Vol. 35 (2004) ACTA PHYSICA POLONICA B No 2

# POSSIBLE SIGNAL FOR CRITICAL POINT IN HADRONIZATION PROCESS

M. Rybczyński[†], Z. Włodarczyk[‡]

Institute of Physics, Świętokrzyska Academy
Świętokrzyska 15, 25-406 Kielce, Poland

and G. Wilk[§]

A. Sołtan Institute of Nuclear Studies
Hoża 69, 00-681 Warszawa, Poland

*(Received December 29, 2003)*

We argue that recent data on fluctuations observed in heavy ion collisions show non-monotonic behaviour as function of number of participants (or "wounded nucleons") $N_{\rm W}$. When interpreted in thermodynamical approach this result can be associated with a possible structure occurring in the corresponding equation of state (EoS). This in turn could be further interpreted as due to the occurrence of some characteristic points (like *softest point* or *critical point*) of EoS discussed in the literature and therefore be regarded as a possible signal of the QGP formation in such collisions. We show, however, that the actual situation is still far from being clear and calls for more investigations of fluctuation phenomena in multiparticle production processes to be performed.

PACS numbers: 24.60.Ky, 25.75.Nq, 05.20.–y

## 1. Introduction

The primary goal of all high energy heavy ion experiments is to find the phase transition between hadronic matter and quark–gluon plasma (QGP) as predicted by the lattice QCD[1]. It is natural then to look at experimental results from such perspective and to scrutinize them for the possible signals of QGP formation. In this work we would like to concentrate on one characteristic feature, which could signal such phase transition, namely it

[†] e-mail: `mryb@pu.kielce.pl`

[‡] e-mail: `wlod@pu.kielce.pl`

[§] e-mail: `wilk@fuw.edu.pl`

[1] For relevant references see, for example, proceedings on recent QM conference [1].

should be accompanied by enhanced fluctuations in some variables [2–5]. For it turns out that, when measuring in final states only hadronic probes produced with small transverse momenta, the only observables surviving the possible phase transition (and, in principle, actually depending on its form) is the observed fluctuation pattern in properly selected variables. Following [2] we shall be interested in the quantity being the ratio of fluctuations in entropy to fluctuations in energy. The reason is that, in thermodynamical approach to be pursued here, both quantities are well defined for any form of matter (confined, mixed and deconfined) both in early stage of collisions and in the final state of the hadronizing system under consideration. When produced matter can be treated as an isolated system, energy is conserved and entropy is also expected to be conserved during the expansion and freezeout stages of the hadronization. Therefore simultaneous measurements of both quantities are likely to provide us with information on the equation of state (EoS) of the hadronizing system. This ratio can be written (see Appendix A for details) as

$$R=\frac{\left(\frac{\delta S}{S}\right)^2}{\left(\frac{\delta E}{E}\right)^2}=\left(1+\frac{\alpha}{1+\alpha\phi}\right)^{-2}, \qquad \text{where} \qquad \phi=\frac{d\ln V}{d\ln T}=\frac{T}{V}\frac{dV}{dT}\,. \quad (1)$$

Here $\alpha = dp/d\varepsilon = p/\varepsilon$ characterizes EoS, $p$ denotes the pressure and $\varepsilon = E/V$ energy density with $V$ being the volume of our system and $T$ its temperature (statistical equilibrium is assumed). The entropy of the system, $S$, should be connected with the multiplicity of the produced secondaries, $S \sim N$ and, similarly, energy released in the production process, $E$, should be connected with the measured sum of transverse momenta, $E \sim \sum_{i=1}^{N} p_{\rm T}$. When variations of the reaction volume (identified with the volume of system) can be neglected, *i.e.*, for $\phi = 0$, one gets formula proposed in [2] as the right quantity the energy dependence of which should be investigated experimentally[2]:

$$R=\frac{\frac{(\delta S)^2}{S^2}}{\frac{(\delta E)^2}{E^2}}=\left(1+\frac{p}{\varepsilon}\right)^{-2}=\frac{1}{(1+\alpha)^2}\,. \quad (2)$$

Notice that $R$ is directly sensitive to the equation of state (EoS) of hadronizing matter because of occurrence of parameter $\alpha$. In what follows we shall assume that

$$\frac{\delta S}{S}=\frac{\delta N}{N} \quad \text{and} \quad \frac{\delta E}{E}=\frac{\delta\sum p_{\rm T}}{\langle\sum p_{\rm T}\rangle}\,. \quad (3)$$

We identify throughout this work $\delta N$ and $\delta\sum p_{\rm T}$ with fluctuations caused by *all possible sources* (as only such are accessible in experiment), *i.e.*, also

[2] For recent reviews on this matter see [6].

all our results concerning different estimations of $p/\varepsilon$ by means of Eq. (2) include influences of all possible sources of fluctuations. In [2] $R$ has been investigated by using some specific statistical model of nuclear collisions. The predicted very characteristic shape of energy dependence of this quantity (non-monotonic, with a single "shark fin"-like maximum) was then proposed as a possible signal of QGP phase transition which should therefore be subjected to future experimental tests [6].

In this work we would like to bring attention to the fact that apparently similar kind of fluctuations (but in function of $N_{\rm W}$, which can be connected with the reaction volume, rather than energy, as is the case in [2]) have been already observed and reported in [8] and [9]. In what follows, using some minimal input, we shall in the next section rewrite results of [8, 9] in terms of variable $R$ defined by (2) and (3), clearly demonstrating that it has similar non-monotonic character as that obtained in [2]. It is therefore tempting to argue that these data show us either what has been called in the literature the *softest point* of the corresponding EoS [10] or what is named in other investigations the *critical point* of EoS [11]. However, at this stage one not only cannot distinguish between both possibilities but even one cannot take them very seriously, for when confronting these data with similar data obtained recently in [13], which use different measure of fluctuations, one discovers that no such feature are seen there[3]. Still, there are some other possibilities offered recently, as, for example, spinodial decomposition occuring at the hadronization stage [12], which deserve also attention.

In Section 3 we shall also analyse (with the same aim as above, *i.e.*, looking for a possible signal of QGP) recent interferometric data, which provide us information on the interaction volume [15][4]. It turns out that they lead to similar results for EoS as data from [8,9] (albeit this time at different energy). The last section contains our summary whereas Appendices contain derivation of our basic formulas.

## 2. Nonmonotonical dependences observed in data on fluctuations

In [8] the following measure of fluctuations has been presented as a function of the number of participants $N_{\rm W}$ (or "wounded" nucleons)[5]:

$$F_{\rm T} = \frac{\Omega_{\rm data} - \Omega_{\rm random}}{\Omega_{\rm random}}; \quad \Omega = \frac{\sqrt{\langle \bar{p}_{\rm T}^2\rangle - \langle \bar{p}_{\rm T}\rangle^2}}{\langle \bar{p}_{\rm T}\rangle}\,. \tag{4}$$

[3] Ref. [13] provides data for parameter $\Phi$ (defined in [14]) at energies 40, 80 and 158 $A$ GeV for the following centrality bins: 0–5%, 0–6.5%, 5–10%, 10–15% and 15–20%.

[4] The volume $V$ measured here is the freeze-out volume but in our discussion we identify it with the production volume.

[5] For proper definition of different types of averages and mean values used here see [16].

It increases with $N_{\rm W}$, reaches maximum at $N_{\rm W} \sim 200$, and decreases for higher values of $N_{\rm W}$. Whereas in [7] such behaviour has been attributed to the peculiar feature of the hadronization model used, namely to the percolation of hadronizing strings produced in collision process[6], here we shall connect it directly, in a similar way as in [2], to the behaviour of EoS of the matter produced at the early stage of the collision. This can be done by rewriting $F_{\rm T}$ in terms of $R$ defined in Eq. (2). To do so let us first notice that, because

$$\frac{{\rm Var}\left(\sum p_{\rm T}\right)}{\langle \sum p_{\rm T}\rangle^2} = \frac{{\rm Var}\left(p_{\rm T}\right)}{\langle p_{\rm T}\rangle^2}\frac{1}{\langle N\rangle} + \frac{{\rm Var}(N)}{\langle N\rangle^2}\,, \tag{5}$$

one can write $R$ as

$$R = \frac{\frac{{\rm Var}(N)}{\langle N\rangle^2}}{\frac{{\rm Var}(\sum p_{\rm T})}{\langle \sum p_{\rm T}\rangle^2}} = \frac{1}{1+\frac{1}{r}}\,, \tag{6}$$

where

$$r = \frac{1}{\omega}\frac{{\rm Var}(N)}{\langle N\rangle} \quad \text{and} \quad \omega = \frac{{\rm Var}(p_{\rm T})}{\langle p_{\rm T}\rangle^2}\,. \tag{7}$$

On the other hand, as shown in Appendix B, $F_{\rm T}$ can be expressed in terms of $r$ and parameter $\rho$,

$$F_{\rm T} = \sqrt{1+2r-2\rho\sqrt{r^2+r}} - 1\,, \tag{8}$$

where $\rho \in [-1,1]$, the correlation coefficient between number of particles $N$ and $\sum p_{\rm T} = \sum_{i=1}^{N} p_{\rm T}$, is our minimal input mentioned above[7]. Using (6) it is now straightforward to rewrite Eq. (8) in the following form:

$$\frac{(1+F_{\rm T})^2-1}{2} = \frac{R}{1-R}\left[1-\frac{\rho}{\sqrt{R}}\right]\,, \tag{9}$$

from which our main result follows:

$$R = \frac{\rho^2+2y(1+y) \pm \rho\cdot\sqrt{\rho^2+4y(1+y)}}{2(1+y)^2}\,, \tag{10}$$

[6] Without introducing notion of EoS or phase transitions, unless phenomenon of percolation itself is regarded as a kind of phase transition. But even then it would be phase transition between bigger and smaller number of strings only, not between hadronic matter and quark–gluon plasma.

[7] It appears because, whereas $R$ depends on fluctuations in $N$ *and* on fluctuations in $\sum_{i=1}^{N} p_{\rm T}$, so far in experiment one measures quantities like $\Phi$ or $F_{\rm T}$ in which both type of fluctuations occur together. Parameter $\rho$ describes therefore their mutual (unknown *a priori*) correlations.

where

$$y = \frac{1}{2}\left[(1+F_{\mathrm{T}})^2 - 1\right]. \tag{11}$$

In addition to the *a priori* unknown correlation coefficient $\rho$ there is also some freedom in the choice of sign in (10). To fix both of them let us notice the following:

*(a)* It is known that purely statistical or broader fluctuations encountered in all multiparticle production processes, *i.e.*, the fact that $\mathrm{Var}(N) \geq \langle N\rangle$, lead to the condition that[8]

$$\frac{\mathrm{Var}(N)}{\langle N\rangle} = \frac{\omega}{\frac{1}{R} - 1} \geq 1\,. \tag{12}$$

*(b)* Furthermore, analysis of recent CERES data [13] shows clearly, see Fig. 4 below, that parameter $\omega$ is very slowly varying function of energy and multiplicity (essentially $\omega \simeq 0.43$). This practical constancy of fluctuations in $p_{\mathrm{T}}$ means that fluctuations of energy $E$, which are given by fluctuations of $\sum p_{\mathrm{T}}$, is not so much given by fluctuations in $p_{\mathrm{T}}$ but by fluctuations in multiplicity $N$, *cf.*, Eq. (5).

These two observations mean therefore that for the multiplicity distributions of the poissonian type and broader, $R$ is limited from below:

$$R > \frac{1}{1+\omega} > 0.699 \tag{13}$$

(for $\omega = 0.43$). Using this limit together with Eq. (10) one obtains that also $\rho^2$ is limited from below, namely

$$\rho^2 > \frac{(\omega y - 1)^2}{\omega + 1}\,. \tag{14}$$

Because, without any additional correlations between $x_i$ and $N$, the $\sum_{i=1}^{N} x_i$ increases with $N$, the positive correlation coefficient, $\rho > 0$, seems to be the only natural choice. In addition, results of PHENIX [8] show that $F_{\mathrm{T}}$ is very small (actually not exceeding $F_{\mathrm{T}} \sim 0.04$). Neglecting therefore term with $y$ in (14) one can estimate that $\rho > 0.83$ (in our calculations we have put it slightly above this limit using value $\rho = 0.85$). It means that $R$ corresponds to a stronger than poissonian fluctuations. To be consistent with limitations on $R$ imposed by (13) we shall choose to the solution with positive sign in (10) (the other one would lead to unacceptable small values of $R$).

[8] See, for example, [18]. Multiplicity distributions broader than poissonian distributions are also observed in hadronic collisions [19].

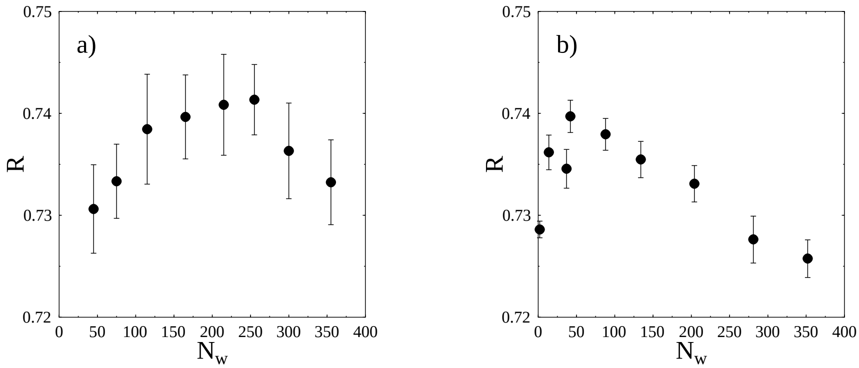


Fig. 1. Left panel: results of transforming PHENIX data from [8] (*i.e.*, $F_{\mathrm{T}}$ *versus* $N_{\mathrm{W}}$) to $R$ as function of the number of participant (or number of "wounded") nucleons, $N_{\mathrm{W}}$ by using Eq. (10). As explained in the text the positive sign in (10) has been chosen and the correlation coefficient set to $\rho = 0.85$ (see text for details). Right panel: $R$ obtained from $\Phi$ as measured by NA49 [9] (for the same value of $\rho$).

We are now prepared to translate, by using Eq. (10), the experimental knowledge of $F_{\mathrm{T}}$ and $\Phi$ as a function of number of participants provided in [8, 9] into similar dependence of $R$. The results are presented in Fig. 1. The only unknown feature there is the choice of parameter $\rho$. In fact for $\rho =$ const. different values result in essentially the same shape of $R$, only shifted accordingly paralelly up or down. The case of $\rho = \rho(N_{\mathrm{W}})$ would lead to some changes, however. Unfortunately at the moment there are no data available to estimate the functional form of $\rho(N_{\mathrm{W}})$. We expect, however, that our choice of $\rho$ used here corresponds to a lower limit for a possible effect observed in Fig. 1. To substantiate this we present in Fig. 2 parameter $R$ as function of $N_{\mathrm{W}}$ calculated directly from definition (6), *i.e.*, using directly measured information on $\mathrm{Var}(N)/\langle N\rangle$ and $\mathrm{Var}(p_{\mathrm{T}})/\langle p_{\mathrm{T}}\rangle^2$ obtained by [9][9].

So far we have not yet used connection of $R$ with EoS variables $p$ and $\varepsilon$ as given by Eq. (2). According to it we can interpret the peculiar shape of curve obtained in Fig. 1 in terms of the type of EoS admitting it, in particular as due to a specific behaviour of $p/\epsilon$. Following therefore discussion in [2] we

[9] This could be regarded as a possible suggestions for experimentalists that everything needed for such discussion as presented here is measurable directly, without resorting to quantities like $F_{\mathrm{T}}$ or $\Phi$, see also [17] for similar conclusions obtained at different circumstances. However, care must be exercised when following such approach because usually bins in centrality are large and one can expect therefore contributions to fluctuations coming from fluctuations in read-offs of the ZDC calorimeter, which for a time being are not known.

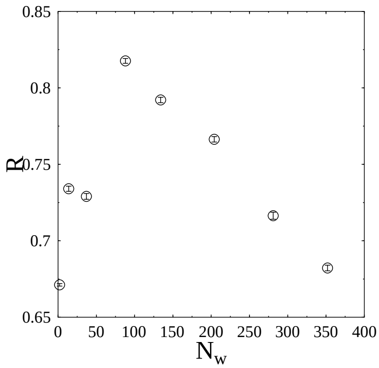


Fig. 2. The expected shape of $R$ as function of $N_{\mathrm{W}}$ this time given directly by Eq. (6) and data [9]; no parameter $\rho$ enters here.

argue that this shape could be regarded as a signal of the existence of either the *softest point* of the corresponding EoS [10] or its *critical point* [11]. This time it would show up as function of the number of participants (which can be translated into a volume of the reaction) rather than energy, as discussed in [2].

Actually, because of relation between $F_{\mathrm{T}}$ and measure $\Phi$ of fluctuations [14] (see (B.9) in Appendix B), measurements of variable $\Phi$ is equally good for the kind of analysis performed here. This is clearly seen in Fig. 1(b) and 2 using NA49 data [9] and in Fig. 3 where recent data on $\Phi$ (*cf.*, [13]) were used as our input[10]. In Fig. 4 we show that data from [13] clearly indicate that, as was already mentioned before, fluctuations of transverse momentum defined by variable $\omega$ introduced in Eq. (7) depend very weakly both on the energy and on the centrality of the collision (*i.e.*, on the number of struck nucleons $N_{\mathrm{W}}$). Therefore fluctuations of transverse momenta are practically irrelevant for the problem considered here, *i.e.*, for the apparent structure seen in the EoS. Main effect is provided by fluctuations of multiplicity $N$, as presented by Eq. (6).

Let us close this Section with the following remark. Because fluctuations in $\sum p_{\mathrm{T}}$ are tightly connected with fluctuation of temperature $T$, therefore one can write that $\frac{\delta E}{E} = \frac{\delta T}{T}$ and express parameter $R$ in yet another form:

$$R = \frac{\frac{\mathrm{Var}(N)}{\langle N\rangle^2}}{\frac{\mathrm{Var}(T)}{T^2}}\,. \tag{15}$$

[10] Notice that in both cases the lack of fluctuations in $N$, *i.e.*, $\mathrm{Var}(N) = 0$ would immediately result in $\Phi = F_{\mathrm{T}} = 0$. Lack of fluctuations means that also $\rho = 0$ what (together with $y = 0$, *cf.* (11)) results in $R = 0$ as well.

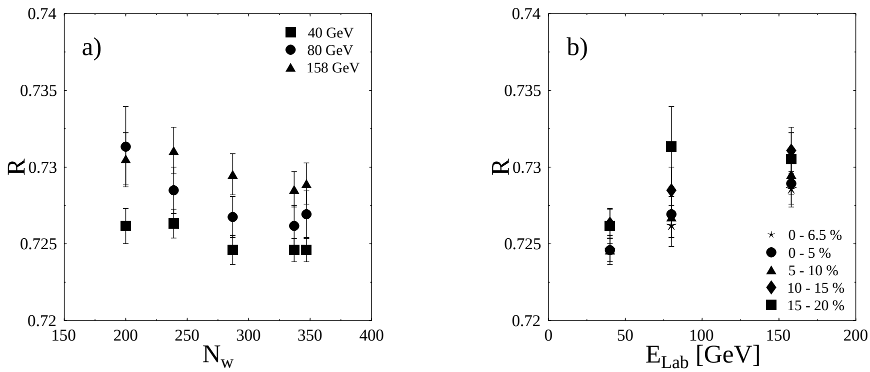


Fig. 3. The same as in Fig. 1(b) but this time using recent data on $\Phi$ parameter obtained by CERES Collaboration [13]. The results are presented both as function of participants (a) and as function of energy (b).

As shown in [20][11] relative fluctuations of temperature $T$ are connected with the heath capacity and can be parameterized by the nonextensivity parameter $q$,

$$\frac{\mathrm{Var}(T)}{T^2} = \frac{N_{\mathrm{W}}}{C_V} = q - 1\,, \tag{16}$$

which in our case is given by

$$q - 1 = \frac{1}{\langle N \rangle}\left[\omega + \frac{\mathrm{Var}(N)}{\langle N \rangle}\right]\,. \tag{17}$$

Notice that results shown in Fig. 4, *i.e.*, that essentially $\omega = \mathrm{const.}$, means that $q$ depends only on the multiplicity (or on the number of struck nucleons), in fact $q-1$ diminishes as $1/\langle N \rangle$. Parameter $q$ allows us then (by using Eq. (7)) to express parameter $R$ in the form stressing its vital dependence on fluctuations of multiplicity, *i.e.*, on $\mathrm{Var}(N)/\langle N \rangle$:

$$R = \frac{1}{q-1}\frac{\mathrm{Var}(N)}{\langle N \rangle^2}\,. \tag{18}$$

As is well known these fluctuations lead to deviations from the poissonian form of multiplicity distributions in multiparticle production processes and result in broader distributions usually expressed by the so called Negative Binomial (NB) form and characterized by the parameter $k$ [12], which in our

[11] For other hints on nonextensivity in hadronic production processes and references to nonextensive statistics, see [21].

[12] *Cf.* [19] for details. Connection between NB and nonextensivity in hadronic collisions represented by $q > 1$ has been discussed in [22].

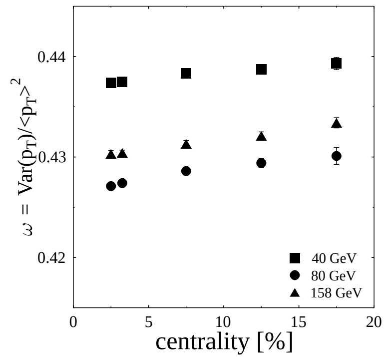


Fig. 4. Fluctuations of the transverse momentum as defined in Eq. (7) as function of centrality and for different energies (data are from [13], they are presented before removal of short range correlations).

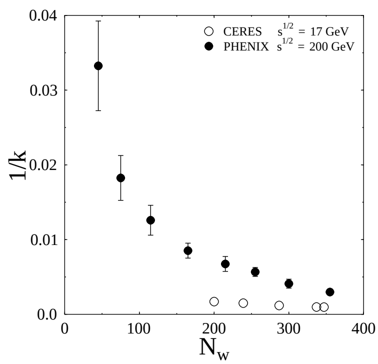


Fig. 5. Dependence of the NB [19] multiplicity distribution parameter $1/k$ on the number of participants $N_{\rm W}$ for PHENIX [8] and CERES [13] data.

case is given by:

$$\frac{1}{k} = \frac{\mathrm{Var}(N)}{\langle N\rangle^2} - \frac{1}{\langle N\rangle} = R(q-1) - \frac{1}{\langle N\rangle}\,. \tag{19}$$

Its dependence on $N_{\rm W}$ for PHENIX data (at $\sqrt{s} = 200$ GeV) and for CERES data (at $\sqrt{s} = 17$ GeV) is shown in Fig. 5. As seen there parameter $1/k$ decreases with increasing number of participants. It means that (*cf.* [19]) with increasing centrality fluctuations of the multiplicity become weaker and the respective multiplicity distributions approach poissonian form. Notice that because $R < 1$ the following condition,

$$\frac{1}{k} + \frac{1}{\langle N\rangle} < q - 1\,, \tag{20}$$

must be satisfied.

## 3. Nonmonotonicity and phase transition

Let us now continue discussion of the parameter $R$ from the point of view of its possible connection with the phase transition. Using known thermodynamical identities [23] one can rewrite $R$ (see Appendix C, all derivatives are for $T =$ const. and $p = \alpha\varepsilon$) as

$$R = \frac{\left(\frac{\partial \ln V}{\partial \ln E}\right)^2}{1 - C_V T \frac{\alpha}{E} \frac{\partial \ln V}{\partial \ln E}} \, . \tag{21}$$

Notice that because in the vicinity of critical point $(\partial p/\partial V)_T \to 0$ the parameter $R$ reaches its maximum there:

$$R \to \frac{\varepsilon^2}{\left(\frac{\partial E}{\partial V}\right)_T^2} = \left(\frac{\partial \ln V}{\partial \ln E}\right)_T^2 . \tag{22}$$

Let us discuss Eq. (21) in more detail concentrating on two cases important for us. First of all notice that for fixed energy $\sqrt{s}$ we have $\xi dE/E = dV/V = dN_{\rm W}/N_{\rm W}$ and with increasing $N_{\rm W}$ the value of $R$ decreases for $\xi = 1$ and increases for $\xi = -1$. It means then that for fixed $N_{\rm W}$ parameter $R$ goes through its maximum when $\partial \ln V/\partial \ln E$ changes sign. Actually exactly such change of sign is observed in compilation of heavy ion data on central collisions (for which $N_{\rm W}$ remains approximately fixed) [15], see Fig. 6(a), from which one can deduce that

$$\frac{\partial \ln V}{\partial \ln E} = \begin{cases} c_1 = -0.73 & \text{for} \quad s < s_c \\ c_2 = +0.15 & \text{for} \quad s > s_c \end{cases} . \tag{23}$$

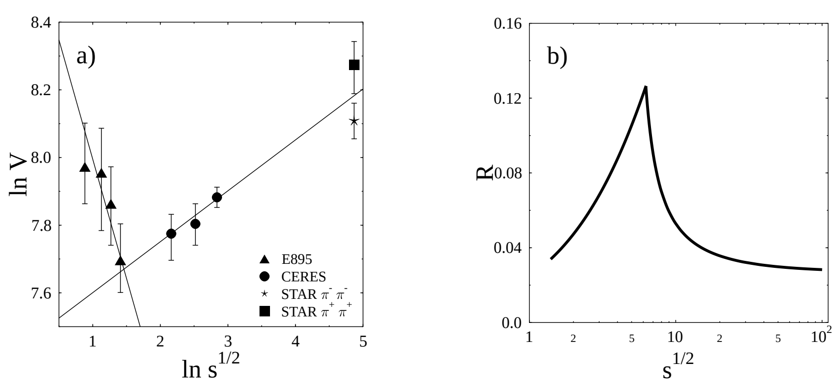


Fig. 6. (a) Our fits to data on volume of the interaction region $V$ for different energies [15]; (b) the corresponding $R(\sqrt{s})$ dependence.

Using now these values together with Eq. (21) we have obtained dependence of $R$ on energy $\sqrt{s}$, *cf.* Fig. 6(b). The parameters used were such that

$$\frac{N_{\rm W}\alpha T}{q-1}\frac{2}{N_{\rm W}} = \frac{2\alpha T}{q-1} = 30$$

what corresponds to the reasonable values of $\alpha = 1/3$, $T = 0.14$ GeV and $q - 1 \approx 1/k = 0.003$ [13]. Notice that $R$ shown there reaches its maximum for energy $\sqrt{s_c} \simeq 6$ GeV. This "critical" value of energy is given by

$$\sqrt{s_c} = \frac{\alpha T}{q-1}\frac{2}{N_{\rm W}}\frac{c_1 c_2}{c_1 + c_2}\,, \tag{24}$$

where $c_i$ denote the corresponding values of derivatives $\partial \ln V/\partial \ln E$ as given in (23). Actually, although the shape of $R$ in Fig. 6(b) resembles that expected in [2], the height of its maximum is much smaller than expected there (to get the value observed in [2] one would have to shift it upward by $\sim 0.7$). The only possible explanation we could offer at the moment is the observation that $R$ displayed in Fig. 6(b) does not contain contribution from statistical fluctuations of multiplicity (which would give precisely the seek for value of 0.69 for the poissonian fluctuations). It would then mean that what we are calculating here are only changes in $V$. For $V =$ const. we are therefore getting $R = 0$. On top of that one has poissonian fluctuations, which for $V =$ const. provide, as mentioned above, the lacking $\sim 0.7$ on top of what is observed. In obtaining it we have increased fluctuations of multiplicity $N$ by poissonian component (adding to $\mathrm{Var}(N)$ the value of $\langle N\rangle$ for constant volume $V$), *i.e.*, we have used here $R'$ instead of $R$ as our input information, where $1/(1-R') = 1/\omega + 1/(1-R)$. It is worth to notice that recent lattice calculations show behaviour of $R$ similar to obtained by us here [11].

## 4. Summary and conclusions

We have shown that the recently proposed method of (almost) direct investigations of the EoS, especially finding a possible traces of phase transitions to QGP phase of matter [2] by analyzing energy dependence of some specific fluctuations observed in the multiparticle production data obtained in heavy ion collisions, can be further extended to include also fluctuations of different types than those proposed in [2]. Three examples were discussed here: (*i*) recent PHENIX Collaboration data [8] on $F_{\rm T}$, (*ii*) recent NA49 Collaboration data [9] on $\Phi$, and (*iii*) CERES Collaboration data [13, 15] on $\Phi$. In particular, we have derived formula (our Eq. (10)) expressing parameter $R$ introduced in [2] by the measured quantity $F_{\rm T}$ (defined in (4)

[13] One should notice that in Eq. (21) we have total energy: $E = \sqrt{s}N_{\rm W}/2$. This is the origin of appearance of $N_{\rm W}$ in presenting $R$ as function of $\sqrt{s}$.

and measured in [8]). Our results (see Fig. 1(a)) show that one observes similar characteristic structure in $R$ as that expected in [2] but now being present at given fixed energy and for different centralities (expressed by the number of participants $N_{\rm W}$). As is seen in Fig. 3 and Fig. 1(b), the same type of analysis can be performed using as input the so called $\Phi$ measure of fluctuations as provided by recent NA49 [9] and CERES data [13]. We would like to stress here that because, as is witnessed by results shown in Fig. 4, fluctuations in $p_{\rm T}$ are rather irrelevant here, our results are almost entirely due to the fluctuations in multiplicity. It means therefore that they are not sensitive to flow phenomena.

We have also established connection of $R$ with fluctuations of temperature $T$ described by nonextensivity parameter $q$ [20]. The fact that data [15] clearly indicate that in the measured centrality range fluctuations in transverse momentum are essentially constant, see Fig. 4, allows us to express parameter $R$ by combination of fluctuations of $T$ (*i.e.*, by parameter $q$) and fluctuations of multiplicity as given by the parameter $1/k$ of NB distribution [19], see Eq. (19) and Fig. 5.

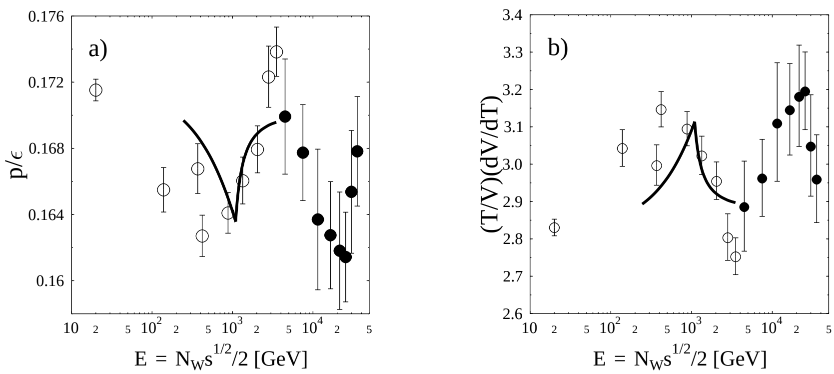


Fig. 7. EoS corresponding to results obtained in Fig. 6(b)(solid line) and to those obtained from Fig. 1 (points). Open points correspond to data from NA49 [9] and full points to data from PHENIX [8]. See text for details.

Finally, using some thermodynamical identities, we have expressed $R$ in terms of parameter $\alpha = p/\varepsilon$ defining form of EoS used and energy dependence of the reaction volume, *cf.* Eq. (21). Using recent data on the later [15] (see Fig. 6(a)) we have obtained the characteristic "shark fin" structure of the energy dependence of the parameter $R$ shown in Fig. 6(b). It corresponds to the shape of EoS as given in Fig. 7. Such structure of $R$ was predicted at [2] (albeit for $\sqrt{s}$ replaced by the so called Fermi collision energy measure) as reaching value of $R \sim 0.8$ at its maximum. In our case the maximum is much lower. Actually, what is shown in Fig. 7 are two situations, which can emerge from definition of $R$ as given in Eq. (1). Namely, as one can notice variation of $R$ can originate either in variations of $p/\varepsilon$ or in variations $dV/dT$ (in extremal cases). For $dV/dT = 0$ we are then getting from data $p/\varepsilon$, and this is the case of Fig. 7(a). On the other hand, for $p/\varepsilon = 1/3$, we are getting $dV/dT$, as in the Fig. 7(b).

We would like to close with the following remark. Our result presented in Fig. 7 shows not one but two "soft points" or "critical points" of EoS (so far we cannot distinguish here between these two possibilities), located at different energies: one at $\sqrt{s} \sim 6$ GeV (obtained from data on $V$ at different energies [15] and from data on $\Phi$ for different centralities [9]) and another one at $\sqrt{s} \sim 100$ GeV (obtained from data on fluctuations measured by quantity $F_{\mathrm{T}}$ for different centralities [8]). We cannot so far offer explanation of this result. Interpreting it, however, in the original spirit of looking for the possible signals of QGP (but keeping in mind our reservation concerning this point expressed above) we could say that it looks like, with increasing energy, first occurs a kind of QGP composed mostly of dressed quarks, which at higher energies is followed by a true QGP containing also liberated gluons[14]. What we see here most probably indicates some additional change in energy dependence of $V$ presented in Fig. 6(a), which would take place at $\sqrt{s} \sim$ 100 GeV. If true it would indicate that $V$ is not increasing with energy since that point, on the contrary, it probably decreases a little (what seems to be confirmed by the first data from RHIC at 200 GeV [25]). One should also keep in mind, before further speculations, that the assumed here access to the EoS by use of fluctuations in hadronic production neglects altogether the possible evolution processes between the QGP and state of freely streaming hadrons observed experimentally. This remark applies also to all works of this kind, like [2]. It means therefore that final conclusions could only be drawn when other approaches, as for example that discussed in [7], are also confronted with all available data, as has been done here.

Partial support of the Polish State Committee for Scientific Research (KBN) (grant 2P03B04123 (MR and ZW) and grants 621/E-78/SPUB/ CERN/P-03/DZ4/99 and 3P03B05724 (GW)) is acknowledged.

## Appendix A

We shall, for completeness, derive here formula (1). We shall consider in what follows our hadronizing system as statistical system in equilibrium without specifying its details. Thermodynamical potential of such system is

$$\Omega = E + pV - TS = \mu N\,. \tag{A.1}$$

[14] Notice that data from [15] indicate that volume of interaction grow much slower than linearly with energy, *i.e.*, the energy densities obtained at different energies increase substantially: $\varepsilon \sim 0.1 s^{0.41}$.

It is then connected with chemical potential $\mu$ with $N$ denoting number of particles in the system under consideration. For system in equilibrium but with varying number of particles $N$ is given by condition $d\Omega/dN = 0$ or, equivalently, $\mu = 0$. For conditions of statistical equilibrium we have then (here $\varepsilon$ and $s$ are the corresponding densities of energy and entropy)

$$\varepsilon + p - Ts = 0\,. \tag{A.2}$$

Using (A.2) together with thermodynamic identity

$$dE = -pdV + TS \tag{A.3}$$

we get

$$d\varepsilon + dp = Tds + sdT\,. \tag{A.4}$$

Denoting $\alpha = c_0^2 = dp/d\varepsilon = p/\varepsilon$ we obtain from (A.2) and (A.4)

$$\frac{ds}{s} = \frac{1}{\alpha}\frac{dT}{T} \qquad \text{and} \qquad d\varepsilon = \frac{s}{\alpha}dT\,. \tag{A.5}$$

Because from (A.2) we get $\varepsilon = Ts - p = T\alpha(d\varepsilon/dT) - \alpha\varepsilon$, then

$$\frac{d\varepsilon}{\varepsilon} = \frac{1+\alpha}{\alpha}\frac{dT}{T}\,. \tag{A.6}$$

From equations (A.5) and (A.6) one gets that ratio of fluctuations of *densities* is given by

$$R = \frac{(ds/s)^2}{d\varepsilon/\varepsilon)^2} = \frac{1}{(1+\alpha)^2}\,. \tag{A.7}$$

Because $S = sV$ and $E = \varepsilon V$ and, respectively, $dS = dsV + sdV$ and $dE = Vd\varepsilon + \varepsilon dV$, then

$$\frac{dS}{s} = \frac{1}{\alpha}\frac{dT}{T}\left(1 + \alpha\frac{T}{V}\frac{dV}{dT}\right) \quad \text{and} \quad \frac{dE}{E} = \frac{1+\alpha}{\alpha}\frac{dT}{T}\left(1 + \frac{\alpha}{1+\alpha}\frac{T}{V}\frac{dV}{dT}\right), \tag{A.8}$$

and the corresponding ratio of fluctuations of entropy and energy is equal

$$R = \frac{(dS/S)^2}{(dE/E)^2} = \left(1 + \frac{\alpha}{1+\alpha\phi}\right)^{-2}, \quad \text{where} \quad \phi = \frac{T}{V}\frac{dV}{dT} = \frac{d\ln V}{d\ln T}\,. \tag{A.9}$$

For $dV = 0$ one gets result obtained already in [2]. However, in general $\phi \neq 0$ and fluctuations of the interaction volume are also important. Eq. (A.9) can be also rewritten as

$$\frac{1}{\alpha} = -\phi + \frac{\sqrt{R}}{1-\sqrt{R}}\,, \tag{A.10}$$

which connects EoS (represented by $\alpha$) with changes in the reaction volume and with fluctuations described by the parameter $R$.

## Appendix B

We shall derive here Eq. (8). Let us first notice that $F_{\mathrm{T}}$ defined in (4) is related to other measure of fluctuations, the so called $\Phi$-measure introduced in [14] and used in [9, 13],

$$\Phi = \sqrt{\frac{\langle Z^2\rangle}{\langle N\rangle}} - \sqrt{\langle z^2\rangle}\,. \tag{B.1}$$

For a measured quantity $x$ (here identified with transverse momentum of produced particles, $x_i = p_{\mathrm{T}i}$) one has:

$$\begin{aligned} z &= x - \langle x\rangle\,, \\ \left\langle z^2\right\rangle &= \mathrm{Var}(x) = \left\langle x^2\right\rangle - \langle x\rangle^2\,, \\ Z &= \sum_{i=1}^{N} z_i = \sum_{i=1}^{N} x_i - N\langle x\rangle\,, \\ \left\langle Z^2\right\rangle &= \left\langle\left(\sum_{i=1}^{N} x_i\right)^2\right\rangle - 2\langle x\rangle\left\langle N\sum_{i=1}^{N} x_i\right\rangle + \left\langle N^2\right\rangle\langle x\rangle^2\,. \end{aligned} \tag{B.2}$$

To get this result one uses following reasoning. If variables $x$ and $N$ are described by distributions characterized by the respective generating functions $f(t)$ and $h(t)$ then and variable $N$ by generating function $h(t)$ then variable $\xi = \sum_{i=1}^{N} x_i$ is described by generating function $G(t) = h[f(t)]$. It means therefore that

$$\langle\xi\rangle = G'(1) = \langle N\rangle\langle x\rangle \tag{B.3}$$

and

$$\langle\xi^2\rangle = G''(1) + G'(1) = \langle N^2\rangle\langle x\rangle^2 + \langle N\rangle\mathrm{Var}(x) \tag{B.4}$$

leading to

$$\mathrm{Var}(\xi) = \langle N\rangle\mathrm{Var}(x) + \langle x\rangle^2\mathrm{Var}(N)\,. \tag{B.5}$$

To characterize correlations between variables $N$ and $\xi$ one has to introduce a correlation coefficient $\rho \in [-1, 1]$, which will be, in what follows, our free parameter (our minimal input). One can therefore write:

$$\left\langle N\sum_{i=1}^{N} x_i\right\rangle = \langle N\rangle\left\langle\sum_{i=1}^{N} x_i\right\rangle + \rho\sqrt{\mathrm{Var}(N)\mathrm{Var}\left(\sum_{i=1}^{N} x_i\right)}\,. \tag{B.6}$$

The last term in (B.6) can be written as

$$\sqrt{\mathrm{Var}(N)\mathrm{Var}\left(\sum_{i=1}^{N} x_i\right)} = \sqrt{\langle N\rangle\mathrm{Var}(N)\mathrm{Var}(x) + \langle x\rangle^2\mathrm{Var}^2(N)}\,. \tag{B.7}$$

Substituting now (B.6) to (B.2) and making use of Eq. (B.3)–(B.5) one gets

$$\langle Z^2\rangle = \langle N\rangle \mathrm{Var}(x) + 2\langle x\rangle^2 \mathrm{Var}(N)\left[1-\rho\sqrt{\frac{\langle N\rangle}{\mathrm{Var}(N)}\frac{\mathrm{Var}(x)}{\langle x\rangle^2}+1}\right]. \quad \text{(B.8)}$$

Substituting this to Eq. (B.1) and making use of the relation between $F_{\rm T}$ and $\Phi$ derived in [16], namely that

$$F_{\rm T} = \frac{\Phi}{\sqrt{\langle z^2\rangle}}, \quad \text{(B.9)}$$

one gets

$$F_{\rm T} = -1 + \sqrt{1+2\frac{\langle x\rangle^2}{\mathrm{Var}(x)}\frac{\mathrm{Var}(N)}{\langle N\rangle}\left[1-\rho\sqrt{\frac{\langle N\rangle}{\mathrm{Var}(N)}\frac{\mathrm{Var}(x)}{\langle x\rangle^2}+1}\right]}, \quad \text{(B.10)}$$

which is Eq.(8) we were looking for.

## Appendix C

Let us start with change in the energy $E$ of the system, which can be written as ($C_V$ is corresponding heat capacity)

$$\Delta E = \left(\frac{\partial E}{\partial V}\right)_T \Delta V + \left(\frac{\partial E}{\partial T}\right)_V \Delta T = \left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right]\Delta V + C_V\Delta T\,. \quad \text{(C.1)}$$

Squaring it and averaging while remembering that [24] fluctuations of the volume and temperature are given by, respectively,

$$\overline{(\Delta V)^2} = -T\left(\frac{\partial V}{\partial p}\right)_T \quad \text{and} \quad \overline{(\Delta T)^2} = \frac{T^2}{C_V} \quad \text{(C.2)}$$

and are statistically independent, *i.e.*, $\overline{\Delta T\delta V} = 0$, one gets

$$\mathrm{Var}(E) = \overline{(\Delta E^2)} = -\left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right]^2 T\left(\frac{\partial V}{\partial p}\right)_T + C_V T^2\,. \quad \text{(C.3)}$$

Similarly, fluctuations of number of particles can be described by formula

$$\mathrm{Var}(N) = -\frac{T\langle N\rangle^2}{V^2}\left(\frac{\partial V}{\partial p}\right)_T. \quad \text{(C.4)}$$

From (C.3) and (C.4) one gets

$$R = \frac{\frac{\mathrm{Var}(N)}{\langle N\rangle^2}}{\frac{\mathrm{Var}(E)}{\langle E\rangle^2}} = \frac{1}{\left[\frac{T}{\varepsilon}\left(\frac{\partial p}{\partial T}\right)_V - \frac{p}{\varepsilon}\right]^2 - \frac{C_V T}{\varepsilon^2}\left(\frac{\partial p}{\partial V}\right)_T}. \tag{C.5}$$

Because

$$T\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial E}{\partial V}\right)_T + p \tag{C.6}$$

then (from now on all derivatives are for $T = \mathrm{const.}$)

$$\begin{aligned} R &= \frac{\varepsilon^2}{\left[\left(\frac{\partial E}{\partial V}\right)^2 - C_V T\left(\frac{\partial p}{\partial V}\right)\right]} = \frac{\varepsilon^2\left(\frac{\partial V}{\partial E}\right)^2}{1 - C_V T\frac{\partial p}{\partial V}\frac{\partial V}{\partial E}\frac{\partial V}{\partial E}} \\ &= \frac{\varepsilon^2\left(\frac{\partial V}{\partial E}\right)^2}{1 - C_V T\frac{\partial p}{\partial E}\frac{\partial V}{\partial E}}\,. \end{aligned} \tag{C.7}$$

Because of EoS one has that

$$p = \alpha\varepsilon \qquad \text{and} \qquad \frac{\partial p}{\partial E} = \frac{\partial p}{\partial V}\frac{\partial V}{\partial E} = \frac{\alpha}{V} > 0\,, \tag{C.8}$$

what immediately leads to Eq. (21):

$$R = \frac{\varepsilon^2\left(\frac{\partial V}{\partial E}\right)^2}{1 - C_V T\frac{\alpha}{V}\frac{\partial V}{\partial E}}\,. \tag{C.9}$$

## REFERENCES


[1] H. Gutbrod, J. Aichelin, K. Werner (Eds), *Nucl. Phys.* **A715**, 1c (2003).

[2] M. Gaździcki, M.I. Gorenstein, S. Mrówczyński, Fluctuations and Deconfinement Phase Transition in Nucleus–Nucleus Collisions, `hep-ph/0304052`.

[3] L. Stodolsky, *Phys. Rev. Lett.* **75**, 1044 (1995); S. Mrówczyński, *Phys. Lett.* **B430**, 9 (1998); M. Stephanov, K. Rajagopal, E. Shuryak, *Phys. Rev. Lett.* **81**, 4816 (1998) and *Phys. Rev.* **D60**, 114028 (1999).

[4] T.A. Trainor, Event-by-Event Analysis and the Central Limit Theorem, `hep-ph/0001148`; H. Heiselberg, *Phys. Rep.* **351**, 161 (2001); B. Müller, *Nucl. Phys.* **A702**, 281c (2002).

[5] L. McLerran, RHIC Physics: The Quark Gluon Plasma and the Color Glass Condensate: 4 Lectures, `hep-ph/0311028`.

[6] M. Gaździcki, Energy Scan Program at the CERN SPS and an Observation of the Deconfinement Phase Transition in Nucleus–Nucleus Collisions, presented at 7th Int. Conf. on Strange Quarks in Matter, SQM 2003, March 2003, Atlantic Beach, USA, `hep-ph/0305176` and *Acta Phys. Pol. B* **34**, 5771 (2003); M.I. Gorenstein, Signals of Deconfinement Transition in Nucleus–Nucleus Collisions, `hep-ph/0310269`.

[7] E.G. Ferreiro, F. del Moral, C. Pajares, Transverse Momentum Fluctuations and Percolation of Strings, `hep-ph/0303137`.

[8] J. Nystrand (PHENIX Collab.), *Nucl. Phys.* **A715**, 603c (2003).

[9] C. Blume *et al.* (NA49 Collab.), *Nucl. Phys.* **A715**, 55c (2003); T. Anticic *et al.* (NA49 Collab.), Transverse Momentum Fluctuations in Nuclear Collisions at 158 AGeV, `hep-ex/0311009`.

[10] C.M. Hung, E.V. Shuryak, *Phys. Rev. Lett.* **75**, 4003 (1995).

[11] C.R. Alton *et al.*, *Phys. Rev.* **D68**, 014507 (2003); see also Z. Fodor, *Nucl. Phys.* **A715**, 319c (2003).

[12] J. Randrup, Spinodial decomposition during the hadronization stage at RHIC?, `hep-ph/0308271`.

[13] D. Adamova *et al.*, (CERES Collab.) *Nucl. Phys.* **A727**, 97 (2003).

[14] M. Gaździcki, S. Mrówczyński, *Z. Phys.* **C54**, 127 (1992).

[15] D. Adamova *et al.* (CERES Collab.), *Phys. Rev. Lett.* **90**, 022301 (2003).

[16] K. Adcox *et al.* (PHENIX Collab.), *Phys. Rev.* **C66**, 024901 (2002).

[17] O.V. Utyuzh, G. Wilk, Z. Włodarczyk, *Phys. Rev.* **C64**, 027901 (2001).

[18] M.M. Aggarwal *et al.*, (WA98 Collab.), *Phys. Rev.* **C65**, 054912 (2002).

[19] See, for example, P. Carruthers, C.C. Shih, *Int. J. Mod. Phys.* **A2**, 1447 (1986); or C. Geich-Gimbel, *Int. J. Mod. Phys.* **A4**, 1527 (1989).

[20] G. Wilk, Z. Włodarczyk, *Phys. Rev. Lett.* **84**, 2770 (2000).

[21] G. Wilk, Z. Włodarczyk, *Chaos, Solitons and Fractals* **13**, 581 (2002); *Physica A* **305**, 227 (2002).

[22] F.S. Navarra, O.V. Utyuzh, G. Wilk, Z. Włodarczyk, *Phys. Rev.* **D67**, 114002 (2003).

[23] *Cf.*, for example, K. Huang, *Statistical Mechanics*, John Wiley and Sons, New York 1987.

[24] L.D. Landau, E.M. Lifshitz, *Course of Theoretical Physics: Statistical Physics*, Pergamon Press, New York 1958.

[25] M. López Noriega (STAR Collab.), *Nucl. Phys.* **A715**, 623c (2003).