

FITS TO DATA ON POLARIZED STRUCTURE FUNCTIONS AND SPIN ASYMMETRIES WITH POWER LAW CORRECTIONS

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We have compared polarized parton densities determined in the NLO QCD fits to polarized structure functions and spin asymmetries. We consider models of such distributions based on MRST 99 and MRST 2001 fits to non-polarized data. Simple power law corrections corresponding to higher twists are taken into account and their importance is analyzed. The role of positivity conditions for parton densities and their influence on the values of χ^2 is discussed.

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In [1] we got parton densities using NLO QCD fit from the data on polarized structure function g_1 and spin asymmetries. We have used in our fits functional form for parton densities corresponding to unpolarized distributions obtained in the Martin, Roberts, Stirling and Thorne (MRST 98) fit [2]. The results for integrated parton densities were very similar in both cases: for ones gotten from the polarized structure functions and spin asymmetries. In that comparison we have not taken into account power law corrections connected with higher twists contributions. It has been pointed out [3] that these corrections can play a role in the determination of parton densities. It is an aim of the present paper to consider the influence of the power law corrections in the different methods of extracting polarized parton distributions from fits to the experimental data.

As before we will use all available experimental data on polarized structure functions and corresponding spin asymmetries [4–10]. Functional form of our polarized parton densities will be taken from corresponding unpolarized parton densities from the fits made by Martin, Roberts, Stirling and Thorne (called MRST 99 [11] and MRST 2001 [12]). Results of our fits to spin asymmetries based on these models were given in [13]. We will use the same method of extracting such densities as in our previous papers [14], where the experimental data for spin asymmetries were used to obtain polarized parton densities. Many experimental groups gave also experimental results for polarized structure functions g_1 measured for proton, neutron and deuteron. The asymptotic behaviour of our polarized parton distributions is determined (up to the condition that the corresponding spin densities are integrable) by the fit to unpolarized data. Experiments on unpolarized targets provide information on the spin averaged quark and gluon densities $q(x, Q^2)$ and $G(x, Q^2)$ inside the nucleon.

Our quark distributions (at $Q^2 = 1 \text{ GeV}^2$) are parametrized as follows:

$$\Delta q_i(x) = A_i x^{\lambda_i} (1-x)^{\eta_i} (1 + \epsilon_i \sqrt{x} + \mu_i x). \quad (1)$$

The similar form one has for gluon density

$$\Delta G(x) = B_G x^{\lambda_G} (1-x)^{\eta_G} (1 + \epsilon_G \sqrt{x} + \mu_G x). \quad (2)$$

The values of constants λ_i and η_i are given in [11, 12].

We have for total quark distributions

$$\begin{aligned} \Delta u &= \Delta u_v + 2\Delta \bar{u}, \\ \Delta d &= \Delta d_v + 2\Delta \bar{d}, \\ \Delta s &= 2\Delta \bar{s}, \end{aligned} \quad (3)$$

and for axial charges

$$\begin{aligned} \Delta \Sigma &= \Delta u + \Delta d + \Delta s, \\ a_8 &= \Delta u + \Delta d - 2\Delta s, \\ a_3 &\equiv g_A = \Delta u - \Delta d. \end{aligned} \quad (4)$$

In order to determine the unknown parameters in the expressions for polarized quark and gluon distributions we calculate the spin asymmetries (starting from initial value $Q^2 = 1 \text{ GeV}^2$) for measured values of Q^2 and make a fit to the experimental data on spin asymmetries for proton, neutron and deuteron targets. The spin asymmetry $A_1(x, Q^2)$ can be expressed via the polarized structure function $g_1(x, Q^2)$ as

$$A_1(x, Q^2) \cong \frac{(1 + \gamma^2) g_1(x, Q^2)}{F_1(x, Q^2)} = \frac{g_1(x, Q^2)}{F_2(x, Q^2)} [2x(1 + R(x, Q^2))] , \quad (5)$$

(In the case of SLAC data [8, 10] g_1/F_1 values were given experimentally) where $R = [F_2(1 + \gamma^2) - 2xF_1]/2xF_1$, whereas $\gamma = 2Mx/Q$ (M stands for proton mass). We will take the value of R from the [15] (of course, formula for R determined experimentally already includes corrections from higher twists). In calculating $g_1(x, Q^2)$ and $F_2(x, Q^2)$ in the next to leading order we use procedure described in [1, 14, 16] (power law corrections are not included). Having calculated the asymmetries according to Eq. (5) for the value of Q^2 obtained in experiments we can make a fit to asymmetries on proton, neutron and deuteron targets.

As usual in our previous papers the value of a_3 is not constrained in such fit. We also do not fix a_8 but we put a constraint on its value. Simply we will add it as an extra experimental point (from hyperon decays one has $a_8 = 0.58 \pm 0.03$, but we enhance an error to 3σ , *i.e.* to 0.1). It is also possible to use experimentally determined $F_2(x, Q^2)$, for example by SMC group from CERN, where the power law corrections are included in the fit [17]. It is also possible to use directly experimental results for polarized structure functions g_1 given by many experimental groups (the problem is that g_1 could be determined in a different way in different experiments). We will try to use these three methods of obtaining polarized quark and gluon densities taking into account very simple (h/Q^2) power law (with no x dependence) correction to the NLO QCD expression for the polarized structure function $g_1(x, Q^2)$ ($g_1(x, Q^2) \rightarrow g_1(x, Q^2) + h/Q^2$). Simple x dependence of coefficient $h(x)$, as well as terms of order $1/Q^4$, give negligible corrections to χ^2 .

TABLE I

fit	Δu	Δd	Δs	$\Delta \Sigma$	g_A	ΔG
A99	0.74	-0.48	-0.16	0.10	1.22	0.47
B99	0.73	-0.49	-0.17	0.08	1.22	0.49
C99	0.78	-0.48	-0.14	0.16	1.26	-0.43
	h_{1p}	h_{1n}	χ^2	χ_0^2	$\chi_{(\text{np})}^2$	$\chi_{0(\text{np})}^2$
A99	0.04	0.02	165.04	177.58	153.83	156.45
B99	0.03	0.03	159.35	165.94	152.01	154.02
C99	0.0008	0.008	150.77	150.86	149.34	149.58

In Table I we present our results for integrated parton densities with corresponding χ^2 for the fit (subscript 0 corresponds to the fit with no higher twists corrections, whereas (np) stands for the fit with no positivity conditions taken into account). Power law corrections are taken in the simple form (suggested in [18]) h_{1p}/Q^2 for g_1^p and h_{1n}/Q^2 for g_1^n (and corresponding

combination for g_1^d). Fit A99 corresponds to the fit to experimental data on polarized structure functions, B99 to g_1 but with F_2 taken from formula from [17] and C99 to the fit to spin asymmetries where power law corrections are negligible. In Table I we also give χ^2 for corresponding fits without power law corrections. By comparing corresponding columns we see how the introduction of these corrections reduces χ^2 . It is seen from Table I that integrated parton densities do not differ much for the three fits (It is also true in the case of fits without power law corrections) however there is essential increase in first two models relatively to the third. It seems that the increase in χ^2 of fits A99 and B99 in comparison with fit C99 is connected with positivity conditions assumed for the parton distributions.

To show the influence of positivity conditions we present also the values for χ^2 for the fits without positivity conditions taken into account with power corrections and without such corrections. The values of integrated quark densities do not change significantly (the strongest change is in gluon contributions which become positive) in the fits without positivity conditions comparing with those where positivity conditions for parton densities were assumed. From Table I we see that in the models without positivity conditions the effect of taking into account power law corrections is rather small, *i.e.* 2.6 in the first ($h_{1p} = 0.02 \pm 0.01$, $h_{1n} = 0.01 \pm 0.04$) and 2 in the second model ($h_{1p} = 0.02 \pm 0.02$, $h_{1n} = 0.03 \pm 0.04$) for 2 degrees of freedom. That means that in those fits the parameters h_{1p} and h_{1n} are at the border of being relevant. It is not very surprising that the fits with assumed positivity conditions (that means with additional restrictions on the form of the fitted parton densities) give χ^2 higher than fits where the positivity conditions are not assumed. On the other hand we probably should not expect in NLO QCD very strong violation of positivity conditions. The values for the integrated parton densities given in Table I for the third model can be compared with the values obtained using MRST 98 model

$$\begin{aligned}
 \Delta u &= 0.77, \\
 \Delta d &= -0.59, \\
 \Delta s &= -0.20, \\
 \Delta \Sigma &= -0.02, \\
 g_A &= 1.36, \\
 \Delta G &= 0.01, \\
 \chi^2 &= 150.47.
 \end{aligned} \tag{6}$$

Our fit to spin asymmetries where 431 points were considered (without averaging over Q^2) was discussed in [1] and similar values for integrated parton densities were obtained.

In Table II we present in a similar way as in Table I results obtained from the fit corresponding to the fit MRST 2001.

TABLE II

fit	Δu	Δd	Δs	$\Delta \Sigma$	g_A	ΔG
A01	0.79	-0.32	-0.04	0.47	1.14	56.6
B01	0.81	-0.35	-0.06	0.40	1.16	42.6
C01	0.86	-0.38	-0.05	0.44	1.24	29.9
	h_{1p}	h_{1n}	χ^2	χ_0^2	$\chi_{(\text{np})}^2$	$\chi_{0(\text{np})}^2$
A01	0.06	0.00	199.73	217.44	155.10	158.51
B01	0.05	0.02	188.88	197.73	152.77	154.41
C01	-0.0005	-0.002	158.40	158.41	148.14	148.23

The values of χ^2 corresponding to the model with positivity condition for parton densities in this case are much higher than for the solution corresponding to MRST 99 presented in Table I. For the spin asymmetries one has $\chi^2=158.41$, which have to be compared with 150.75. These solutions have more singular behaviour for valence u quark and very different gluon behaviour for small x . Therefore it is more difficult to fit (with positivity conditions for quark densities) the experimental data and χ^2 for fits A01 and B01 and C01 is higher. On the other hand the solutions without assuming positivity conditions are not very different from the solutions corresponding to MRST 99 (χ^2 corresponding to spin asymmetries 148.23 is smaller than 149.58). The reduction in χ^2 coming from power law corrections for solutions with assumed positivity conditions for quark densities is significant (for B01 $h_{1p} = 0.05 \pm 0.02$, $h_{1n} = 0.02 \pm 0.04$) but similarly to the situation presented in Table I for the solutions without assuming positivity conditions the changes in χ^2 caused by these corrections are rather small (for A01(np) $h_{1p} = 0.03 \pm 0.02$, $h_{1n} = -0.01 \pm 0.04$ and for B01(np) $h_{1p} = 0.02 \pm 0.02$, $h_{1n} = 0.01 \pm 0.04$). It is difficult to draw a conclusion that taking into account our simple power law corrections of the form h/Q^2 is very important for the fits. The integrated quark densities for different fits do not differ much (like in Table I there are some changes for integrated gluon distributions). In general taking into account positivity conditions for parton densities, in spite of relatively big changes in χ^2 , does not change significantly the integrated parton densities.

There are changes when we compare solutions corresponding to MRST 98, MRST 99 and MRST 2001. From the Eq. (6), Table I and Table II we see that Δu and Δd increase, Δs decrease and as consequence $\Delta \Sigma$ increase.

The values of $\Delta\Sigma$ are not very different from $g_8 = 0.58$. We have the situation when the solutions give different integrated parton densities and the corresponding χ^2 values are very close (especially when we take solutions corresponding to the case when we do not assume positivity conditions for parton densities). There is a dependence on the form of assumed parton densities but that does not influence strongly χ^2 values. It is tempting to choose (in spite of higher χ^2 value when positivity condition for quark densities are assumed) solution corresponding to MRST 2001 (recent fit to unpolarized data) where integrated parton densities give relatively high value of $\Delta\Sigma$ and small value of Δs (or at least consider it seriously).

We have compared polarized parton distributions, corresponding to MRST 99 and MRST 2001 determination, using three different methods of obtaining polarized parton densities. Simplest power law correction corresponding to higher twists have been taken into account. As expected these power law corrections are negligible in the fits to spin asymmetries. At the first sight they seem to be important in the first two models. For comparison we have also considered the solutions where positivity conditions for parton densities have not been assumed. We observe strong increase in χ^2 in models A99, A01 and B99, B01 (fits to g_1) connected with positivity conditions. Reduction in χ^2 by taking into account power law corrections is significant in the models where positivity conditions were assumed and rather marginal in models without positivity conditions for parton densities. Hence, it seems that the determination of power law corrections is not very reliable. There are some differences in integrated parton densities corresponding to the models MRST 98, MRST 99 and MRST 2001. Within the definite model three different methods of fitting give very similar results for integrated quark densities (but not very much for a gluon one) also in the case when positivity conditions for parton densities are not assumed. The latest model (C01) even if not by the values of χ^2 (with slightly higher χ^2 value with positivity conditions for parton densities assumed) is preferred by the reasons of interpretation.

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