SCHWINGER TUNNELING AND THERMAL CHARACTER OF HADRON SPECTRA

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It is shown that an oscillatory character of the solutions of the collisionless kinetic equations describing production of the quark–gluon plasma in strong color fields leads to the exponential (thermal-like) transverse-momentum spectra of partons produced in the soft region ($100\,\mathrm{MeV} < p_\perp < 1\,\mathrm{GeV}$). In addition, the production of partons in the very soft region ($p_\perp < 100\,\mathrm{MeV}$) is clearly enhanced above the thermal-like background.

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1. Introduction

The transverse-momentum spectra of hadrons measured at RHIC are very well reproduced by the thermal model [1]. Since the thermal-like spectra appear also in the collisions of more elementary systems, the question arises if the thermal behavior observed at RHIC may be truly attributed to the rescattering processes or is it of a completely different origin connected, *e.g.*, with a trivial phase-space dominance effect (for a recent discussion of this and similar issues see Refs. [2–5]).

In this paper we follow the idea formulated by Bialas and argue that the thermal shape of the transverse-momentum spectra of hadrons may have its origin in the fluctuations of the string tension. In Ref. [6] Bialas showed that the thermal character of the measured transverse-momentum spectra,

$$\frac{dN_{\mathrm{exp}}}{d^2p_\perp} \sim \exp\left(\frac{-m_\perp}{T}\right), \qquad m_\perp = \sqrt{m^2 + p_\perp^2}\,, \tag{1}$$

may be understood as an effect of the fluctuations of the string tension κ^2 which appears in the Schwinger formula [7–10],

$$\frac{dN_{\text{Schwinger}}}{d^2p_\perp} \sim \exp\left(\frac{-\pi m_\perp^2}{\kappa^2}\right) . \tag{2}$$

Although the $m_\perp$-dependence in Eqs. (1) and (2) is different, the appropriate averaging of formula (2) over κ may produce indeed an exponential function,

$$\int d\kappa\, P(\kappa) \exp\left(\frac{-\pi m_\perp^2}{\kappa^2}\right) \sim \exp\left(\frac{-m_\perp}{T}\right) . \tag{3}$$

The explicit (gaussian) form of the distribution $P(\kappa)$ as well as a relation connecting T with the average value of κ^2,

$$T = \sqrt{\frac{\langle \kappa^2 \rangle}{2\pi}} , \tag{4}$$

was given and discussed in Ref. [6].

In this paper we show that the situation described above appears naturally in the kinetic equations describing production of the quark-gluon plasma in strong color fields. In this case, due to the screening effects, the color fields change in time and may even oscillate [11]. As a consequence, the transverse-momentum spectra acquire a form very similar to Eq. (3). The only difference is that κ^2 should be treated now as a function of time

$$\int dt\, P'(t) \exp\left(\frac{-\pi m_\perp^2}{\kappa^2(t)}\right) \sim \exp\left(\frac{-m_\perp}{T}\right) . \tag{5}$$

The form of the distribution $P'(t)$ is uniquely determined by the kinetic equations and, as we shall see, formula (5) yields effectively the exponential spectra in the soft region, 100 MeV $< p_\perp <$ 1GeV. For larger values of $p_\perp$ the model based on the Schwinger formula gives the spectrum which decays faster than the exponential function. However, in this region the production of particles becomes a hard process and the use of the Schwinger formula is inadequate. On the other hand, for very small values of $p_\perp$ we find an enhancement above the exponential background, which is a desirable effect in view of the experimental measurements of the pion spectra which consistently show such an increase.

Although there is a formal similarity between our formulas and those used by Bialas, there is also an important physical difference between the two approaches. In our calculations we consider the values of the string tension which are larger than the elementary string tension. This may be a realistic situation in heavy-ion collisions [12]. On the other hand, Bialas considers possible fluctuations of the elementary string tension, which may appear due to stochastic nature of the QCD vacuum [13]. Thus, our approach may explain the origin of the thermal spectra observed in heavy-ions but it is not capable of describing the thermal features observed in, *e.g.*, electron-positron annihilations. It is conceivable, however, that the effect of the stochastic vacuum plays an additional role in the heavy-ion collisions leading to even more pronounced thermalization effects.

2. Tunneling of partons in oscillatory chromoelectric fields

In our approach we use the semi-classical kinetic equations for the quark-gluon plasma written in the abelian dominance approximation [11, 14–17]

$$\left(p^\mu \partial_\mu \pm g\boldsymbol{\epsilon}_i \cdot \boldsymbol{F}^{\mu\nu} p_\nu \partial^p_\mu\right) G^\pm_i(x,p) = \frac{dN^\pm_i}{d\Gamma}, \tag{6}$$

$$\left(p^\mu \partial_\mu + g\boldsymbol{\eta}_{ij} \cdot \boldsymbol{F}^{\mu\nu} p_\nu \partial^p_\mu\right) \tilde{G}_{ij}(x,p) = \frac{d\tilde{N}_{ij}}{d\Gamma}\,, \tag{7}$$

where $G^+_i(x,p)$, $G^-_i(x,p)$ and $\tilde{G}_{ij}(x,p)$ are the phase-space densities of quarks, antiquarks and gluons, respectively. Here g is the strong coupling constant and $i,j = (1,2,3)$ are color indices. The terms on the left-hand-side describe the free motion of the particles as well as their interaction with the mean color field $\boldsymbol{F}_{\mu\nu}$. The terms on the right-hand-side describe production of quarks and gluons due to the decay of the field. In our present calculations we include only the two lightest flavors and neglect the quark masses ($m_\perp = p_\perp$).

We note that Eqs. (6) and (7) do not include any thermalization effects. The latter can be taken into account if the collision integrals are incorporated on the right-hand-side of Eqs. (6) and (7). So far, most of the approaches have included the collision integrals in the relaxation-time approximation [18–20]. A more recent and elaborated treatment of the collision integrals may be found in Ref. [21]. We note also that the semi-classical kinetic equations may be derived within a field-theoretic approach if a separation of different time scales can be achieved: the time scales associated with quantum phase oscillations and amplitudes of pair creation should be much smaller than the time scales associated with the oscillations of the fields [22–25].

In the next sections we shall consider a one-dimensional (*i.e.*, uniform in the transverse direction) boost-invariant system. In this case it is convenient to use the boost-invariant variables introduced in Refs. [26]

$$u = \tau^2 = t^2 - z^2\,, \qquad w = tp_\parallel - zE\,, \qquad \boldsymbol{p}_\perp\,, \tag{8}$$

and also

$$v = Et - p_{\parallel}\; z = \sqrt{w^2 + m_\perp^2 u}\,. \tag{9}$$

From these two equations one can easily find the energy and the longitudinal momentum of a particle

$$E = p^0 = \frac{vt + wz}{u} = p_\perp \cosh y\,, \qquad p_\parallel = \frac{wt + vz}{u} = p_\perp \sinh y\,. \tag{10}$$

Besides the rapidity y, we also introduce the quasirapidity variable η which is related to the space-time coordinates t and z by equations

$$t = \tau \cosh \eta\,, \qquad z = \tau \sinh \eta\,. \tag{11}$$

The invariant measure in the momentum space is

$$dP = d^2 p_\perp \frac{dp_\parallel}{p^0} = d^2 p_\perp \frac{dw}{v}\,, \tag{12}$$

whereas in the Minkowski space-time the appropriate measure has the form

$$d^4 x = \tau \sinh \eta \, d\tau \, d\eta \, dx \, dy\,. \tag{13}$$

The invariant measure in the phase-space is $d\Gamma = d^4x d^3p/p^0$. In the considered situation, the only non-zero components of the tensor $\boldsymbol{F}_{\mu\nu} = (F^3_{\mu\nu}, F^8_{\mu\nu})$ are those corresponding to the color field $\boldsymbol{\mathcal{E}} = \boldsymbol{F}^{30}$. The quarks and gluons couple to the field $\boldsymbol{\mathcal{E}}$ through the charges $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\eta}_{ij}$ defined in [11, 27].

3. Transverse-momentum spectra

For one-dimensional boost-invariant systems the production rates appearing on the right-hand-side of Eqs. (6) and (7) have a general form [28][1]

$$\frac{dN}{d\Gamma} = p^0 \frac{dN}{d^4x\; d^3p} = \frac{F}{4\pi^3} \left| \ln \left(1 \mp \exp \left(-\frac{\pi p_\perp^2}{F} \right) \right) \right| \delta\left(w - w_0\right) v\,, \tag{14}$$

where F is the force acting on a parton (for the boost-invariant systems F depends only on τ and the color charge of a quark or a gluon), w_0 is the longitudinal momentum gained by a parton during the tunneling process [28, 29],

$$w_0 = -\frac{p_\perp^2}{2F}\,, \tag{15}$$

[1] We neglect here the finite-size effects in the pseudorapidity space taken into account in Ref. [28], since they have a negligible effect on the time evolution of the system.

and the plus/minus sign is connected with the statistics of the tunneling particles (plus for bosons and minus for fermions). Introducing the notation

$$\frac{dN}{d\Gamma} = \mathcal{R}(\tau, p_\perp)\delta\left(w \mp w_0\right) v\,, \tag{16}$$

we find that the transverse-momentum spectra of partons are given by the formula

$$\begin{aligned}\frac{dN}{dy\, d^2p_\perp} &= \int d^4x \frac{dN}{d\Gamma} = \pi R^2 \int\limits_0^\infty d\tau'\,\tau' \int\limits_{-\infty}^{+\infty} d\eta\, \mathcal{R}(\tau', p_\perp)\delta\left(w \mp w_0\right) v \\ &= \pi R^2 \int\limits_0^\infty d\tau'\,\tau'\,\mathcal{R}(\tau', p_\perp)\,,\end{aligned} \tag{17}$$

or more explicitly

$$\frac{dN}{dy\, d^2p_\perp} = \frac{R^2}{4\pi^2} \sum_{\text{all partons}} \int\limits_0^\infty d\tau'\,\tau'\,F(\tau') \left| \ln\left(1 \mp \exp\left(-\frac{\pi p_\perp^2}{F(\tau')}\right)\right)\right| . \tag{18}$$

Here we have introduced the sum over all tunneling partons, *i.e.*, quarks and gluons, and R is the transverse radius of our system. However, for simplicity of notation we skip the indices denoting different quantum numbers of partons. The explicit expressions for F and all other details are given in Ref. [28]. In the numerical calculations we use the value $\pi R^2 = 1\,\mathrm{fm}^2$, hence our results describe the production of partons per unit transverse area. Eq. (18) is the counterpart of Eq. (3) studied by Bialas.

We note that formula (18) may be alternatively obtained from the Cooper–Frye formalism outlined shortly in the Appendix. We also note that the transverse-momentum spectra have not been calculated so far in the formalism outlined in Section 2, only the mean $p_\perp$ was studied in Ref. [16].

4. Results

The starting point of our calculation is Eq. (18). The time dependence of the forces F is known (in the numerical form) from the studies performed in Ref. [28]. In practice, the integration range over τ' is always finite; the forces $F(\tau')$ are different from zero only at the initial stage of the evolution of the system ($\tau' < 1.5$ fm). The initial condition for the color field is obtained from the Gauss law

$$\boldsymbol{\mathcal{E}}_0 = \sqrt{\frac{2\sigma_g}{\pi R^2}}\, k\, \boldsymbol{\eta}_{12}\,. \tag{19}$$

Here the string tension $\sigma_g = 3\sigma_q = 3$ GeV/fm, and the number of color charges which span the initial field is denoted by k (note that $\boldsymbol{\eta}_{12} = (1,0)$).

In Figs. 1 and 2 we show our results obtained for $k = 2$ and $k = 3$. The transverse-momentum spectra are represented by the solid lines. In both cases one can observe that the spectra may be well approximated by the exponential function, especially in the soft region $100\,\mathrm{MeV} < p_\perp < 1\,\mathrm{GeV}$. Hence, the Schwinger tunneling mechanism in time-dependent fields indeed leads to the thermal-like spectra, although no rescattering processes are taken into account in this picture.

The dashed lines in Figs. 1 and 2 represent the exponential functions with the inverse-slope parameter T and the normalization fitted at $p_\perp =$ 350 MeV. The inverse-slope parameters are $T = 220$ MeV and $T = 270$ for $k = 2$ and $k = 3$, respectively. We thus see that larger initial fields lead to higher effective temperatures. This is already expected from Eq. (4), since larger initial fields lead to larger fluctuations (changes) of the field in time and, finally, to larger values of the parameter T. We may even try to apply Eq. (4) in our case, replacing the average value of κ^2 by the time average of F. In this way we find $T = 258$ MeV for $k = 2$ and $T = 307$ MeV for $k = 3$. As we can see, the rough estimate based on Eq. (4) gives the correct magnitude of T. In the case $k = 2$ we find the mean transverse momentum $\langle p_\perp \rangle = 376$ MeV and the rapidity density (per unit transverse area) dN/dy $= 1.2$. In the case $k = 3$ we find $\langle p_\perp \rangle = 426$ MeV and $dN/dy = 1.9$. These results are consistent with the earlier reported values [17].

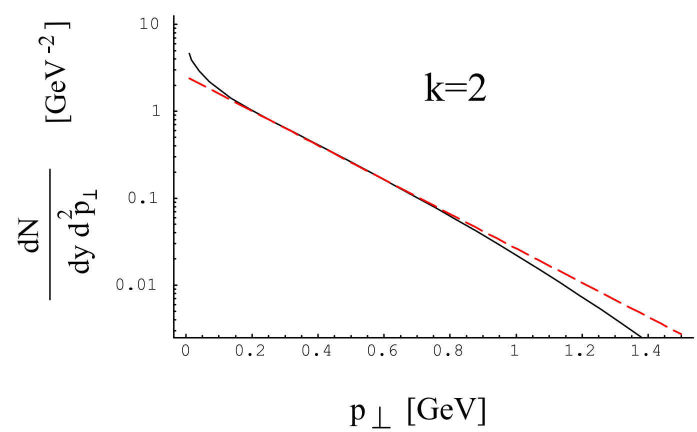

Fig. 1. The transverse-momentum spectrum of quarks and gluons obtained in the case $k = 2$ (solid line) and the exponential function const$\times \exp[-p_\perp/(220\mathrm{MeV})]$ (dashed line). The inverse slope parameter T was fitted at $p_\perp = 350$ MeV.

Another interesting feature of the spectra shown in Figs. 1 and 2 is the enhancement of the particle production above the thermal-like background

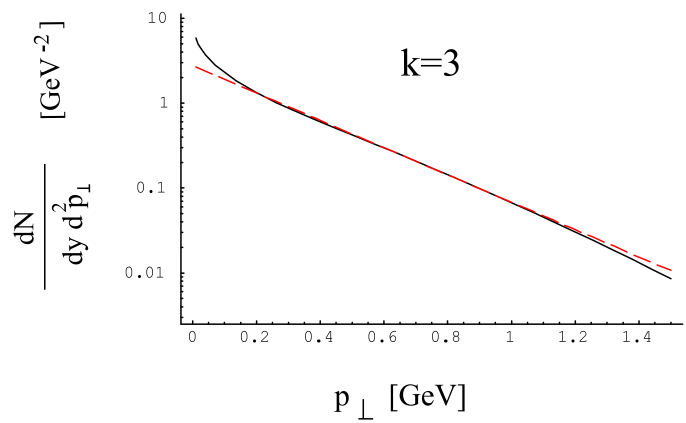

Fig. 2. The spectrum obtained in the case $k = 3$ (solid line) and the exponential function const$\times \exp[-p_\perp/(270\text{MeV})]$ (dashed line). The inverse slope parameter T was fitted also at $p_\perp = 350$ MeV.

in the very soft region $p_\perp < 100$ MeV. This type of the behavior is observed in the pion spectra measured by various experiments at CERN and RHIC, and is usually explained as the effect of the resonance decays which give contributions mainly in the low-$p_\perp$ region. The production of such very soft partons occurs in our model at later times when the forces F are small and the production of the particles with large $p_\perp$ is strongly suppressed. In other words, this may be treated as a phase-space effect – when the initial string breaks into many small pieces, only particles with small $p_\perp$ can tunnel and they contribute mainly to the low-$p_\perp$ peak. Nota bene, such type of the behavior was also found in the simulations of the sequential decays of the color-flux tubes [30].

We conclude that the Schwinger tunneling mechanism in strong varying fields offers an appealing explanation of a very fast formation of the thermal-like system in heavy-ion collisions.

Appendix A

Cooper–Frye formula

The transverse-momentum spectra may be calculated from the Cooper–Frye formula [31]

$$\frac{dN}{dy\, d^2p_\perp} = \int d\Sigma_\mu(x) p^\mu\, f(x,p)\,. \tag{A.1}$$

In Eq. (A.1) the quantity $d\Sigma_\mu(x)$ is the element of the freeze-out hypersurface and $f(x,p)$ denotes the phase-space distribution function. Assuming that the system is boost invariant in the longitudinal (z) direction and uniform in the transverse (x, y) directions, we may rewrite Eq. (A.1) in the form

$$\frac{dN}{dy\, d^2p_\perp} = \int dx\, dy\, d\eta\, v\, f(\tau, w, p_\perp)\,. \tag{A.2}$$

Here we used the property $d\Sigma_\mu = u_\mu \tau\, d\eta\, dx\, dy$, which follows from the condition that freeze-out occurs at a constant value of the invariant time τ. We also used the explicit form of the boost-invariant four-velocity, $u^\mu = (t, 0, 0, z)/\tau$, which gives $p^\mu u_\mu = v/\tau$. Now using the explicit form for the solutions of the kinetic equations (6) and (7) obtained in Ref. [28]

$$f(\tau, w, p_\perp) = \int\limits_0^\tau d\tau'\, \tau' \mathcal{R}(\tau', p_\perp) \delta\left(\Delta h(\tau, \tau') \pm w - w_0(\tau', p_\perp)\right) \tag{A.3}$$

we arrive at formula (18).

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