ERRATIC FLUCTUATIONS IN RAPIDITY GAPS IN RELATIVISTIC NUCLEUS–NUCLEUS COLLISIONS

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Fluctuations of the spatial pattern are investigated by analyzing $14.5\,A$ GeV/c ^{28}Si-nucleus interactions on event-by-event basis. For this the nearest neighbor rapidity spacings are analyzed. The two entropy like quantities, S_q and Σ_q, estimated from the two moments of the rapidity gap distributions G_q and H_q, are observed to deviate significantly from 1. This would indicate the presence of erratic nature of event-by-event fluctuations in rapidity gap distributions. The variations of $\ln S_q$ and $\ln \Sigma_q$ with q and their dependence on multiplicity of relativistic charged particles are investigated. A similar analysis is carried out for a Monte Carlo generated event sample using the event generator, Hijing 1.33. The results obtained for the simulated data are observed to compare well with the corresponding experimental values.

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1. Introduction

The method of scaled factorial moments, F_q [1] has been extensively used to investigate the fluctuations and chaotic behavior of multiparticle production in high energy e^+e^-, pp, p–nucleus (pA) and nucleus–nucleus (AA) collisions [2–6]. However, owing to the averaging procedure used in these studies, some interesting effects or information on the chaotic behavior, if any, in a part of an event, might be overlooked [7, 8]. To account for these fluctuations, a new method of analysis, referred to as the erraticity analysis, has been introduced [9] and successfully applied by some workers to explain multiparticle production phenomenon [8, 10–13]. In these investigations positive values for the entropy index, μ_q are found, indicating the presence of

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erraticity or the event-by-event(e-by-e) fluctuations in multiparticle system. However, it is not yet clear whether these fluctuations have some dynamical origin. It has, however, been pointed out [14, 15] that such a study is not very effective for analyzing the low multiplicity events, because when event multiplicity is small and the number of bins are large, then most of the non-empty bins would have only one particle and, therefore, only a few bins would have the particles, $n \geq q$, where q represents the order of moment and hence contribute to the event factorial moment F_q^e. To overcome such a limitation, a new method of analysis, based on the rapidity spacings between the neighboring particles, has recently been proposed [14]. This type of analysis is rather more suitable for studying the fluctuations and chaos when the event multiplicity is small. A few attempts have been made [15, 16] to study the erratic nature of fluctuations in multiparticle system using the method of rapidity gaps. It was, therefore, considered worthwhile to investigate the e-by-e fluctuations in 14.5 A GeV/c ^{28}Si-nucleus collisions.

2. Method of analysis

Method of erraticity analysis based on the rapidity gaps, has been presented in detail in Ref. [14]. However, a brief description about the technique is considered important and hence discussed here. It may be interesting to mention that single particle density distribution in pseudorapidity (η) space is non-flat. In order to have a flat distribution, a new cumulative variable, $X(\eta)$ has been introduced [17, 18], which is defined as:

$$X(\eta) = \frac{\int\limits_{\eta_{\min}}^{\eta} \rho(\eta) d(\eta)}{\int\limits_{\eta_{\min}}^{\eta_{\max}} \rho(\eta) d(\eta)}, \tag{1}$$

where, $\eta_{\min}$ and $\eta_{\max}$ denote respectively the minimum and maximum values of η interval considered, whereas $\rho(\eta)$ is the single particle pseudorapidity density. In $X(\eta)$ space, the charged particles are uniformly distributed between $0 \leq X(\eta) \leq 1.0$. For an event with multiplicity N, the nearest neighbor spacing is determined from:

$$x_i = X_{i+1} - X_i \quad i = 0, \ldots, N \tag{2}$$

with $X_0 = 0$ and $X_{N+1} = 1$, being the boundaries of the $X(\eta)$ space, *i.e.*, each event is described by a set, S_{e}, of $N + 1$ numbers such that $S_{\mathrm{e}} = \{x_i | i = 0, 1, \ldots, N\}$ such that $\sum_{i=0}^{N} x_i = 1$. For a given event S_{e} contains significant information and hence e-by-e fluctuations in S_{e} is expected to provide useful information regarding the multiparticle production mechanism. The existence of clusterization and correlation amongst the produced particles indicates that the particles are not uniformly distributed in the η space. Some of these will be rather closely spaced while some will be much farther apart [14, 15]. Thus, presence of small as well large gaps is expected in an event. A moment that emphasizes large gaps, may too provide quite important information about an event. One such moment is defined as

$$G_q = \frac{1}{N+1}\sum_{i=0}^{N} {x_i}^q\,, \tag{3}$$

such that $G_0 = 0$ and $G_1 = \frac{1}{N+1}$. As G_q fluctuates from event to event, it would have a distinct distribution for a given set of events. The shape of the G_q distribution would characterize the nature of e-by-e fluctuations in the rapidity gap distributions. A moment to quantify the degree of these fluctuation is defined as [15],

$$C_p^q = \frac{1}{N_{\rm evt}}\sum_{i=1}^{N_{\rm evt}} (G_q)^p\,. \tag{4}$$

As $C_1^q = \langle G_q\rangle$ gives no information about the degree of fluctuations, but the derivative at $p = 1$,

$$S_q = -\frac{d}{dp}C_p^q\Big|_{p=1} = -\langle G_q \ln G_q\rangle \tag{5}$$

is envisaged to yield maximum information regarding the e-by-e fluctuations. Unlike F_q, G_q moments, expressed by Eq. (3), do not filter out statistical fluctuations. The contribution of statistical fluctuations are, therefore, estimated by evaluating $G_q^{\rm st}$ and hence $S_q^{\rm st}$ from a correlation-free Monte Carlo (MC) event sample. Deviation in the ratio,

$$S_q = \frac{s_q}{s_q^{\rm st}} \tag{6}$$

from unity is regarded as a measure of erraticity of rapidity gaps in multiparticle production. Another moment, which like G_q, receives dominant contributions from the large gaps, is defined as

$$H_q = \frac{1}{N+1}\sum(1-x_i)^{-q}, \qquad\qquad N \geq q+1 \tag{7}$$

where N denotes the event multiplicity. Using the values of H_q, an entropy like quantity is defined as

$$\sigma_q = \langle H_q \ln H_q\rangle \tag{8}$$

while for measuring contribution from the statistical fluctuations, a similar quantity,

$$\sigma_q{}^{\mathrm{st}} = \langle H_q^{\mathrm{st}} \ln H_q^{\mathrm{st}} \rangle \tag{9}$$

is estimated from correlation-free MC data. Thus, a quantity Σ_q which is regarded as another measure of erraticity is calculated from:

$$\Sigma_q = \frac{\sigma_q}{\sigma_q^{\mathrm{st}}} \,. \tag{10}$$

3. Experimental details

A stack of G5 emulsion horizontally exposed to $14.5\,A$ GeV/c Silicon beam from BNL-AGS has been used. The events were scanned by along the track method, while the angles of the emitted relativistic charged particles, θ, were measured by using coordinate method. The events which were selected for the measurement, satisfy the following criteria:

1. The incident beam should not be inclined by an angle more than 3° with respect to the mean beam direction, so as to ensure that a particular event is caused by the genuine primary.

2. The events lying within 20 μm from the top and bottom surfaces of the emulsion pellicle were rejected.

In emulsion experiments [19], tracks of emitted particles are classified as black, grey and shower tracks on the basis of their ionization.
Black tracks: Ionization, I, produced by the particles forming black tracks is, $I \geq 10\,I_0$, where I_0 being the minimum ionization produced by a singly charged particle, the range of the tracks $\leq$ 3 mm and relative velocities, $\beta \leq 0.3$. They are usually the target fragments.
Grey particles: They are mostly recoiling target protons. The values of ionization produced by such tracks lie in the range $1.4\,I_0 \leq I \leq 10\,I_0$.
Relativistic particles: They are mostly pions having relative velocities, $\beta \geq 0.7$. The ionization produced by these particles satisfy the condition $I \leq 1.4\,I_0$.
By adopting these criteria, a sample of 505 events, having shower particle multiplicity, $N \geq 4$, are analyzed.
Hijing data: In order to compare the experimental results with the predictions of QCD-based theoretical models on multiparticle production, Monte Carlo event generator, Hijing 1.0 [20], is used to simulate 15000 ^{28}Si-emulsion interactions at $14.5\,A$ GeV/c; these events are also analyzed.
Correlation-free MC data: For estimating the magnitude of fluctuations due to the statistical reasons, a sample of correlation-free Monte Carlo (MC)

events (statistical data) corresponding to the experimental and the Hijing event samples are simulated by applying the following criteria:

(i) Multiplicity distribution of the simulated data sample should be similar to the experimental one,

(ii) for events with multiplicity N, $X(\eta)$ values of the N particles should be randomly distributed in the range 0–1, and

(iii) the emitted particles should be uncorrelated.

4. Experimental results

The η values of the relativistic charged particles produced in each event lying in the interval $\eta_0 \pm 3.0$, where η_0 is the central hadron–nucleon rapidity, are transformed into the variable $X(\eta)$ through Eq. (1) and rapidity gaps x_i between adjacent particles are estimated. Distribution of rapidity gaps between two adjacent particles, x_i for the experimental, Hijing and statistical data sets are displayed in Fig. 1. It is observed that distributions corresponding to the experimental and the statistical data samples acquire almost similar shapes, while that obtained for the Hijing data is somewhat narrower as compared to the other two distributions. This might be due to the higher value of mean multiplicity, $\langle N\rangle$, for the Hijing data as compared to the experimental one: The values of $\langle N\rangle$ for the experimental and Hijing data are respectively 20.26 ± 1.04 and 28.74 ± 0.15.

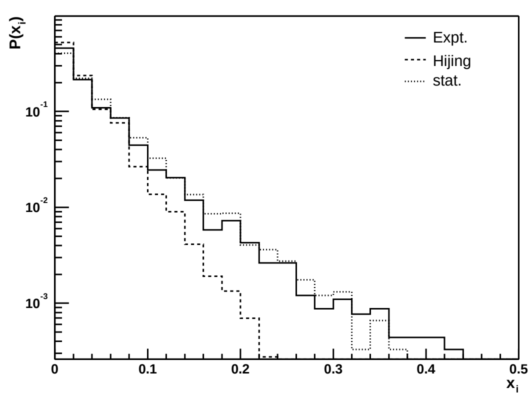

Fig. 1. Distributions of rapidity gaps, x_i for the three data samples.

Variations of $\ln S_q$ and $\ln \Sigma_q$ with the order of the moment q are shown for both the experimental and Hijing events in Figs. 2–3. Since mean multiplicity, $\langle N\rangle$, for the Hijing events is comparatively larger than those for the experimental data, the Hijing events having multiplicity, $N > 70$ are not considered in order to keep the value of $\langle N \rangle$ close to the experimental value. It may be interesting to note that the values of $\langle N \rangle$ for the experimental and Hijing data sets are found to be 20.26 ± 1.04 and 23.04 ± 0.17, respectively. For determining S_q and Σ_q for the Hijing events an equal number of correlation free MC events has been simulated by the criteria as discussed earlier.

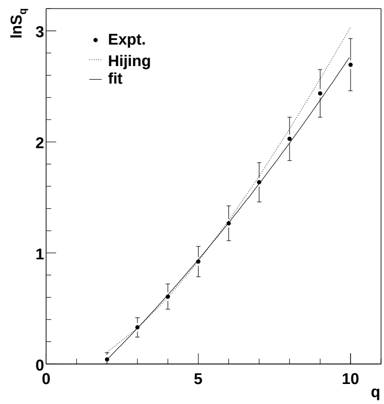

Fig. 2. Variation of $\ln S_q$ with q. Solid line is due to the 2nd order polynomial fit to the experimental data.

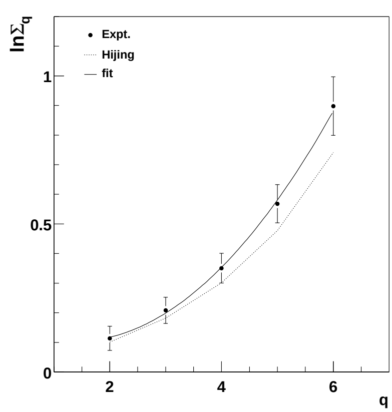

Fig. 3. Variation of $\ln \Sigma_q$ with q. Solid line is due to the 2nd order polynomial fit to the experimental data.

The following observations may be made from Figs. 2 and 3:

1. The values of both S_q and Σ_q deviate significantly from 1, revealing thereby the existence of erraticity behavior in multiparticle production in relativistic AA collisions.
2. Values of S_q for the Hijing data are slightly higher while that of Σ_q are somewhat smaller in comparison to the corresponding values for the experimental data. This difference might arise due to the difference in the mean multiplicities of relativistic charged particles for the two data sets.
3. The trends of variations of both $\ln S_q$ and $\ln \Sigma_q$ with q are nicely reproduced by the second order polynomial of the type

$$y = a + bq + cq^2\,; \quad y = \ln S_q\,, \quad \ln \Sigma_q\,. \tag{11}$$

The values of the parameters a, b and c obtained in the present study, are listed in Table I. Power law behavior of the type $S_q \sim q^{\alpha}$, as suggested in Ref. [14] is not observed in our case. Exponential behavior of the type S_q (or Σ_q) $\sim e^{\alpha q}$ may, however, be approximated for $q \geq 4$.

TABLE I

Values of parameters a, b, c and χ^2/D.F.obtained using Eq. (11)

	N	a	b	c	χ^2/D.F.
S_q	≥ 4	-0.47 ± 0.17	0.24 ± 0.08	0.08 ± 0.01	0.28
	≥ 10	-0.36 ± 0.17	0.20 ± 0.08	0.018 ± 0.01	0.22
	≥ 20	-0.44 ± 0.36	0.25 ± 0.15	0.02 ± 0.01	0.09
	≥ 30	-0.34 ± 0.25	0.20 ± 0.11	0.02 ± 0.01	0.09
Σ_q	≥ 4	-0.171 ± 0.200	-0.100 ± 0.118	0.036 ± 0.016	0.123
	≥ 10	0.027 ± 0.143	0.006 ± 0.079	0.008 ± 0.010	0.020
	≥ 20	0.16 ± 0.001	0.016 ± 0.001	0.0013 ± 0.0001	0.013
	≥ 30	0.0008 ± 0.0008	0.013 ± 0.001	0.0006 ± 0.0001	0.005

In order to investigate the dependence of S_q and Σ_q on the multiplicity of relativistic charged particles, the two parameters are calculated using the events characterized by, *(i)* $N > 10$, *(ii)* $N > 20$ and *(iii)* $N > 30$ for different values of q. The results are exhibited in Figs. 4–5. It is interesting to notice in the figures that S_q exhibits a weak dependence while Σ_q shows a strong dependence on the multiplicity of relativistic charged particles. The dependences of $\ln S_q$ and $\ln \Sigma_q$ on q for each group of data are nicely fitted by a 2nd order polynomial in q given by Eq. (11). The values of the constants a, b, c appearing in Eq. (11) obtained in the present study are given in Table I. It may be noticed that the two quantities, S_q and Σ_q, are different from unity, indicating thereby the presence of e-by-e erratic fluctuations in the rapidity gaps in both the experimental and Hijing data. It may be of interest to mention that in Ref. [14] it has been stressed that exact nature of the dependence of S_q and Σ_q on q has no physical significance, whatsoever. Hence, the conclusions arrived on the basis of the trends of variations of S_q and Σ_q with q would just help compare the results obtained for different types of data. However, an important conclusion drawn from such an analysis is that no binning is required and consequently, all the events are included in the analysis.

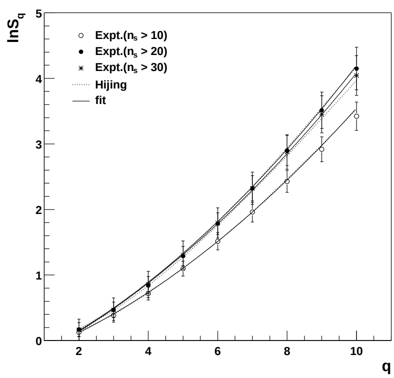

Fig. 4. $\ln S_q$ *versus* q for different n_s groups of events. Solid lines represent best fit to the experimental data.

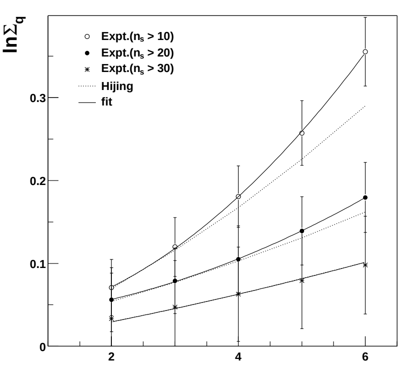

Fig. 5. Dependence of $\ln \Sigma_q$ on q for three n_s groups of events. Solid lines represent best fit to the experimental data.

5. Conclusions

Erraticity analysis using the method of rapidity spacings is carried out using the experimental data on 14.5 A GeV/c ^{28}Si-nucleus collisions and the findings are compared with those obtained for the Hijing simulated data. It is observed that S_q and Σ_q have values greater than 1, indicating that both the quantities are equally useful for studying the erratic nature of e-by-e fluctuations in rapidity gap distributions. The trends of the variations of S_q and Σ_q with q, seen in the present study, are somewhat different to that reported earlier [8, 15, 16]. The findings, however, give a positive indication for the occurrence of erratic nature of e-by-e fluctuations in rapidity gap distributions.

REFERENCES

[1] A. Bialas, R. Peschanski, *Nucl. Phys.* **B273**, 703 (1986).

[2] B. Buschbeck *et al.*, *Phys. Lett.* **B215**, 788 (1988); W. Braunschweig *et. al.*, *Phys. Lett.* **B231**, 548 (1989).

[3] I. Derado *et al.*, *Z. Phys.* **C47**, 23 (1990).

[4] I.V. Azhinenko *et al.*, (NA22 Collaboration), *Phys. Lett.* **B222**, 306 (1989).

[5] R. Holynski *et al.*, (KLM Collaboration), *Phys. Rev. Lett.* **62**, 733 (1989); M.I. Adamovich et.al., (EMU01 Collaboration), *Phys. Rev. Lett.* **263**, 539 (1991); *Nucl. Phys.* **B388**, 3 (1992).

[6] E.A. De Wolf *et al.*, *Phys. Rep.* **270**, 48 (1996).

[7] R.A. Janik, B. Ziaja, *Acta Phys. Pol. B* **30**, 259 (1999).

[8] D. Ghosh, *et al.*, *Phys. Rev. Lett.* **B540**, 52 (2002); *J. Phys. G* **29**, 2087 (2003); *Phys. Rev.* **C68**, 024808 (2003).

[9] R.C. Hwa, *Acta Phys. Pol. B* **27**, 1789 (1996).

[10] W. Shaoshun, W. Zhaomin, *Phys. Rev.* **D57**, 3036 (1998).

[11] W. Shaoshun, W. Chong, W. Zhaomin, *Phys. Lett.* **B458**, 505 (1999).

[12] M.R. Atayan *et al.*, (NA22 Collaboration), *Phys. Lett.* **B558**, 22 (2002).

[13] M.K. Ghosh, A. Mukhopadhyay, *Phys. Rev.* **C68**, 03490 (2003).

[14] R.C. Hwa, Q. Zhang, *Phys. Rev.* **D62**, 014003 (2000).

[15] W. Shaoshun, W. Chong, *Phys. Lett.* **B505**, 43 (2001).

[16] M.R. Atayan *et al.*, (EHS/NA22 Collaboration), *Phys. Lett.* **B558**, 29 (2002).

[17] A. Bialas, M. Gazdzicki, *Phys. Lett.* **B252**, 483 (1990).

[18] W. Ochs, *Z. Phys.* **C50**, 339 (1991).

[19] C.F. Powell, P.H. Fowler, D.H. Perkins, *The Study Elementary Particles by Photographic Methods*, Pergamon Press, 450 (1959).

[20] M. Gyulassy, Xin-Nian Wang, *Comput. Phys. Commun.* **83**, 307 (1994).