COMPARISON OF δ- AND GOGNY-TYPE PAIRING INTERACTIONS*

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The matrix elements of the zero-range δ-force and the finite range Gogny-type pairing force are compared. The strengths of the δ-interaction for rare-earth nuclei are adjusted. Pairing gaps resulting from different pairing interactions are compared to experimental ones.

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The pairing interaction plays a crucial role in the nuclear mean-field model calculations. The most frequently used pairing type is the so-called monopole pairing interaction (g = const model). More realistic residual interactions are those with state-dependent matrix elements, *e.g.* δ- and Gogny-forces. Previously the Gogny-type pairing force were studied within conventional Hartree–Fock–Bogoliubov (HFB) method [1] and in fully self-consistent Relativistic Hartree–Fock–Bogoliubov theory (RHB) [2]. Here we investigate different kinds of pairing forces using the Nilsson mean-field model and BCS approximation.

Contrary to the finite-range Gogny force, the δ-interaction is the zero-range force. The zero-range nature of the pairing interaction tends to overestimate the coupling to continuum states [3]. This defect does not occur when using the Gogny force which is, however, cumbersome to handle; it involves sophisticated numerical techniques while the δ-force is relatively simple for numerical calculations. The effect of the finite range for the δ-force can be easily simulated by introducing an energy cutoff which plays the role of an additional parameter similarly like in the case of the monopole pairing model except that the δ-interaction is less sensitive to the cutoff.

These seemingly different pairing interactions should produce in fact similar results- the coherence length of nucleonic Cooper pair is of the order of

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size of a nucleus, therefore its structure should be insensitive to the details of the interaction in the particle–particle channel [4]. The aim of this paper is to show strong correlations and similarity between matrix elements of the pairing part of the Gogny force and of the δ-pairing interaction. The pairing strengths of the δ force for rare-earth nuclei are determined from empirical odd–even mass differences. Additionally it is shown that the diagonal matrix elements of the δ-pairing interaction fulfill some dependences known from the monopole pairing strength analysis, therefore an alternative way of adjusting δ-pairing strengths is given. In order to simplify the calculations we assume the Nilsson single particle potential as a generator of single particle states and wave functions.

The finite range part of the Gogny force has the form [5]

$$\hat{V}_{\mathrm{G}}(\vec{r}_{12}) = \sum_{i=1,2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau) \mathrm{e}^{-\vec{r}_{12}^{\,2}/\mu_i^2}\,, \tag{1}$$

where P_σ and P_τ are the spin and isospin exchange operators respectively. In all calculations the D1S parameters [7] have been used and it is assumed that only the isovector part of the interaction is active, *i.e.*, $P_\tau = 1$ (proton–proton and neutron–neutron channel). The zero-range δ interaction reads [3]

$$\hat{V}_\delta(\vec{r}_{12}) = V_0 \frac{1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2}{4} \delta(\vec{r}_{12})\,. \tag{2}$$

Both interactions can be written in a similar form

$$\hat{V}(\vec{r}_{12}) = V(|\vec{r}_1 - \vec{r}_2|)(A + BP_\sigma)\,, \tag{3}$$

where the radial part is given by (see *e.g.* [6])

$$V(|\vec{r}_1 - \vec{r}_2|) = \sum_{\lambda=0}^{\infty} V_\lambda(r_1 - r_2) \frac{4\pi}{2\lambda + 1} \sum_\mu Y^*_{\lambda\mu}(r_1) Y_{\lambda\mu}(r_2)\,. \tag{4}$$

It is convenient to calculate the matrix elements of the interaction (3) in J basis [2]:

$$V_{JMJ'M'} = \langle (ab)JM|\hat{V}(\vec{r}_{12})|(a'b')J'M'\rangle\,, \tag{5}$$

where $a = (l_a j_a)$. In the case of the same kind of nucleons and for the pairing channel one obtains

$$\begin{aligned} V_{\mathrm{G}} = \langle (ab)00|\hat{V}_{\mathrm{G}}(\vec{r}_{12})|(a'b')00\rangle &= \delta_{j_a j_b}\delta_{j_{a'} j_{b'}}\delta_{l_a l_b}\delta_{l_{a'} l_{b'}} \sum_{S=0,1;\lambda} (-1)^S (2S+1) \\ &\times (2l_a+1)(2l_b+1)\sqrt{2j_{a'}+1}\sqrt{2j_{b'}+1}\, V_\lambda(r,r')(A + B(2S-1)) \\ &\times \begin{pmatrix} l_a & \lambda & l_{a'} \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} l_a & l_{a'} & \lambda \\ l_{a'} & l_a & S \end{Bmatrix} \begin{Bmatrix} l_a & l_a & S \\ \frac{1}{2} & \frac{1}{2} & j_a \end{Bmatrix} \begin{Bmatrix} l_{a'} & l_{a'} & S \\ \frac{1}{2} & \frac{1}{2} & j_{a'} \end{Bmatrix}. \end{aligned} \tag{6}$$

$$V_\delta = \langle (ab)00|\hat{V}_{\rm G}(\vec{r}_{12})|(a'b')00\rangle = \delta_{j_a j_b}\delta_{j_{a'} j_{b'}}\delta_{l_a l_b}\delta_{l_{a'} l_{b'}}\delta(r-r')\frac{V_0}{r^2}$$
$$\times\sqrt{2j_{a'}+1}\sqrt{2j_{b'}+1}\sum_\lambda(-1)^{\lambda+j_a-j_{a'}}\frac{2\lambda+1}{8\pi}\begin{pmatrix} l_a & \lambda & l_{a'} \\ 0 & 0 & 0\end{pmatrix}^2 . \quad (7)$$

The allowed integer values for λ in the sums are $|l_a - l_{a'}| \leq \lambda \leq l_a + l_{a'}$.

The simplest pairing model, *i.e.*, the monopole pairing model, assumes that all of the matrix elements (5) in the pairing Hamiltonian are equal and state independent.

In order to simplify our studies the calculations were performed in Nilsson model [8]. Formula (6) consists of two spin terms ($S = 0$ and $S = 1$). Numerical calculations have shown that the $S = 1$ term contributes less than 0.1% of the total value and can be neglected in further calculations. Figure 1 shows diagonal matrix elements ($g_{aa} = (a\bar{a}|\hat{V}_{{\rm G}|\delta}\bar{a}a)$) of the Gogny force and of the δ interaction for protons in ^{138}Nd, in cases of equilibrium and zero deformations. In figure 2 non-diagonal matrix elements are shown for neutrons in ^{130}Nd. We can obtain approximately the same values of matrix elements for both interactions considered here by a proper choice of the δ force strength (V_0) which was not done in case of results shown in figures 1 and 2 for the transparency. Strong correlations between diagonal and non-diagonal matrix elements of both interactions are common for all considered nuclei, either for protons or for neutrons.

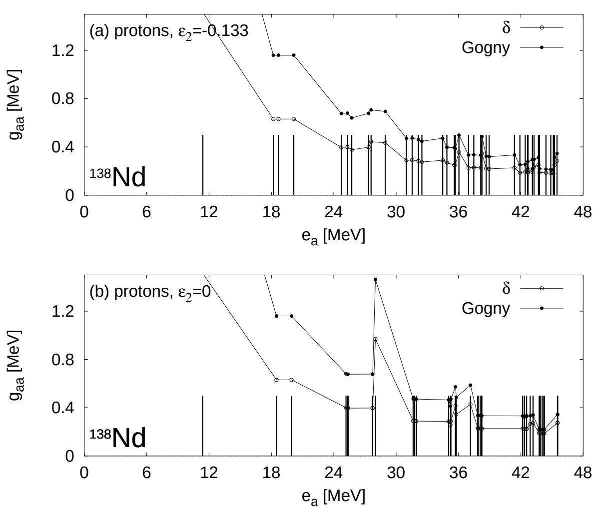

Fig. 1. Diagonal matrix elements $g_{aa} = (a\bar{a}|\hat{V}_{{\rm G}|\delta}|\bar{a}a)$ *versus* single particle energies for protons in ^{138}Nd. Vertical lines represent the single particle spectrum. The (a) part of the figure corresponds to the equilibrium deformation while the part (b) to the spherical case.

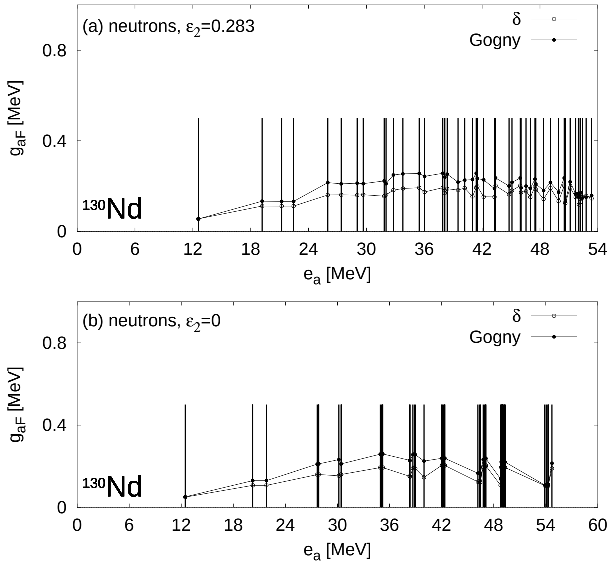

Fig. 2. Same as in Fig. 1 but for non-diagonal matrix elements which couple to the Fermi level F in the case of neutrons in ^{130}Nd.

Usually the pairing strength is determined from odd–even mass differences for separate nuclei. In Ref. [10] an explicit formula expressing the monopole pairing interaction strength g by the average gap parameter is derived and a simple dependence of g on the neutron and proton numbers is found. In the case of state-dependent pairing interactions the diagonal matrix elements follow the same particle number dependence as the monopole pairing strength.

Figure 3 shows the dependence of diagonal matrix elements g_{aa} of the δ-force on the particle number $2n$ equal to the doubled number of levels n (the possible number of particles occupying all of the levels up to the level n) for the spectrum ^{148}Ce. The dependence is of the type

$$g_{aa}(2n) = \mathrm{const}\frac{(\hbar\omega_{0\ell})}{(2n)^{2/3}}\,, \tag{8}$$

where $\hbar\omega_{0\ell} = 41.0[1 \pm (N-Z)/A]/A^{1/3}\,\mathrm{MeV}$. The plus sign holds here for neutrons ($\ell = N$) and the minus for protons ($l = Z$). The constant values found after the numerical fit of the δ-pairing strength to the mass differences are 0.310 $\hbar\omega_{0N}$ for neutrons and 0.300 $\hbar\omega_{0Z}$ for protons. This behavior resembles the results of Ref. [10] where it was shown that the monopole constants are $g_N = 0.284\hbar\omega_{0N}$ and $g_Z = 0.290\hbar\omega_{0Z}$. This leads to 5% differences in the matrix elements and may serve as an alternative way of adjusting an approximate δ-pairing strength which is especially useful in case of nuclei far from the beta stability line, where no experimental masses are known.

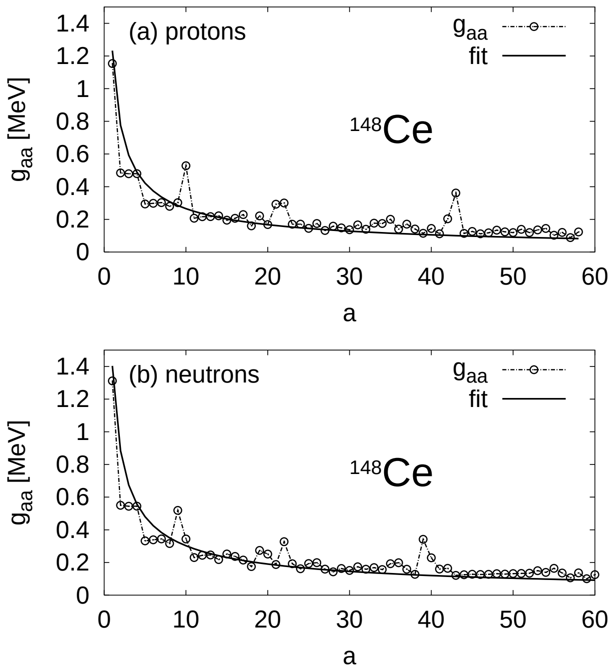

Fig. 3. Diagonal matrix elements g_{aa} of the δ interaction as a function of the level number a for protons (a) and neutrons (b) in ^{148}Ce. The solid lines represent the fit of the matrix elements according to Eq. (8).

Pairing gaps measure the lowest excitation energies of even–even nuclear systems and determine the binding energy of the coupled pair of nucleons. There are a few different ways of evaluating the pairing indicators from experimental nuclear masses. The three-point $\Delta^{(3)}$ indicator was proved to be a proper estimate of the pairing gap parameter [11]. It is defined as follows

$$\Delta^{(3)}(\mathcal{N}) = \frac{\pi_{\mathcal{N}}}{2}[B(\mathcal{N}-1) + B(\mathcal{N}+1) - 2B(\mathcal{N})]\,, \tag{9}$$

where $\pi_{\mathcal{N}} = (-1)^{\mathcal{N}}$ is the parity number and $B(\mathcal{N})$ is the binding energy of a system consisting of $\mathcal{N}$ particles. Theoretical pairing gaps are calculated in BCS model. State dependent BCS equations for the pairing gaps and the occupation probabilities are

$$\Delta_i = \frac{1}{2}\sum_j g_{ij}\frac{\Delta_j}{\sqrt{(e_j-\lambda)^2+\Delta_j^2}}\,,$$
$$v_i^2 = \frac{1}{2}\left(1-\frac{e_i-\lambda}{\sqrt{(e_i-\lambda)^2+\Delta_i^2}}\right)\,. \tag{10}$$

The set of BCS equations is solved numerically using a suitable iteration procedure. The strengths V_0 of the δ force for protons and neutrons were adjusted for the pairing window containing $2\sqrt{15Z(N)}$ levels, respectively. The values of V_0 were determined from the condition of the equality of the experimental pairing indicator $\Delta^{(3)}$ and the lowest quasiparticle energy for each nucleus and then the values of V_0 were averaged for the whole region. The strengths of the δ-pairing force for rare-earth nuclei obtained this way are [9]: $V_0^{\rm protons} = 240$ MeV fm^3, $V_0^{\rm neutrons} = 230$ MeV fm^3.

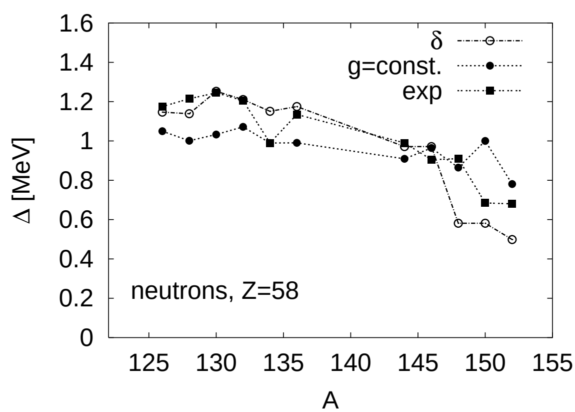

Fig. 4. Neutron pairing gaps calculated with the δ-pairing interaction (opened circles) and in the monopole pairing model (filled circles) in comparison to $\Delta^{(3)}$ pairing indicators (squares).

Figure 4 shows the pairing gaps for the δ interaction (opened circles) and the monopole pairing interaction (filled circles) in comparison to experimental pairing gaps (Eq. (9)) for Cerium isotopes. The monopole pairing strengths used in the calculations are those of Ref. [10]. The state dependent pairing results in theoretical pairing gaps which differ from $\Delta^{(3)}$ indicators on 10% while the monopole pairing gaps differ on 20%.

In conclusion, both the zero range δ force and the finite range part of the Gogny force have been investigated. It was shown that after proper renormalization both forces give the same matrix elements therefore they produce the same results. Thus the use of the δ force is justified and will reduce numerical efforts in various studies.

The $2n^{-2/3}$ dependence of the diagonal matrix elements of the δ-pairing force can be used in adjusting approximate δ-pairing strengths.

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