MICROSCOPIC THEORY OF THE TWO-PROTON RADIOACTIVITY*

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We develop the realistic continuum shell model which includes the coupling between many-particle (quasi-)bound states and the continuum of one- and two-particle scattering states. This microscopic approach is applied to the description of the two-proton radioactivity from the excited state 1^-_2 in ^{18}Ne.

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1. Introduction

Nuclear decays with three fragments in the final state are very exotic processes. The diproton radioactivity, predicted by Goldansky [1] and called by him the "true three-body decay", can occur for even-Z nuclei beyond a proton drip line. If the sequential decay is energetically forbidden due to the pairing energy, a simultaneous two-proton ($2p$) decay becomes the only possible decay branch. In spite of long lasting experimental efforts [2], no fully convincing finding has been reported (see however recent data on $2p$ radioactivity of ^{45}Fe [3]). In light nuclei, it happens often that there are broad intermediate states in the nucleus $^{A-1}(Z-1)$ available to the one-proton ($1p$) decay which yield a combination of sequential and direct modes for the $2p$ decay. Recently, the $2p$ decay from 1^-_2 state at 6.15 MeV in ^{18}Ne to the ground state (g.s.) of ^{16}O has been observed [4]. Since there are no

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intermediate states in ^{17}F available for a one-proton decay, this case was advocated as a candidate for the diproton decay. In the following, we shall present an analysis of this decay using a new microscopic approach which takes into account the configuration mixing and uses the S-matrix formalism to calculate the asymptotic states [5].

2. Shell Model Embedded in the Continuum

Our approach extends the Shell Model Embedded in the Continuum (SMEC) [5, 6] for the description of $2p$ decay. The Hilbert space is divided in three subspaces: Q, P and T. In Q subspace, A nucleons are distributed over (quasi-)bound single-particle (qbsp) orbits. In P, one nucleon is in the non-resonant continuum and $A-1$ nucleons occupy qbsp orbits. In T, two nucleons are in the non-resonant continuum and $(A-2)$ are in qbsp orbits. The coupling between Q, P and T subspaces changes the 'unperturbed' Hamiltonian in Q into the effective Hamiltonian:

$$\begin{aligned} H_{QQ}^{(\mathrm{eff})} &= H_{QQ} + H_{QT}G_T^{+}(E)H_{TQ} \\ &\quad + \left[H_{QP}+H_{QT}G_T^{+}(E)H_{TP}\right]\tilde{G}_P^{(+)}(E)\left[H_{PQ}+H_{PT}G_T^{(+)}(E)H_{TQ}\right], \end{aligned} \tag{1}$$

where

$$\tilde{G}_P^{(+)}(E) = \left[E - H_{PP} - H_{PT}G_T^{(+)}(E)H_{TP}\right]^{-1}$$

is the Green's function in P modified by the coupling to T, and

$$G_T^{(+)}(E) = [E - H_{TT}]^{-1}$$

is the Green's function in T. In the above equations, H_{PP}, H_{TT} are the unperturbed Hamiltonians in P, T subspaces, respectively, and H_{QP}, H_{PQ}, H_{PT}, H_{TP} are the corresponding coupling terms between Q, P, and T subspaces. The second term on the r.h.s of Eq. (1) describes a 'pure' diproton emission, and the third term describes the modification due to the mixing of sequential $2p$, diproton and $1p$ decay modes. In the following, we shall discuss separately two limits of a general process (1): *(i)* $H_{TQ} = H_{QT} = 0$, and *(ii)* $H_{TP} = H_{PT} = 0$, which correspond to pure sequential and pure diproton decays, respectively. In both limits, the interference with one-proton emission is included through the mixing of SM wave functions in Q.

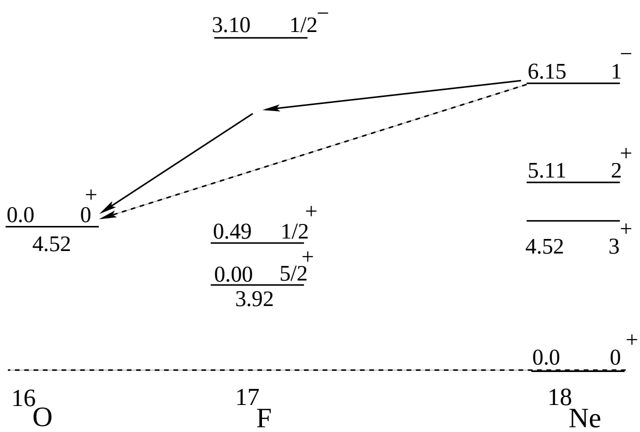

Fig. 1. Two-proton decay pattern scheme from 1_2^- state of ^{18}Ne.

2.1. Sequential emission

We consider $2p$ decay mode from 1_2^- state in ^{18}Ne at the excitation energy 6.15 MeV. Since there are no intermediates resonance states in ^{17}F, the only possible sequential process is through the 'resonant halo' of bound states [5]: the g.s. $5/2_1^+$ and the weakly bound ($S_p \sim 105$ keV) first excited state $1/2_1^+$ of ^{17}F. The effective Hamiltonian in Q for this process reads:

$$\begin{aligned} H_{QQ}^{(\mathrm{eff})} &= H_{QQ} + H_{QP} G_P^{+}(E) H_{PQ} \\ &\quad + H_{QP} \tilde{G}_P^{(+)}(E) H_{PT} G_T^{(+)}(E) H_{TP} G_P^{(+)}(E) H_{PQ} \,, \end{aligned} \tag{2}$$

where

$$G_P^{(+)}(E) = [E - H_{PP}]^{-1}$$

is the unperturbed Green's function in P. To obtain (2), one neglects H_{QT} and H_{TQ} in (1) and rewrites (1) in a form which explicitly exhibits a Q–P coupling term: $H_{QP} G_P^{+} H_{PQ}$.

First we calculate the contribution due to the coupling with one proton in the continuum of ^{17}F (Q–P coupling):

$$\langle 1^- | H_{QQ} + H_{QP} G_P^{+}(E) H_{PQ} | 1^- \rangle \,,$$

which yields a 'mixed' state $\left| 1^-_{(\mathrm{mix})} \right\rangle$. Then, we calculate the contribution of the sequential $2p$ emission:

$$\left\langle 1^-_{(\mathrm{mix})} \right| H_{QP} \tilde{G}_P^{(+)}(E) H_{PT} G_T^{(+)}(E) H_{TP} G_P^{(+)}(E) H_{PQ} \left| 1^-_{(\mathrm{mix})} \right\rangle \,.$$

The first emitted proton is assumed to be a spectator of the second emission. This implies a following identification:

- $H_{PP} \longrightarrow H_{Q'Q'} + h_0$, *i.e.* H_{PP} is splitted into $H_{Q'Q'}$ which acts on $A-1$ nucleons in qbsp orbits *after* the first proton emission, and h_0 which is a one-body potential describing an average effect of $A-1$ particles on the emitted proton,

- $H_{TT} \longrightarrow H_{P'P'} + h_0$, *i.e.* H_{TT} is splitted into $H_{P'P'}$ which acts on $A-2$ nucleons in qbsp orbits and 1 nucleon in the continuum *after* the first proton emission, and h_0,

- $H_{QT} \longrightarrow H_{Q'P'}$, *i.e.* H_{QT} becomes a coupling between newly defined Q' and P' subspaces.

In solving SMEC problem with $H_{QQ}^{(\mathrm{eff})}$ given in Eq. (2), the radial single-particle (s.p.) wave functions in Q and the scattering wave functions in P and T are generated by a self-consistent procedure starting with the average potential of Woods–Saxon (WS) type with the spin–orbit and Coulomb parts included, and taking into account the residual coupling between Q, P and Q, T subspaces. (Details of the calculations will be published elsewhere [7].) This procedure yields new orthonomalized wave functions in Q, P and T and new self-consistent potentials for each many-body state in Q [6]. For the effective interaction in H_{QQ} and in $H_{Q'Q'}$, we take either WBT Hamiltonian [8] or USD Hamiltonian for the (sd)-shell [9] and the KB' interaction for the (pf)-shell [10]. The cross-shell interaction is the G matrix [11]. The latter interaction will be called ($psdfp$)-Hamiltonian. The residual couplings H_{QP}, H_{PT} between different subspaces are given by the contact force [6]: $V_{12} = -V_{12}^{(0)}[\alpha + \beta P_{12}^{\sigma}]\delta(r_1 - r_2)$, where $\alpha + \beta = 1$ and P_{12}^{σ} is the spin exchange operator.

The sequential $2p$ emission from 1_2^- in ^{18}Ne is passing through a resonant continuum of weakly bound states $5/2_1^+$ and $1/2_1^+$ at energies above the $1p$ emission threshold in ^{17}F. Fig. 2 shows the dependence of the density of width $\Gamma^{(\mathrm{seq})}$ on the energy $\varepsilon \equiv \varepsilon_{p1}$ taken away by the first emitted proton, *i.e.*, the sharing of the total energy available for the sequential $2p$ decay. This curve resembles a resonance, even though no intermediate resonance exists in ^{17}F. The dominant contribution to the peak $\Gamma^{(\mathrm{seq})}(\varepsilon)$ comes from the resonant continuum of $1/2_1^+$ state bound by $\sim$105 keV. For ($psdfp$)-Hamiltonian, the ratio of the sequential decay through the $1/2_1^+$ continuum to the total decay width $\Gamma_{\mathrm{tot}}^{(\mathrm{seq})}$ is 85.2% or 74.1% depending on whether the strength $V_{12}^{(0)}$ of the residual coupling equals 700 or 900 MeV$\times$fm^3. For these two coupling strengths, the calculated $\Gamma_{\mathrm{tot}}^{(\mathrm{seq})}$ is 20.8 or 23.8 eV, respectively, whereas the experimental estimate obtained assuming a pure sequential decay is [4]: $\Gamma^{(\mathrm{seq})} = 57 \pm 6$ eV. WBT Hamiltonian gives much smaller values for $\Gamma_{\mathrm{tot}}^{(\mathrm{seq})}$. We have found a strong dependence of $\Gamma_{\mathrm{tot}}^{(\mathrm{seq})}$ on the SMEC

Hamiltonian and, to a somewhat smaller extent, on the radial features of s.p. resonances ($0d_{3/2}$, $0f_{7/2}$, $0f_{5/2}$, $1p_{1/2}$, $1p_{3/2}$) included in Q (details of the regularization procedure for resonances can be found in Ref. [5]). Interestingly, the ratio of $1p$ partial decay width to $1/2^+_1$ state and the sequential $2p$ decay width is nearly constant in different calculations and equals ~ 20.

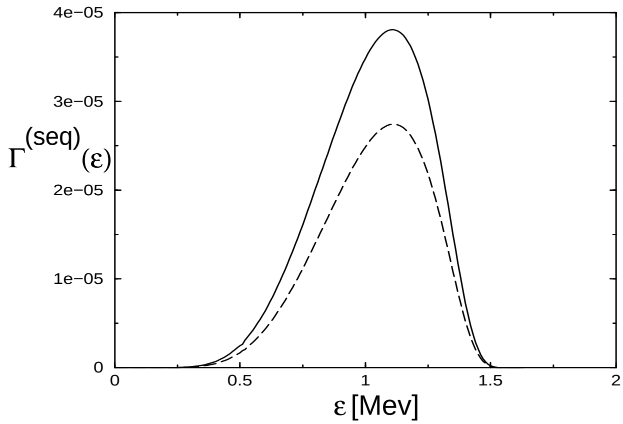

Fig. 2. Dependence of the density of width $\Gamma^{(\mathrm{seq})}$ on the energy ε taken by the first emitted proton is shown for the (*psdfp*)- (solid line) and WBT- (dashed line) Hamiltonians. The strength of the residual coupling is -900 MeV$\times$fm^3 and -700 MeV$\times$fm^3 in the calculations using (*psdfp*)- and WBT-interactions, respectively.

2.2. Diproton decay

In this case, the effective Hamiltonian in Eq. (1) reduces to:

$$H_{QQ}^{(\mathrm{eff})} = H_{QQ} + H_{QP}G_P^{+}(E)H_{PQ} + H_{QT}G_T^{(+)}(E)H_{TQ}\,. \tag{3}$$

Intermediate couplings, *i.e.* H_{TP} (H_{PT}), are neglected. As for the sequential decay process, first we calculate the contribution due to the coupling of 1^-_2 state to $1p$ continuum in ^{17}F: which yields a 'mixed' state $\left|1^-_{(\mathrm{mix})}\right\rangle$. Then, we calculate

$$\left\langle 1^-_{(\mathrm{mix})}\right| H_{QT}G_T^{(+)}H_{TQ}\left|1^-_{(\mathrm{mix})}\right\rangle\,,$$

i.e., the contribution of the diproton emission. This can be rewritten formally as $\langle w \mid \omega\rangle$, where

$$|w\rangle = H_{TQ}\left|1^-_{(\mathrm{mix})}\right\rangle$$

is the source term and ω, which represents the continuation in T of the Q-space wave function, is given by the solution of an inhomogeneous coupled channel equation:

$$(E - H_{TT})|\omega\rangle = |w\rangle\,.$$

In the decay to the g.s. of ^{16}O, only one channel is open:

$$\left|t_{\rm (int)}, \Theta; L=1, l=0, S=0\right\rangle\,,$$

where $|t_{\rm (int)}\rangle$ denotes the internal state of ^{16}O, L is the relative angular momentum between a cluster and ^{16}O, Θ represents the internal motion of the cluster, and l its internal angular momentum. Protons in the cluster are coupled to the spin $S=0$.

The proton–proton interaction is taken fully into account in Q. On the other hand, in T, the final state interaction between two protons is taken into account phenomenologically in terms of the s-wave phase shift:

$$\Gamma^{\rm (dip)} = \frac{\Gamma(Q_{2p})}{\tilde{\gamma}^2(Q_{2p})}\int\limits_0^{Q_{2p}} \tilde{\gamma}^2(E)\rho(Q_{2p}-E)dE\,, \tag{4}$$

where

$$\Gamma(Q_{2p}) = -2{\rm Im}\left[\int\limits_0^{+\infty} \omega(r,Q_{2p})w(r,Q_{2p})dr\right] \tag{5}$$

and Q_{2p} equals 1.63 MeV. $\tilde{\gamma}(E)$ in (4) is the partial width:

$$\tilde{\gamma}(E) = \int\limits_0^{+\infty} \xi(E,r)w(r)dr\,,$$

where ξ is the solution of the homogeneous coupled channel equation:

$$(E - H_{TT})|\xi\rangle = 0\,.$$

$\rho(Q_{2p}-E)$ in (4) is the $2p$ density-of-states function [9], whereas $w(r,U)$ in (5):

$$w(r,U) = \left\langle t_{\rm (int)}, \Theta; L=1, l=0, S=0; r\left|H_{TQ}\right|1^-_{\rm (mix)}\right\rangle \tag{6}$$

is the probability amplitude that two protons leave nucleus as a cluster. Similarly, $\omega(r,U)$ in (5):

$$\omega(r,U) = \left\langle t_{\rm (int)}, \Theta; L=1, l=0, S=0; r\left|G_T^+(U)H_{TQ}\right|1^-_{\rm (mix)}\right\rangle$$

describes the extention of $\left|1^-_{(\mathrm{mix})}\right\rangle$ into T subspace. One should notice that $w(r,U)$ and $\omega(r,U)$ are calculated in the relative coordinates, *i.e.*, r is the distance between ^{16}O and a diproton. $\omega(r,U)$ is calculated with a WS potential acting on a particle (diproton) with $Z = 2$ and mass $m = 2m_p$. Since the source term $w(r,U)$ is localized, we expand it in a harmonic oscillator basis with the help of the Moshinsky transformation. In this basis, an explicit dependence of $w(r,U)$ on U drops out. The U-dependence reappears in $\Gamma^{(\mathrm{dip})}$ (see Eq. (4)) through a $2p$ density-of-states function $\rho(U)$.

The residual interaction H_{QT} (H_{TQ}) in (3) is given by a contact force described above. In the absence of an external mixing in Q (Q–P mixing), the diproton source is real. In all studied cases, we found that an imaginary part of the source $w(r,U)$ is about two orders of magnitude smaller than the real part. Fig. 3 shows a real part of the diproton source for different Hamiltonians in Q and for different strengths of the external coupling. One can see that the dependence on the strength of the coupling terms H_{QP}, H_{QT}, and on the chosen shell-model Hamiltonian H_{QQ} is relatively weak (*cf.* Fig. 3). The source function strongly oscillates in the interior region and

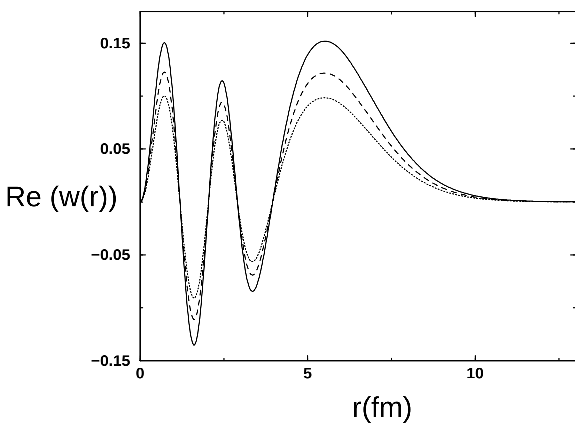

Fig. 3. A real part of the diproton source function $w(r,U)$ is shown for (*psdfp*)-Hamiltonian and two different strengths of the residual coupling: $V_{12}^{(0)} = -900$ MeV×fm^3 (solid line) and $V_{12}^{(0)} = -700$ MeV×fm^3 (dashed line). The dotted line shows results for WBT Hamiltonian and a residual interaction with: $V_{12}^{(0)} = -700$ MeV×fm^3. In this plot, U is equal to the total energy available for $2p$ decay: $Q_{2p} = 1.63$ MeV.

the major part of the diproton width comes from a narrow region ($\Delta R \simeq 2$ fm) in the outer part of the Coulomb barrier. The diproton emission width is a convolution of the $\omega(r,U)$ function with the source function $w(r)$. As said before, $w(r)$ depends weakly on the choice of H_{QQ}, H_{QP} or H_{QT}.

On the contrary, function $\omega(r,U)$ depends strongly on the choice of the effective interaction in H_{QQ}. Consequently, the diproton emission width depends strongly on the shell-model Hamiltonian in Q and weakly on the strength of the coupling terms H_{QP} and H_{QT}. For the (*psdfp*)-Hamiltonian, $\Gamma^{(\rm dip)}$ equals 2.83 or 3.31 eV, depending on the value of the coupling strength of the residual interaction $V_{12}^{(0)} = -700$ or -900 MeV$\times$fm^3. For WBT Hamiltonian, $\Gamma^{(\rm dip)}$ is smaller and equals 1.99 eV for $V_{12}^{(0)} = -700$ MeV$\times$fm^3. These numbers are about one order of magnitude smaller than the value deduced experimentally under an assumption of a pure diproton emission mechanism [4].

3. Conclusions

Our analysis excludes a diproton emission from 1_2^- state of ^{18}Ne as a dominant $2p$ emission mode. We have found, on the contrary, that the calculated sequential emission through the resonant continuum of bound states in ^{17}F agrees qualitatively with the experimental data [4]. A strong correlation between the rate of a sequential $2p$ decay and the rate of a one-proton partial decay to the 'halo state' $1/2_1^+$ in ^{17}F gives an access to the properties of the 'ghost' of $1/2_1^+$ state in the continuum. The measurement of this partial decay width would be helpful to reduce the uncertainty associated with the choice of the effective interaction in Q.

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