doublet with the same spin in the delta spectrum; these doublets would belong to different representations of Eq. (26), *i.e.* to distinct multiplets and their energies are not related. On the other hand, if  $(1/2, 1) \oplus (1, 1/2)$  were realized, then the highly lying states in  $N$  and  $\Delta$  spectrum would have a  $N$  parity doublet and a  $\Delta$  parity doublet with the same spin and which are degenerate in mass. In either of cases the highly lying spectrum must systematically consist of parity doublets.

If one looks carefully at the nucleon spectrum, see Fig. 4, and the delta spectrum one notices that the systematic parity doubling in the nucleon spectrum appears at masses of 1.7 GeV and above, while the parity doublets in the delta spectrum insist at masses of 1.9 GeV<sup>10</sup>. This fact implies that at least those nucleon doublets that are seen at  $\sim 1.7$  GeV belong to  $(1/2, 0) \oplus (0, 1/2)$  representation. Below we show doublets of different spin in the energy range of 1.9 GeV and higher:

$$J = \frac{1}{2} : N^+(2100) (*), N^-(2090) (*), \Delta^+(1910) \quad , \quad \Delta^-(1900)(**);$$

$$J = \frac{3}{2} : N^+(1900)(**), N^-(2080)(**), \Delta^+(1920) \quad , \quad \Delta^-(1940) (*);$$

$$J = \frac{5}{2} : N^+(2000)(**), N^-(2200)(**), \Delta^+(1905) \quad , \quad \Delta^-(1930) \quad ;$$

$$J = \frac{7}{2} : N^+(1990)(**), N^-(2190) \quad , \quad \Delta^+(1950) \quad , \quad \Delta^-(2200) (*);$$

$$J = \frac{9}{2} : N^+(2220) \quad , \quad N^-(2250) \quad , \quad \Delta^+(2300)(**), \Delta^-(2400)(**);$$

$$J = \frac{11}{2} : \quad ? \quad , \quad N^-(2600) \quad , \quad \Delta^+(2420) \quad , \quad ? \quad ;$$

$$J = \frac{13}{2} : N^+(2700)(**), \quad ? \quad , \quad ? \quad , \quad \Delta^-(2750)(**);$$

$$J = \frac{15}{2} : \quad ? \quad , \quad ? \quad , \quad \Delta^+(2950)(**), \quad ? \quad .$$

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<sup>10</sup> This means that the parity doubling in both cases is seen at approximately the same excitation energy with respect to the corresponding ground state.

If approximate mass degeneracy between  $N$  and  $\Delta$  doublets at  $M \geq 1.9$  GeV is accidental, then the baryons in this mass region are organized according to  $(1/2, 0) \oplus (0, 1/2)$  for  $N$  and  $(3/2, 0) \oplus (0, 3/2)$  for  $\Delta$  parity–chiral doublets. If not, then the high lying spectrum forms  $(1/2, 1) \oplus (1, 1/2)$  multiplets. It can also be possible that in the narrow energy interval more than one parity doublet in the nucleon and delta spectra is found for a given spin. This would then mean that different doublets belong to different parity–chiral multiplets. Systematic experimental exploration of the high-lying states is required in order to assign unambiguously baryons to the multiplets.

### 10. Can simple potential models explain parity doubling?

Before discussing a model for highly excited hadrons that is compatible with the chiral symmetry restoration and parity doubling it is useful to answer a question whether the potential models like the traditional constituent quark model can explain it. Consider first mesons. Within the potential model the mesons are considered to be systems of two constituent quarks which interact via linear confinement potential plus some perturbation from the one gluon exchange [32] or instanton induced interaction [33]. Within the potential description the parity of the state is unambiguously prescribed by the relative orbital angular momentum  $L$  of the constituent quarks. For example, all the states on the radial pion Regge trajectory, see Fig. 9, are  $^1S_0$   $\bar{Q}Q$  states, while the members of the  $f_0$  trajectory are the  $^3P_0$  states. Hence the centrifugal repulsion for the states of opposite parity is different. Then it is clear that such a model cannot explain a systematic approximate degeneracy of the states of opposite parity. A fine tuning of the perturbation can in principle provide an *accidental* degeneracy of some of the states, but then there will be no one-to-one pairing and degeneracy for the other states. As a consequence the potential models of mesons cannot accomodate a lot of experimentally observed highly excited mesons. For example, while the parameters within the model of Ref. [32] are fitted to describe the two lowest pion states and it still can accomodate the third radial state of the pion, it does not predict at all the existence of  $\pi(2070)$  and  $\pi(2360)$ ; the fourth and the fifth radial states of the pion do not appear in this picture up to 2.4 GeV. A similar situation occurs also in other channels. The failure of the potential description is inherently related to the fact that it cannot incorporate chiral symmetry restoration as a matter of principle. The latter phenomenon is intrinsically a relativistic phenomenon which is a consequence of the fact that the ultrarelativistic valence quarks in the highly excited hadrons must necessarily be chiral (*i.e.* they have definite helicity and chirality). It is a generic property of the ultrarelativistic fermions which cannot be simulated within the  $^{2S+1}L_J$  type potential description.

If one uses instead a relativistic description within the Dirac or Bethe–Salpeter equations frameworks, then the parity doubling and chiral symmetry restoration is incompatible with the Lorentz scalar potential which is often used to simulate confinement. The reason is that the Lorentz scalar potential manifestly breaks chiral symmetry and is equivalent to introduction of some effective mass which increases with the excitation and size of hadrons. With the Lorentz vector confining potential and assuming that there is no constituent mass of quarks one can obtain parity doubling [34].

A few comments about the parity doubling within the potential models that attempt to describe the highly lying baryons are in order. The models that rely on confinement potential cannot explain an appearance of the systematic parity doublets. This is apparent for the harmonic confinement. The parity of the state is determined by the number  $N$  of the harmonic excitation quanta in the  $3q$  state. The ground states ( $N = 0$ ) are of positive parity, all baryons from the  $N = 1$  band are of negative parity, baryons from the  $N = 2$  band have a positive parity irrespective of their angular momentum, *etc.* However, the number of states in the given band rapidly increases with  $N$ . This means that such a model cannot provide an equal amount of positive and negative parity states, which is necessary for parity doubling, irrespective of other residual interactions between quarks in such a model. Similar problem persists with the linear confinement in the  $3q$  system.

While all vacancies from the  $N = 0$  and  $N = 1$  bands are filled in nature, such a model, extrapolated to the  $N = 3$  and higher bands predicts a very big amount of states, which are not observed (the so-called missing resonance problem). The chiral restoration transition takes place at excitation energies typical for the highest states in the  $N = 2$  band and in the  $N = 3$  bands. If correct, it would mean that description of baryons in this transition region in terms of constituent quarks becomes inappropriate.

The model that relies on the pure color Coulomb interaction between quarks also cannot provide the systematical parity doubling. While it gives an equal amount of the positive and negative parity single quark states in the  $n = 2, 4, \dots$  bands (*e.g.*  $2s$ – $2p$ , or  $4s$ – $4p$ ,  $4d$ – $4f$ ), the number of the positive parity states is always bigger in the  $n = 1, 3, 5, \dots$  bands.

## 11. Chiral symmetry restoration and the string (flux tube) picture

A question arises what is a microscopical mechanism of chiral symmetry restoration in excited hadrons and what is a relevant physical picture? We have already mentioned before that a possible scenario is related to the fact that at large space-like momenta the dynamical (constituent) mass of quarks must vanish. If in the highly excited hadrons the momenta of valence quarks

are indeed large, then the effects of spontaneous breaking of chiral symmetry should be irrelevant in such hadrons [3, 35].

Here we will discuss a possible fundamental origin for this phenomenon. We will show below that both chiral and  $U(1)_A$  restorations can be anticipated as a direct consequence of the semiclassical regime in the highly excited hadrons.

At large  $n$  (radial quantum number) or at large angular momentum  $L$  we know that in quantum systems the *semiclassical* approximation (WKB) *must* work. Physically this approximation applies in these cases because the de Broglie wavelength of particles in the system is small in comparison with the scale that characterizes the given problem. In such a system as a hadron the scale is given by the hadron size while the wavelength of valence quarks is given by their momenta. Once we go high in the spectrum the size of hadrons increases as well as the typical momentum of valence quarks. This is why a highly excited hadron can be described semiclassically in terms of the underlying quark degrees of freedom.

A physical content of the semiclassical approximation is most transparently given by the path integral. The contribution of the given path to the path integral is regulated by the action  $S(q)$  along the path  $q(x, t)$

$$\sim e^{iS(q)/\hbar}. \quad (49)$$

The semiclassical approximation applies when  $S(q) \gg \hbar$ . In this case the whole amplitude (path integral) is dominated by the classical path  $q_{cl}$  (stationary point) and those paths that are infinitesimally close to the classical path. All other paths that differ from the classical one by an appreciable amount do not contribute. These latter paths would represent the quantum fluctuation effects. In other words, in the semiclassical case the quantum fluctuations effects are strongly suppressed and vanish asymptotically.

The  $U(1)_A$  symmetry of the QCD Lagrangian is broken only due to the quantum fluctuations of the fermions. The  $SU(2)_R \times SU(2)_L$  spontaneous (dynamical) breaking is also pure quantum effect and is based upon quantum fluctuations. To see the latter we remind the reader that most generally the chiral symmetry breaking (*i.e.* the dynamical quark mass generation) is formulated via the Schwinger–Dyson (gap) equation. It is not yet clear at all which specific gluonic interactions are the most important ones as a kernel of the Schwinger–Dyson equation (*e.g.* instantons<sup>11</sup>, or gluonic exchanges, or perhaps other gluonic interactions, or a combination of different interactions). But in any case the quantum fluctuations effects of the quark fields

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<sup>11</sup> The instanton itself is a semiclassical gluon field configuration. But chiral and  $U(1)_A$  symmetry breakings by instantons is a quark field quantum fluctuations process. This is because these breakings are due to chiral quark pair creation from the vacuum by the instanton.

are very strong in the low-lying hadrons and induce both chiral and  $U(1)_A$  breakings. As a consequence we do not observe any chiral or  $U(1)_A$  multiplets low in the spectrum. However, if the quantum fluctuations effects are relatively suppressed, then the dynamical mass of quarks must vanish as well as the effects of the  $U(1)_A$  anomaly.

We have just mentioned that in a bound state quantum system with large enough  $n$  or  $L$  the effects of quantum fluctuations must be suppressed and vanish asymptotically<sup>12</sup>. Hence at large hadron masses (*i.e.* with either large  $n$  or large  $L$ ) we should anticipate symmetries of the classical QCD Lagrangian. Then it follows that in such systems both the chiral and  $U(1)_A$  symmetries must be restored. This is precisely what we see phenomenologically. In the nucleon spectrum the doubling appears either at large  $n$  excitations of baryons with the given small spin or in resonances of large spin. Similar features persist in the delta spectrum. In the meson spectrum the doubling is obvious for large  $n$  excitations of small spin mesons and there are signs of doubling of large spin mesons (the data are, however, sparse). It would be certainly interesting and important to observe systematically multiplets of parity-chiral and parity- $U(1)_A$  groups (or, sometimes, when the chiral and  $U(1)_A$  transformations connect *different* hadrons, the multiplets of the  $U(2)_L \times U(2)_R$  group). The high-lying hadron spectra must be systematically explored.

The strength of the argument given above is that it is very general. Its weakness is that we cannot say anything concrete about microscopical mechanisms of how all this happens. For that one needs a detailed microscopical understanding of dynamics in QCD, which is both challenging and very difficult task. But even though we do not know how microscopically all this happens, we can anticipate that in highly excited hadrons we must observe symmetries of the classical QCD Lagrangian. The only basis for this statement is that in such hadrons a semiclassical description is correct.

As a consequence, in highly excited hadrons the valence quark motion has to be described semiclassically and at the same time their chirality (helicity) must be fixed. Also the gluonic field should be described semiclassically. All this gives an increasing support for a string picture [8] of highly excited hadrons. Indeed, if one assumes that the quarks at the ends of the string have definite chirality, see Fig. 10, then all hadrons will appear necessarily in chiral multiplets [9]. The latter hypothesis is very natural and is well

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<sup>12</sup> That the quantum fluctuations effects vanish in the quantum bound state systems at large  $n$  or  $L$  is well known *e.g.* from the Lamb shift. The Lamb shift is a result of the radiative corrections (which represent effects of quantum fluctuations of electron and electromagnetic fields) and vanishes as  $1/n^3$ , and also very fast with increasing  $L$ . As a consequence high in the hydrogen spectrum the symmetry of the classical Coulomb potential gets restored.

compatible with the Nambu string picture. The ends of the string in the Nambu picture move with the velocity of light. Then, (it is an extension of the Nambu model) the quarks at the ends of the string must have definite chirality. In this way one is able to explain at the same time both Regge trajectories and parity doubling.

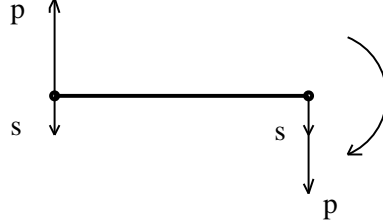


Fig. 10. Rotating string with the right and the left quarks at the ends.

One arrives at the following situation: (i) the hadrons with the different chiral configurations of the quarks at the ends of the string which belong to the same parity–chiral multiplet and that belong to the same intrinsic quantum state of the string must be degenerate; (ii) the total parity of the hadron is determined by the product of parity of the string in the given quantum state and the parity of the specific parity–chiral configuration of the quarks at the ends of the string. There is no analogy to this situation in the nonrelativistic physics where parity is only determined by the orbital motion of particles. Thus one sees that for every intrinsic quantum state of the string there necessarily appears parity doubling of the states with the same total angular momentum.

The spin–orbit operator  $\vec{\sigma} \cdot \vec{L}$  does not commute with the helicity operator  $\vec{\sigma} \cdot \vec{\nabla}$ . Hence the spin–orbit interaction of quarks with the fixed chirality or helicity is absent. In particular, this is also true for the spin–orbit force due to the Thomas precession

$$U_T = -\vec{\sigma} \cdot \vec{\omega}_T \sim \vec{\sigma} \cdot [\vec{v}, \vec{a}] \sim \vec{v} \cdot [\vec{v}, \vec{a}] = 0, \quad (50)$$

where  $U_T$  is the energy of the interaction and  $\vec{\omega}_T$ ,  $\vec{v}$  and  $\vec{a}$  are the angular frequency of Thomas precession, velocity of the quark and its acceleration, respectively.

The absence of the spin–orbit force in the chirally restored regime is a very welcome feature because it is a well-known empirical fact that the spin–orbit force is either vanishing or very small in the spectroscopy in the  $u, d$  sector [39]. This fact is difficult, if impossible, to explain within the potential constituent quark models.

In addition, for the rotating string

$$\vec{\sigma}(i) \cdot \vec{R}(i) = 0, \quad (51)$$



$$\vec{\sigma}(i) \cdot \vec{R}(j) = 0, \quad (52)$$

where the indices  $i, j$  label different quarks and  $\vec{R}$  is the radius-vector of the given quark in the center-of-mass frame. The relations above immediately imply that the possible tensor interactions of quarks related to the string dynamics should be absent, once the chiral symmetry is restored.

## 12. Conclusions

We have demonstrated in these lectures that the chiral symmetry of QCD is crucially important to understand physics of hadrons in the  $u, d$  (and possibly in the  $u, d, s$ ) sector. The low-lying hadrons are mostly driven by the spontaneous breaking of chiral symmetry. This breaking determines the physics and effective degrees of freedom in the low-lying hadrons. For example, it is SBCS which sheds a light on the meaning of the constituent quarks. The latter ones are quasiparticles and appear due to coupling of the valence quarks to the quark condensates of the vacuum. The pion as Nambu–Goldstone boson represents a relativistic bound state of quasiparticles  $Q$  and  $\bar{Q}$  and is a highly collective state in terms of original bare quarks. A strength of the residual interaction between the quasiparticles in the pion is dictated by chiral symmetry and is such that it exactly compensates the mass of the constituent quarks so the pion becomes massless in the chiral limit. In the low-lying baryons the physics at low momenta is mostly dictated by the coupling of constituent quarks and Goldstone bosons. Then a crucially important residual interaction between the constituent quarks in the low-lying baryons is mediated by the pion field, which is of the flavor- and spin-exchange nature.

However, this physics is relevant only to the low-lying hadrons. In the high-lying hadrons the chiral symmetry is restored, which is referred to as effective chiral symmetry restoration or chiral symmetry restoration of the second kind. A direct manifestation of the latter phenomenon is a systematical appearance of the approximate chiral multiplets of the high-lying hadrons. The essence of the present phenomenon is that the quark condensates which break chiral symmetry in the vacuum state (and hence in the low-lying excitations over vacuum) become simply irrelevant (unimportant) for the physics of the highly excited states. The physics here is such as if there were no chiral symmetry breaking in the vacuum. The valence quarks simply decouple from the quark condensates and consequently the notion of the constituent quarks with dynamical mass induced by chiral symmetry breaking becomes irrelevant in highly excited hadrons. Instead, the string picture with quarks of definite chirality at the end points of the string should be invoked. In recent lattice calculations DeGrand has demonstrated that

indeed in the highly excited mesons valence quarks decouple from the low-lying eigenmodes of the Dirac operator (which determine the quark condensate via Banks–Casher relation) and so decouple from the quark condensate of the QCD vacuum [36].

Hence physics of the high-lying hadrons is mostly physics of confinement acting between the light quarks. Their very small current mass strongly distinguishes this physics from the physics of the heavy quarkonium, where chiral symmetry is irrelevant and the string (flux tube) can be approximated as a static potential acting between the slowly moving heavy quarks. In the light hadrons in contrast the valence quarks are ultrarelativistic and their fermion nature requires them to have a definite chirality. Hence the high-lying hadrons in the  $u, d$  sector open a door to the regime of dynamical strings with chiral quarks at the ends. Clearly a systematic experimental exploration of the high-lying hadrons is required which is an interesting and important task and which should be of highest priority at the existing accelerators and at the forthcoming ones like PANDA at GSI and JPARC.

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