ON THE ATOMIC RESONANCES
IN THE 0ν2EC TRANSITIONS*

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The nuclear method to discover Majorana neutrinos is the neutrinoless double β decay. An interesting alternative is offered by the inverse process, neutrinoless radiative double electron capture, accompanied by a photon emission. Two different mechanisms seem plausible: the magnetic type radiation by an initial electron and the resonant electric type radiation by the final atom. The physical background for these processes is calculated.

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1. Introduction

The search for neutrino-less double decay, $0\nu\beta\beta$, decay is a major challenge of to-day's neutrino physics. The process, if found, will prove unambiguously the Majorana nature of neutrino. It will also provide a sensitive measure of the neutrino mass. Nuclear physics, with the studies of $0\nu\beta\beta$ transitions is expected to resolve this question for the electron neutrino, ν_e. If it is a Majorana particle then by definition it is identical to its charge conjugate. Thus the neutrino produced in a weak process on one nucleon may be absorbed by another nucleon. The outcome is a nuclear reaction

$$(A, Z - 2) \rightarrow (A, Z) + e + e. \tag{1}$$

Amplitudes for such a process are proportional to the Majorana neutrino mass. For the description of the problems involved we refer to reviews [1–6].

The experimental check of transition (1) requires detection of two correlated electrons of a given total energy. Such measurements are difficult not only because of low rates and the dominating random background, but

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also as a result of the high physical background. The latter is due to the neutrino accompanied double beta process $2\nu\beta\beta$. Some of these difficulties may be overcome by studying the inverse process of neutrino-less double electron capture accompanied by a photon emission [6,7]. There are several experimental advantages. The monoenergetic photon provides a convenient experimental signature. Other advantages include the favourable ratio of the $0\nu2EC$ to the competing $2\nu2EC$ capture rates as opposed to that of $0\nu\beta\beta$ to $2\nu\beta\beta$. This point is discussed in detail below. Another important advantage of the $0\nu2EC$ process is the existence of the coincidence trigger to suppress the random background. These advantages offset, in part, the longer lifetimes for the $0\nu2EC$ decays.

Chances for the capture process have been calculated in Ref. [7]. The seemingly most likely process, which involves two $1S$ electrons and leads to two $1S$ holes in the final atom, does not conserve spin. Allowed is the capture of $1S, 2S$ electron pair with the photon emitted by one of these electrons in intermediate states. The rate increases roughly with Z^7, high Z atoms are thus required. Still, the experiments would be difficult. There is yet another process which may be more likely at small energy release. This involves the virtual captures of $1S, 1S$ electrons and the radiation process in the final excited atom. A resonance enhancement of the capture rates is predicted [8–10], when the energy release Q is comparable to the $2P$–$1S$ atomic level difference. Away from the resonance the rates depend only slowly on Q, in strong contrast with the $0\nu\beta\beta$ decays. This makes studies of decays to excited states in final nuclei feasible, thus enhancing chances of locating the resonances. Candidates for such studies have been considered [7,9], and the experimental feasibility is found encouraging, [10].

This paper consists of two sections. Section 2 gives a brief presentation of two basic radiation mechanisms that occur in the $0\nu2EC$ transitions. In Section 3 the physical background for these two modes is calculated.

2. Two modes of radiative $0\nu2EC$ transitions

The rate $\Gamma(0\nu\beta\beta)$ for the neutrino-less double beta decay may be factorised into nuclear and leptonic parts (see *e.g.* [4, 11, 12])

$$\Gamma(0\nu\beta\beta) = G^{0\nu} \mid M^{0\nu} \mid^2 (m_\nu/m_e)^2. \tag{2}$$

This equation involves nuclear matrix elements $M^{0\nu}$, the leptonic contributions including final state electron wave functions and final phase space elements are contained in $G^{0\nu}$. The neutrino mass factor m_ν reflects the chance for the left handed neutrino emitted from one nucleon to have the right-handed component required for the subsequent absorption on another

nucleon. The reverse process of double electron capture

$$(A, Z) + e + e \rightarrow (A, Z - 2) \tag{3}$$

is not allowed by the energy-momentum conservation, emission of a third body is necessary. In the following we consider the photon emission to fulfil this requirement:

$$(A, Z) + e + e \rightarrow (A, Z - 2) + \gamma. \tag{4}$$

Again, the capture rate may be factorised into the nuclear, leptonic and photonic terms

$$\Gamma(0\nu\gamma) = 2\pi \int \frac{d\boldsymbol{q}}{(2\pi)^3} \delta(Q - q) G^{2\mathrm{EC}} \left[\frac{M^{0\nu} m_\nu}{4\pi} \right]^2 \mid M^\gamma(q) \mid^2, \tag{5}$$

where Q is the photon energy given by the mass difference of the initial and final atoms reduced by the energies of two electron holes left in the final state. The factors involved in Eq. (5) differ from those of Eq. (2) by the final phase space, the transition energy and electron wave functions. The nuclear transition element is of similar nature though it connects different nuclei. It is given by the terms in brackets which describe the propagation of neutrino between two nucleons. These contain the neutrino mass and the nuclear matrix element of "neutrino exchange potential". For the dominant Gamow–Teller transitions the latter is $M^{0\nu} = \langle Z - 2 \mid \frac{\sigma\sigma' \exp(-rq_\nu)}{r} \mid Z \rangle$ where r is the distance between the two nucleons and q_ν is the average energy needed to generate neutrino of very short propagation range. Calculations yield $M^{0\nu} \approx 1/\mathrm{fm}$, [4, 12].

An additional factor M^γ introduced into Eq. (5) denotes the probability of photon emission. Two cases are likely to offer a good chance for the experimental detection. The first one occurs when an electron from $2S$ or $1S$ state radiates before it is captured by the nucleus. In this case,

$$\mid M^\gamma(q) \mid^2 = \frac{e^2}{2q m_e^2} f_M, \tag{6}$$

where e is the electron charge and $\sqrt{2q}$ is the photon wave normalisation. The first term gives the order of magnitude estimate. Finer calculations based on Glauber–Martin [13, 14], theory for the electron radiating in the Coulomb field give the corrective factor $f_M \approx 1$ for the magnetic transition that takes place in these circumstances, [7, 10].

The second mode of radiative transitions comes from a different scenario, indicated in figure 1. Two $1S$ electrons may be captured in a virtual process

which generates a final atom with two $1S$ electron holes. This final atom radiates and one of the holes is filled. That is a resonant-like situation and close to the resonance

$$| M^\gamma(q) |^2 = \frac{\Gamma^r(2P \to 1S)}{[q - Q_{\text{res}}]^2 + [\Gamma^r/2]^2} \frac{\pi}{q^2} , \tag{7}$$

where Γ^r is the radiative width of the final two-electron-hole atom. There are a number of resonant situations. The most important one happens with $Q_{\text{res}} = E(2P) - E(1S)$ that is when the $0\nu2EC$ transition energy Q coincides with the $2P$–$1S$ electric radiative transition in the final nucleus (the K_α transition indicated in Eq. (7)). These resonant situations may greatly enhance the rate. Practical interest requires $| Q_{\text{res}} - Q | < 1$ keV. There are several targets likely to fulfil this condition, [6,9].

THE RESONANT SITUATION
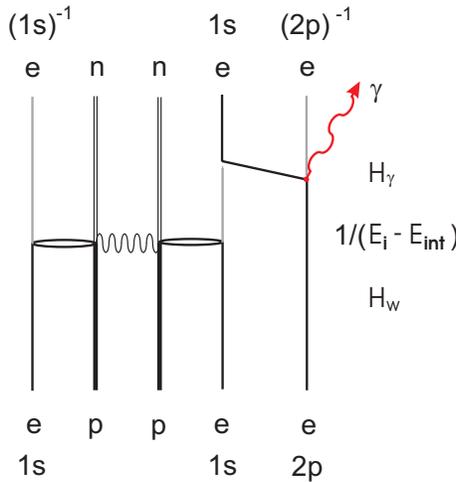
Fig. 1. The diagram for the $0\nu2EC$, double electron capture. Indicated is the resonance situation that occurs in the intermediate state: after the capture — before the radiation process. H_{W}, H_γ are the weak and radiative Hamiltonians respectively, $E_{\text{i}}, E_{\text{int}}$ are energies of the initial and intermediate states, E_{1s}, E_{2p} are negative atomic state energies.

3. The physical background

Assuming the photon energy resolution to be D we calculate now the ratio of the signal from the $0\nu\gamma$ to the physical background due to the $\nu\nu\gamma$ transitions in the double electron capture. This ratio is defined as

$$R_{\text{s/b}} = \frac{\Gamma(0\nu\gamma)}{\Gamma(\nu\nu\gamma)N_D}, \tag{8}$$

where N_D is the fraction of photons from the dominant $\nu\nu\gamma$ decay mode emitted into the region from the end of the spectrum Q down to $Q - D/2$. For the two neutrino process the radiative rate is

$$\Gamma(\nu\nu\gamma) = \int \frac{2\pi d\boldsymbol{q}d\boldsymbol{p}d\boldsymbol{p}'}{(2\pi)^9 2E(p)2E(p')} \delta(Q - E(p) - E(p') - q) G^{2\text{EC}} \mid M^{2\nu}M^\gamma(q) \mid^2, \tag{9}$$

where the nuclear matrix element $M^{2\nu}$ differs from $M^{0\nu}$. For the Gamow–Teller transitions one has $M^{2\nu} = \langle Z - 2 \mid \sigma\sigma' \mid Z \rangle$, and calculations, [4,12], yield $M^{2\nu} \approx 1$. The matrix elements $M^{2\nu}$ for the neutrino and $M^{0\nu}$ for the neutrino-less decays are almost energy independent. The leptonic factor $G^{2\text{EC}}$ is also constant as opposed to the equivalent factor in the double β decays. The required ratio $R_{\text{s/b}}$ is thus given by the phase space. Let us present Eq. (9) as an integral over the photon energy distribution $W(q)$:

$$\Gamma(\nu\nu\gamma) = \int_0^Q W(q)dq. \tag{10}$$

The background contribution follows as

$$\Gamma(\nu\nu\gamma)N_D = \int_{Q-D/2}^Q W(q)dq. \tag{11}$$

From Eq. (9) one obtains

$$W(q) = \frac{G^{2\text{EC}} \mid M^{2\nu} \mid^2}{(2\pi)^5 3} q^2(Q - q)^3 \mid M^\gamma(q) \mid^2. \tag{12}$$

First we consider the magnetic type transition related to the $1S, 2S$ electron capture. In this case M^γ given by Eq. (5) is constant and the ratio of total rates:

$$R_{0\nu/2\nu} = \frac{\Gamma(0\nu\gamma)}{\Gamma(\nu\nu\gamma)} = \frac{120\pi^2 m_\nu^2 \mid M^{0\nu} \mid^2}{Q^4 \mid M^{2\nu} \mid^2} \tag{13}$$

which gives the relative frequency of the non-neutrino to the corresponding two-neutrino radiative transitions. For characteristic values $Q = 1$ MeV and $m_\nu = 1$ eV one obtains $R_{0\nu/2\nu} = 510^{-5}$. The contribution from the physical background to the region of the monoenergetic photon depends on the energy resolution:

$$R_{\text{s/b}} = \frac{3 \times 2^7 \pi^2 m_\nu^2 \mid M^{0\nu} \mid^2}{D^4 \mid M^{0\nu} \mid^2} \,. \tag{14}$$

For $Q = 1$ MeV, $m_\nu = 1$ eV and $D = 3$ keV one obtains very low background $R_{\text{s/b}} = 2 \times 10^6$.

Now we turn to the case of the resonant electron captures. The rate of the signal to the background depends strongly on the relation of the transition energy to the resonance energy. Assume that the situation of a nearby resonance is materialised with $Q > Q_{\text{res}}$, that is the resonance transition is located within the photon spectrum. Also, we assume the line to be within the photon energy resolution band $\mid Q - E_{\text{res}} \mid < D/2$. We exclude for a moment the optimal but highly unlikely situation of $\mid Q - E_{\text{res}} \mid < \Gamma^r$ corresponding to the line coinciding with the end of the photon spectrum. With these limitations one obtains

$$R_{\text{s/b}} \approx \frac{6\pi m_\nu^2 \mid M^{0\nu} \mid^2}{[Q - Q_{\text{res}}]^3 \mid M^{2\nu} \mid^2} \frac{\Gamma^r(2P \to 1S)}{[Q - Q_{\text{res}}]^2 + [\Gamma^r/2]^2} \,. \tag{15}$$

Beyond the range of natural widths (0.05–0.10 keV) the ratio $R_{\text{s/b}}$ falls down as $[Q - Q_{\text{res}}]^{-5}$. For $Q - Q_{\text{res}} = 1$ keV the signal to background rate is still very convenient $R_{\text{s/b}} \approx 10^4$. However, the conditions deteriorate quickly with the increasing separation.

Formula (15) is valid for $Q - Q_{\text{res}} > \Gamma^r$. The other two situations, *i.e.* that of $Q < Q_{\text{res}}$ when the centre of the atomic line is beyond the photon spectrum, or that of a "direct hit" into the line, $\mid Q - E_{\text{res}} \mid < \Gamma^r$, offer the luxurious ratio $R_{\text{s/b}} \approx 10^9$, which means no physical background.

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