# A PROPOSAL OF QUARK MASS FORMULA AND LEPTON SPECTRUM*

Wojciech Królikowski

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00-681 Warszawa, Poland

*(Received September 25, 2003)*

An explicit mass formula for up and down quarks of three generations is proposed. Its structure contains an additional term in comparison to the efficient mass formula for charged leptons found out some time ago. The additional term is conjectured to be proportional to $(3B+Q^{(u,d)})^2 = 25/9$ or $4/9$, where $B = 1/3$ and $Q^{(u,d)} = 2/3$ or $-1/3$ are the baryon number and electric charge of quarks. It is interesting to observe that the analogical term for charged leptons proportional to $(L+Q^{(e)})^2$, where $L = 1$ and $Q^{(e)} = -1$, would vanish consistently (here, $F = 3B+L$ is the fermion number). Under this conjecture, the mass formula *predicts* one quark mass, *e.g.* $m_b$, in accordance with its experimental estimate, if the experimental estimations of five other quark masses are used as an input to determine five free parameters involved.

PACS numbers: 12.15.Ff, 12.15.Hh, 14.60.Pq

## 1. Introduction

Some time ago we observed that the mass spectrum of charged leptons $e_i = e^-, \mu^-, \tau^-$ can be presented with high precision by the formula [1]

$$m_{e_i} = \rho_i \mu^{(e)} \left( N_i^2 + \frac{\varepsilon^{(e)} - 1}{N_i^2} \right) , \tag{1}$$

where

$$\rho_i = \frac{1}{29} , \frac{4}{29} , \frac{24}{29} \tag{2}$$

($\sum_i \rho_i = 1$) and

$$N_i = 1, 3, 5 , \tag{3}$$

* Work supported in part by the Polish State Committee for Scientific Research (KBN), grant 2 P03B 129 24 (2003–2004).

while $\mu^{(e)} > 0$ and $\varepsilon^{(e)} > 0$ are constants. In fact, with the experimental values $m_e = 0.510999$ MeV and $m_\mu = 105.658$ MeV as an input, the formula (1) rewritten explicitly as

$$m_e = \frac{\mu^{(e)}}{29}\varepsilon^{(e)}\,,\ m_\mu = \frac{\mu^{(e)}}{29}\frac{4}{9}\left(80+\varepsilon^{(e)}\right)\,,\ m_\tau = \frac{\mu^{(e)}}{29}\frac{24}{25}\left(624+\varepsilon^{(e)}\right) \quad (4)$$

leads to the *prediction*

$$m_\tau = \frac{6}{125}\left(351 m_\mu - 136 m_e\right) = 1776.80 \text{ MeV} \quad (5)$$

and also determines

$$\mu^{(e)} = \frac{29(9m_\mu - 4m_e)}{320} = 85.9924 \text{ MeV}\,,\ \varepsilon^{(e)} = \frac{320 m_e}{9m_\mu - 4m_e} = 0.172329\,. \quad (6)$$

We can see that the prediction (5) lies really close to the experimental value $m_\tau^{\exp} = 1776.99^{+0.29}_{-0.26}$ MeV [2]. The interested reader may find a discussion about theoretical background of the simple formula (1) in Ref. [1]. In particular, the numbers $\rho_i$ and $N_i$ $(i = 1, 2, 3)$ given in Eqs. (2) and (3) are interpreted there.

In the present note we will try to extend the formula (1), originally worked out for charged leptons, also to up and down quarks, $u_i = u, c, t$ and $d_i = d, s, b$, whose masses are estimated experimentally as [2]

$$m_u \sim 1.5 \ \text{to} \ 4.5 \text{ MeV}\,,\ \ m_c \sim 1.0 \ \text{to} \ 1.4 \text{ GeV}\,,\ \ m_t \sim 174.3 \pm 5.1 \text{ GeV} \quad (7)$$

and

$$m_d \sim 5 \ \text{to} \ 8.5 \text{ MeV}\,,\ \ m_s \sim 80 \ \text{to} \ 155 \text{ MeV}\,,\ \ m_b \sim 4.0 \ \text{to} \ 4.5 \text{ GeV}\,. \quad (8)$$

We will use the average values

$$m_u \sim 3 \text{ MeV}\,,\ \ m_c \sim 1.2 \text{ GeV}\,,\ \ m_t \sim 174 \text{ GeV} \quad (7')$$

and

$$m_d \sim 6.8 \text{ MeV}\,,\ \ m_s \sim 118 \text{ MeV}\,,\ \ m_b \sim 4.3 \text{ GeV}\,. \quad (8')$$

Note that the ratio $m_u/m_d$ is estimated independently as 0.2 to 0.7 with the average 0.45 [2]. In this work we will continue the line of Ref. [3].

## 2. Mass spectrum

Let us try the mass ansatz whose structure differs from the charged-lepton mass formula (1) by the additional term appearing only for the third generation $i = 3$ :

$$m_{u_i} = \rho_i \mu^{(u)} \left( N_i^2 + \frac{\varepsilon^{(u)} - 1}{N_i^2} + \delta_{i\,3} \beta^{(u)} \right) \tag{9}$$

and

$$m_{d_i} = \rho_i \mu^{(d)} \left( N_i^2 + \frac{\varepsilon^{(d)} - 1}{N_i^2} + \delta_{i\,3} \beta^{(d)} \right) , \tag{10}$$

where $\rho_i$ and $N_i$ are given as before in Eqs. (2) and (3), while $\mu^{(u,d)} > 0$, $\varepsilon^{(u,d)} > 0$ and $\beta^{(u,d)} > 0$ are constants. It is seen that *a priori* Eqs. (9) and (10) cannot give us any predictions since there are six quark masses and six free parameters. At most, all parameters can be determined. Let us do it. From Eqs. (9) and (10) as well as (7′) and (8′) we can calculate

$$\begin{aligned} m_{t,b} &= \frac{6}{125}\left(351 m_{c,s} - 136 m_{u,d}\right) + \frac{24}{29}\mu^{(u,d)}\beta^{(u,d)} \\ &\sim \left\{ \begin{array}{l} 20 \quad + 0.81\ \beta^{(u)} \\ 1.94 + 0.078\, \beta^{(d)} \end{array} \right\} \text{GeV} \end{aligned} \tag{11}$$

and

$$\begin{aligned} \mu^{(u,d)} &= \frac{29(9m_{c,s} - 4m_{u,d})}{320} \sim \left\{ \begin{array}{r} 980 \\ 94 \end{array} \right\} \text{MeV}\,, \\ \varepsilon^{(u,d)} &= \frac{320 m_{u,d}}{9m_{c,s} - 4m_{u,d}} \sim \left\{ \begin{array}{l} 0.089 \\ 2.1 \end{array} \right\} . \end{aligned} \tag{12}$$

Thus, from Eq. (11) we obtain

$$\beta^{(u)} \sim 190\,,\ \beta^{(d)} \sim 30 \tag{13}$$

and for their ratio the estimate

$$\frac{\beta^{(u)}}{\beta^{(d)}} \sim 6.3\,. \tag{14}$$

If this ratio was determined theoretically, then the value of $\beta^{(u)}$ or $\beta^{(d)}$ could be a prediction. Let us guess at a theoretical expression for $\beta^{(u)}/\beta^{(d)}$ consistent with the estimation (14). In fact, putting

$$\beta^{(u,d)} \propto \left(3B + Q^{(u,d)}\right)^2 = \left\{ \begin{array}{r} 25/9 \\ 4/9 \end{array} \right\} , \tag{15}$$

where $B = 1/3$ and $Q^{(u,d)} = \left\{ \begin{array}{r} 2/3 \\ -1/3 \end{array} \right\}$ are the baryon number and electric charge of quarks, we get

$$\frac{\beta^{(u)}}{\beta^{(d)}} = \frac{25}{4} = 6.25\,. \tag{16}$$

Under the conjecture (15) taking $\beta^{(u)} \sim 190$ we *predict* from the ratio (16) that $\beta^{(d)} \sim 30$ in accordance with the estimates (13). Then $\beta^{(u)}/(25/9) = \beta^{(d)}/(4/9) \sim 68$ or $\beta^{(u)}/25 = \beta^{(d)}/4 \sim 7.6$ . Thus, under the conjecture (15) leading to the ratio (16), taking as an input the experimental estimates (7′) and (8′) for $m_u, m_c, m_t$ and $m_d, m_s$, we *predict* that $m_b \sim 4.3$ GeV in accordance with its experimental estimate (8′).

It is interesting to notice that, defining for charged leptons the constant

$$\beta^{(e)} \propto \left(L + Q^{(e)}\right)^2 \,, \tag{17}$$

where $L = 1$ is the lepton number, we obtain $\beta^{(e)} = 0$ in consistency with the absence of the $\beta^{(e)}$-term in the charged-lepton mass formula (1).

Thus, generically, we can define for fermions $f = e, u, d$ the constant

$$\beta^{(f)} \propto \left(3B + L + Q^{(f)}\right)^2 = \left\{ \begin{array}{r} 0 \\ 25/9 \\ 4/9 \end{array} \right\} \tag{18}$$

which may play a crucial role in the mechanism of large enhancement of masses $m_t$ and $m_b$ over $m_\tau$ for fermions of the third generation $i = 3$ as well as of these $m_t$ and $m_b$ over $m_u$, $m_c$ and $m_d$, $m_s$, respectively. Here, $F = 3B + L$ is the fermion number as given for quarks and leptons.

For Majorana neutrinos $\nu_{i\,\mathrm{L}} + (\nu_{i\,\mathrm{L}})^c$ and $\nu_{i\,\mathrm{R}} + (\nu_{i\,\mathrm{R}})^c$, built up from the active and sterile mass neutrinos $\nu_{i\,\mathrm{L}}$ and $\nu_{i\,\mathrm{R}}$ $(i = 1,2,3)$, respectively, the averaged $L$ is zero, so for them the corresponding constant $\beta^{(\nu)} = 0$ in the effective Majorana form of neutrino mass matrix arising from the Dirac component $M^{(\mathrm{D})}$ and Majorana righthanded component $M^{(\mathrm{R})}$ of neutrino generic $6 \times 6$ mass matrix (its Majorana lefthanded component $M^{(\mathrm{L})}$ is commonly assumed to be zero). With $M^{(\mathrm{D})}$ and $M^{(\mathrm{R})}$, the neutrino mass term in the Lagrangian reads

$$\sum_{\alpha\beta} \overline{\nu_{\alpha\,\mathrm{L}}} M^{(\mathrm{D})}_{\alpha\beta} \nu_{\beta\,\mathrm{R}} + \frac{1}{2} \sum_{\alpha\beta} \overline{\nu_{\alpha\,\mathrm{R}}} M^{(\mathrm{R})}_{\alpha\beta} \left(\nu_{\beta\,\mathrm{R}}\right)^c + \mathrm{h.c.}\,, \tag{19}$$

where $\nu_{\alpha\,\mathrm{L}} = \sum_i U_{\alpha\,i} \nu_{i\,\mathrm{L}}$ $(\alpha = e, \mu, \tau)$ are active flavor neutrinos and $U = (U_{\alpha\,i})$ denotes the lepton mixing matrix. This $U$ is equal to the active-neutrino diagonalizing matrix, if the flavor representation is used, where the charged-lepton mass matrix $M^{(e)}$ is diagonal. Phenomenologically, $U$ gets the bilarge form, consistent with all confirmed neutrino-oscillation experiments [4]. For simplicity we take $M^{(\mathrm{D})}$ and $M^{(\mathrm{R})}$ real.

In the popular seesaw mechanism [5], $M^{(\mathrm{D})}$ is dominated by $M^{(\mathrm{R})}$. Then, for active neutrinos $\nu_{\alpha\,\mathrm{L}}$ the effective Majorana mass matrix

$$M^{(\nu)} = M^{(\mathrm{D})} \frac{1}{M^{(\mathrm{R})}} M^{(\mathrm{D})\,T} \tag{20}$$

appears leading to the effective mass term in the Lagrangian

$$\frac{1}{2} \sum_{\alpha\beta} \overline{\nu_{\alpha\,\mathrm{L}}} M^{(\nu)}_{\alpha\beta} \left(\nu_{\beta\,\mathrm{L}}\right)^c + \mathrm{h.c.}\,. \tag{21}$$

In the flavor representation mentioned above, we obtain

$$U^\dagger M^{(\nu)} U = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\,. \tag{22}$$

If the neutrino Dirac mass matrix $M^{(\mathrm{D})}$ displays the same spectrum $m_{e_i} = m_e, m_\mu, m_\tau$ as the charged-lepton mass matrix $M^{(e)}$ [6], and if the diagonalizing matrix for $M^{(\mathrm{D})}$ is equal to the mixing matrix $U$ [6], then

$$U^\dagger M^{(\mathrm{D})} U = \mathrm{diag}(m_e, m_\mu, m_\tau)\,. \tag{23}$$

Under these assumptions we obtain from Eq. (20) the following mass spectrum for light active mass neutrinos $\nu_{i\mathrm{L}}$:

$$m_{\nu_i} = \frac{m^2_{e_i}}{M_{\nu_i}} \tag{24}$$

with

$$\frac{1}{M_{\nu_i}} \equiv \left( U^\dagger \frac{1}{M^{(\mathrm{R})}} U \right)_{i\,i} = \sum_{\alpha\beta} U^*_{\alpha i} \left( \frac{1}{M^{(\mathrm{R})}} \right)_{\alpha\beta} U_{\beta i}\,, \tag{25}$$

where $M_{\nu_i} \gg m_{e_i}$. (More generally, the spectrum of $M^{(\mathrm{D})}$ may be proportional to $m_{e_i}$ with a positive coefficient $\eta \leqslant 1$; in this case $\mu^{(\nu)} = \eta\mu^{(e)}$, $\varepsilon^{(\nu)} = \varepsilon^{(e)}$ and $\beta^{(\nu)} = \beta^{(\mathrm{e})} = 0$. Then, $1/M_{\nu_i}$ in Eq. (24) become proportional to $(U^+ M^{(\mathrm{R})-1} U)_{ii}$ with the positive coefficient $\eta^2 \leqslant 1$.) If, in addition, the neutrino Majorana mass matrix $M^{(\mathrm{R})}$ is diagonal in the same basis as $M^{(\mathrm{D})}$ [6],

$$U^\dagger M^{(\mathrm{R})} U = \mathrm{diag}(M_{\nu_1}, M_{\nu_2}, M_{\nu_3})\,, \tag{26}$$

then the mass scales $M_{\nu_i}$ defined in Eq. (25) become the Majorana masses of heavy sterile mass neutrinos $\nu_{i\,\mathrm{R}}$. In general, however, $1/M_{\nu_i}$ as given in

Eq. (25) are the average values of the matrix $1/M^{(\mathrm{R})}$ in the states of light active mass neutrinos $\nu_{i\,\mathrm{L}}$.

In Ref. [6] we make the conjecture of (*weighted*) proportionality between $M_{\nu_i}$ and $m_{e_i}$, namely

$$M_{\nu_i} \propto N_i^2 m_{e_i}\,, \tag{27}$$

where $N_i = 1, 3, 5$ as in Eq. (1). Then, from the seesaw formula (24) we infer that

$$m_{e_i} \propto N_i^2 m_{\nu_i}\,. \tag{28}$$

This gives the *prediction*

$$\frac{m_{\nu_2}}{m_{\nu_3}} = \frac{25}{9}\frac{m_\mu}{m_\tau} = 0.1652\,, \tag{29}$$

where the experimental values $m_\mu = 105.658$ MeV and $m_\tau = 1776.99^{+0.29}_{-0.26}$ MeV [2] are used. The experimental estimates $m^{\exp}_{\nu_2} \sim \sqrt{7\times10^{-5}}$ eV and $m^{\exp}_{\nu_3} \sim \sqrt{2.5\times10^{-3}}$ eV (valid if $m^2_{\nu_1} \ll m^2_{\nu_2} \ll m^2_{\nu_3}$) [4] lead to the ratio

$$\frac{m^{\exp}_{\nu_2}}{m^{\exp}_{\nu_3}} \sim \sqrt{2.8\times10^{-2}} = 0.17 \tag{30}$$

consistent with the prediction (29) (for $m^{\exp}_{\nu_2} \sim \sqrt{7\times10^{-5}}$ eV and $m^{\exp}_{\nu_3} \sim \sqrt{2\times10^{-3}}$ eV this ratio is equal to 0.19; in order to get with $m^{\exp}_{\nu_3} \sim \sqrt{2\times10^{-3}}$ eV the ratio (30) consistent with (29) one ought to have $m^{\exp}_{\nu_2} \sim \sqrt{5.6\times10^{-5}}$ eV). In another way, Eq. (29) *predicts*

$$m_{\nu_3} \sim \sqrt{2.6\times10^{-3}}\,\mathrm{eV} = 5.1\times10^{-2}\ \mathrm{eV}\,, \tag{31}$$

if the input $m_{\nu_2} \sim \sqrt{7\times10^{-5}}$ eV is used. From Eq. (28) we can also estimate $m_{\nu_1}$. In fact, with $m_e = 0.510999$ MeV and $m_\mu = 105.658$ MeV [2] we get

$$\frac{m_{\nu_1}}{m_{\nu_2}} = 9\frac{m_e}{m_\mu} = 0.0435271\,, \tag{32}$$

what leads to the *prediction*

$$m_{\nu_1} \sim \sqrt{1.3\times10^{-7}}\,\mathrm{eV} = 3.6\times10^{-4}\ \mathrm{eV}\,, \tag{33}$$

when the input $m_{\nu_2} \sim \sqrt{7\times10^{-5}}$ eV is applied. Through the seesaw formula (24), $M_{\nu_i} = m^2_{e_i}/m_{\nu_i}$, the estimates $m_{\nu_1} \sim 3.6\times10^{-4}$ eV, $m_{\nu_2} \sim \sqrt{7\times10^{-5}}$ eV$= 8.4\times10^{-3}$ eV and $m_{\nu_3} \sim 5.1\times10^{-2}$ eV give the *prediction*

$$M_{\nu_1} \sim 7.2\times10^{5}\ \mathrm{GeV}\,,\ M_{\nu_2} \sim 1.3\times10^{9}\ \mathrm{GeV}\,,\ M_{\nu_3} \sim 6.2\times10^{10}\ \mathrm{GeV}\,. \tag{34}$$

Thus, $M_{\nu_1}$ is predicted to be $O(10^5)$ times smaller than $M_{\nu_3}$, while $m_{\nu_1}$ is $O(10^2)$ times lighter than $m_{\nu_3}$. Note that the proportionality coefficient in the relations (27) and (28), call it $\zeta$, can be easily estimated. Making use of *e.g.* $m_\mu = 105.658$ MeV and $m_{\nu_2} \sim \sqrt{7 \times 10^{-5}}$ eV we obtain $\zeta = m_\mu / 9 m_{\nu_2} \sim 1.4 \times 10^9$.

## 3. Mixing

In contrast to the mixing matrix for leptons (Maki–Nakagawa–Sakata matrix) that is bilarge:

$$U = (U_{\alpha i}) = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}} s_{12} & \frac{1}{\sqrt{2}} c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} s_{12} & -\frac{1}{\sqrt{2}} c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{35}$$

($\alpha = e, \mu, \tau$ , $i = 1, 2, 3$), where $c_{12}^2$ and $s_{12}^2$ are of similar magnitudes corresponding to $\theta_{12} \sim 33^\circ$ ($s_{12}^2/c_{12}^2 \sim 0.42$), the mixing matrix for quarks (Cabibbo–Kobayashi–Maskawa matrix) is nearly diagonal:

$$V = (V_{ij}) = \begin{pmatrix} c_{12} & s_{12} & s_{13} e^{-i\delta_{13}} \\ -s_{12} & c_{12} & s_{23} \\ s_{12} s_{23} - s_{13} e^{i\delta_{13}} & -s_{23} & 1 \end{pmatrix} \tag{36}$$

($i = u, c, t$ , $j = d, s, b$), where now $c_{ij}^2 \gg s_{ij}^2$ and $s_{12}^2 \gg s_{23}^2 \gg s_{13}^2$ with $s_{12} = 0.2229 \pm 0.0022$, $s_{23} = 0.0412 \pm 0.0020$ and $s_{13} = 0.0036 \pm 0.0007$ [2] (thus, $\theta_{12} \simeq 13^\circ$, $\theta_{23} \simeq 2^\circ$ and $\theta_{13} \simeq 0.2^\circ$). In terms of the Wolfenstein parameters $\lambda, A, \rho, \eta$ we can write

$$s_{12} = \lambda \, , \; s_{23} = A\lambda^2 \, , \; s_{13} e^{-i\delta_{13}} = A\lambda^3 (\rho - i\eta) \tag{37}$$

and so, $c_{12} \simeq 1 - \lambda^2/2$ and $s_{12}s_{23} - s_{13}e^{i\delta_{13}} = A\lambda^3(1 - \rho - i\eta)$ (Eq. (36) works up to $O(\lambda^3)$). The flavor down quarks are $d'_i = \sum_j V_{ij} d_j$ with $d'_i = d', s', b'$, while $d_i = d, s, b$ denote mass down quarks. In the flavor representation, where the up-quark mass matrix $M^{(u)}$ is diagonal, the quark mixing matrix $V = (V_{ij})$ is equal to the down-quark diagonalizing matrix:

$$V^\dagger M^{(d)} V = \mathrm{diag}(m_d, m_s, m_b) \, . \tag{38}$$

The flavor down quarks, denoted traditionally as $d'_i = d', s', b'$, might be labeled (in analogy with the flavor neutrinos $\nu_\alpha = \nu_e \, , \, \nu_\mu \, , \, \nu_\tau$) also as $d_\alpha = d_u, d_c, d_t$ (where the label $\alpha = u, c, t$ refers to the up quarks $u_i = u, c, t$ that are here an analogy of the charged leptons $e_i = e^-, \mu^-, \tau^-$ marking the corresponding flavor neutrinos with the label $\alpha = e, \mu, \tau$). Then,

$d_\alpha = \sum_i V_{\alpha\, i} d_i$ where $d_i = d, s, b$ are the mass down quarks (an analogy of the mass neutrinos $\nu_i = \nu_1, \nu_2, \nu_3$), and $V = (V_{\alpha\, i})$ is the quark mixing matrix (36) (an analogy of the lepton mixing matrix $U = (U_{\alpha\, i})$). In the flavor representation, where the up-quark mass matrix $M^{(u)}$ is diagonal, there are only (mass = flavor) up quarks $u_i = u, c, t$ (like the (mass = flavor) charged leptons $e_i = e^-, \mu^-, \tau^-$ in their flavor representation, where their mass matrix $M^{(e)}$ is diagonal).

Making use of the formula $M^{(\nu)}_{\alpha\beta} = \sum_i U_{\alpha\, i} m_{\nu_i} U^*_{\beta\, i}$ $(\alpha, \beta = e, \mu, \tau)$ and applying Eq. (35) we can calculate the strongly nondiagonal $M^{(\nu)} = \left(M^{(\nu)}_{\alpha\beta}\right)$:

$$\begin{aligned} M^{(\nu)}_{ee} &= m_{\nu_1} c^2_{12} + m_{\nu_2} s^2_{12}\,, \\ M^{(\nu)}_{\mu\mu} &= M^{(\nu)}_{\tau\tau} = \frac{1}{2}(m_{\nu_1} s^2_{12} + m_{\nu_2} c^2_{12} + m_{\nu_3})\,, \\ M^{(\nu)}_{e\mu} &= -M^{(\nu)}_{e\tau} = \frac{1}{\sqrt{2}}(-m_{\nu_1} + m_{\nu_2}) c_{12} s_{12}\,, \\ M^{(\nu)}_{\mu\tau} &= \frac{1}{2}(-m_{\nu_1} s^2_{12} - m_{\nu_2} c^2_{12} + m_{\nu_3})\,. \end{aligned} \tag{39}$$

In contrast, from the formula $M^{(d)}_{ij} = \sum_k V_{ik} m_{d_k} V^*_{jk}$ $(i, j = d, s, b)$ and Eq. (36) we can evaluate the nearly diagonal $M^{(d)} = \left(M^{(d)}_{ij}\right)$ (up to $O(\lambda^3)$):

$$\begin{aligned} M^{(d)}_{dd} &= m_d c^2_{12} + m_s s^2_{12} = m_d + O(\lambda^2)\,, \\ M^{(d)}_{ss} &= m_d s^2_{12} + m_s c^2_{12} = m_s + O(\lambda^2)\,, \\ M^{(d)}_{bb} &= m_b\,, \\ M^{(d)}_{ds} &= (m_s - m_d) c_{12} s_{12} = O(\lambda) + O(\lambda^3)\,, \\ M^{(d)}_{db} &= -(m_s - m_d) s_{12} s_{23} + (m_b - m_d) s_{13} e^{-i\delta_{13}} = O(\lambda^3)\,, \\ M^{(d)}_{sb} &= (m_b - m_s) s_{23} = O(\lambda^2)\,, \end{aligned} \tag{40}$$

where $\lambda \simeq 0.22$, $\lambda^2 \simeq 0.050$, $\lambda^3 \simeq 0.011$.

From Eqs. (40) we infer that

$$M^{(d)}_{dd} + M^{(d)}_{ss} = m_d + m_s\,,\; \left(M^{(d)}_{dd} - M^{(d)}_{ss}\right)^2 + 4 M^{(d)\,2}_{ds} = (m_s - m_d)^2 \tag{41}$$

and hence

$$m_{d,s} = \frac{M^{(d)}_{dd} + M^{(d)}_{ss}}{2} \mp \sqrt{\left(\frac{M^{(d)}_{dd} - M^{(d)}_{ss}}{2}\right)^2 + M^{(d)\,2}_{ds}}\;, \quad m_b = M^{(d)}_{bb}\,. \tag{42}$$

Similarly, we calculate

$$
\begin{aligned}
\frac{1}{2}\sin 2\theta_{12} \;&\equiv\; c_{12}s_{12} = \frac{M^{(d)}_{ds}}{m_s - m_d} = O(\lambda) + O(\lambda^3)\,,\\
\sin\theta_{23} \;&\equiv\; s_{23} \quad\;\, = \frac{M^{(d)}_{sb}}{m_b - m_s} = O(\lambda^2)\,,\\
\sin\theta_{13} \;&\equiv\; s_{13} \quad\;\, = \frac{\left|M^{(d)}_{db} + (m_s - m_d)\, s_{12}s_{23}\right|}{m_b - m_d} = O(\lambda^3)\,. \qquad (43)
\end{aligned}
$$

In the last Eq. (43), $s_{12}$ can be expressed through $c_{12}s_{12}$: $s_{12} = \frac{1}{\sqrt{2}}[1 - \sqrt{1-(2c_{12}s_{12})^2}]^{1/2}$. The relations (42) and (43) present (up to $O(\lambda^3)$) the down-quark masses $m_d, m_s, m_b$ and their mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ in terms of six elements $M^{(d)}_{ij}$ $(i \leq j)$ of the down-quark mass matrix $M^{(d)}$ (up to $O(\lambda^3)$).

Concluding, we have constructed in this note the mass formulae (9) and (10) for up and down quarks that extend the efficient mass formula (1), found out previously for charged leptons [1], by including in their structure an additional term for the third quark generation. While the charged-lepton formula predicts $m_\tau = 1776.80$ MeV, the quark formulae predict $m_b \sim 4.3$ GeV if the specific conjecture (15) is made (masses of all other charged leptons and up and down quarks are reproduced exactly by fitting two charged-lepton and five up- and down-quark parameters). Under the conjecture (15) implying naturally (17), the additional mass term for the third charged-lepton generation vanishes consistently. We have commented also upon the seesaw mass formula (24) for light active neutrinos (relating their masses with charged-lepton masses) under the specific conjecture (27) on the mass scales of heavy sterile (righthanded) neutrinos [6].

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