

MEDIUM EFFECTS OF LOW ENERGY PIONS*

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Fits of pion–nucleus potentials to large sets of pionic atom data reveal departures of parameter values from the corresponding free πN parameters. These medium effects can be quantitatively reproduced by a chiral-motivated model where the pion decay constant is modified in the medium or by including the empirical on-shell energy dependence of the amplitudes. No consistency is obtained between pionic atoms and the free πN interaction when an extreme off-shell chiral model is used. The role of the size of data sets is briefly discussed.

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1. Introduction

Renewed interest in pionic atoms in general, and in the s -wave part of the pion–nucleus potential in particular, stems from three recent developments. The first is the experimental observation of ‘deeply bound’ pionic atom states in the ($d, {}^3\text{He}$) reaction [1–4], the existence of which was predicted a decade earlier [5–7]. The second is the very accurate measurements of the shift and width of the $1s$ level in pionic hydrogen [8] and in pionic deuterium [9, 10] which leads to precise values for the s -wave πN scattering lengths (see also Ref. [11]). The third development is the attempt to explain the ‘anomalous’ s -wave repulsion [12] in terms of a density dependence of the pion decay constant [13], or very recently by constructing the πN amplitude near threshold within a systematic chiral perturbation expansion [14] and in particular imposing on it gauge invariance [15, 16].

The so-called anomalous repulsion of the s -wave pionic atom potential is the empirical finding, from fits of optical potentials to pionic atom data, that the strength of the repulsive s -wave potential inside nuclei is typically twice as large as is expected on the basis of the free πN interaction. This

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enhancement is, to a large extent, due to the value of the potential parameter b_1 , the in-medium *isovector s-wave* πN amplitude. The possible modification in the medium of this amplitude is the topic of the present work. One may define medium effects in this context as unexpected results obtained from analyses of experimental data, usually with rather simple models, based in the present case on the free pion–nucleon interaction.

Section 2 presents briefly the standard pion–nucleus potential [17] where the various parameters are defined. Section 3 shows results of fits of the standard potential to a large-scale (‘global’) set of data where medium effects are presented in a quantitative way. Section 4 shows similar results for the model due to Weise [13] where the pion decay constant is assumed to be modified in the medium. An alternative approach where the medium effects are essentially due to the energy dependence of the amplitudes is discussed in Section 5 both for a chiral (off shell) approach and for an empirical (on-shell) approach. In Section 6 the statistical significance of the results and their relation to the extent of the data base are discussed together with consequences of constraining parameter values. Section 7 is a summary.

2. The pion–nucleus potential

The optical potential V_{opt} used in the Klein–Gordon equation is of the standard form due to Ericson and Ericson [17],

$$2\mu V_{\text{opt}}(r) = q(r) + \vec{\nabla} \cdot \alpha(r) \vec{\nabla} \quad (1)$$

with

$$q(r) = -4\pi \left(1 + \frac{\mu}{M}\right) \{ \bar{b}_0(r)[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)] \} \\ - 4\pi \left(1 + \frac{\mu}{2M}\right) 4B_0 \rho_n(r) \rho_p(r), \quad (2)$$

$$\alpha(r) = \frac{\alpha_1(r)}{1 + \frac{1}{3}\xi\alpha_1(r)} + \alpha_2(r), \quad (3)$$

where

$$\alpha_1(r) = 4\pi \left(1 + \frac{\mu}{M}\right)^{-1} \{ c_0[\rho_n(r) + \rho_p(r)] + c_1[\rho_n(r) - \rho_p(r)] \}, \quad (4)$$

$$\alpha_2(r) = 4\pi \left(1 + \frac{\mu}{2M}\right)^{-1} 4C_0 \rho_n(r) \rho_p(r). \quad (5)$$

In these expressions ρ_n and ρ_p are the neutron and proton density distributions normalized to the number of neutrons N and number of protons Z , respectively, μ is the pion–nucleus reduced mass and M is the mass of the

nucleon, $q(r)$ is referred to as the s -wave potential term and $\alpha(r)$ is referred to as the p -wave potential term. The latter will not be discussed here further as there are essentially no non-trivial medium effects with this term. The function $\bar{b}_0(r)$ in Eq. (2) is given in terms of the *local* Fermi momentum $k_F(r)$ corresponding to the isoscalar nucleon density distribution:

$$\bar{b}_0(r) = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)k_F(r), \quad (6)$$

where b_0 and b_1 are minus the pion–nucleon isoscalar and isovector effective scattering lengths, respectively. The quadratic terms in b_0 and b_1 represent double-scattering modifications of b_0 . In particular, the b_1^2 term represents a sizable correction to the nearly vanishing linear b_0 term.

The nuclear density distributions ρ_p and ρ_n are an essential part of the pion–nucleus potential. Density distributions for protons can be obtained by unfolding the finite size of the charge of the proton from charge distributions obtained from experiments with electrons or muons. This leads to reliable proton densities, particularly in the surface region, which is the relevant region for producing strong-interaction effects in pionic atoms. The neutron distributions are, however, generally not known to sufficient accuracy and we have therefore adopted a semi-phenomenological approach that covers a broad range of possible neutron density distributions. The feature of neutron density distributions which is most effective in determining parameter values of the potential is the radial extent, as represented, for example, by r_n , the rms radius of the neutron density. Other features such as the detailed shape of the distribution have only minor effect on values of potential parameters, although they do affect the quality of fits. For that reason we chose the rms radius as the prime parameter in the present study, and focus attention on values of $r_n - r_p$, the difference between the rms radii. A simple parameterization adopted here that is easy to relate to the results of relativistic mean field (RMF) calculations [18, 19] is

$$r_n - r_p = \alpha \frac{N - Z}{A} + \eta. \quad (7)$$

In order to allow for possible differences in the shape of the neutron distribution, the ‘skin’ and the ‘halo’ forms of Ref. [19] have been used. Note that the results of RMF calculations are described very well [19] by using $\alpha = 1.5$ fm and $\eta = -0.03$ fm. This value of α is probably an upper limit as other sources of information suggest values close to 1–1.2 fm.

3. Results for the standard potential

Results obtained from the use of the standard potential of the previous section, here marked as ‘conventional’, with 100 data points along the periodic table, from ^{20}Ne to ^{238}U , are shown in Fig. 1. These data include also the deeply bound $1s$ states in ^{205}Pb [3] and in $^{115,119,123}\text{Sn}$ [4]. The figure shows that for $\alpha = 1.5$ and for the ‘skin’ type of density a very good fit to the data is obtained (χ^2 per point of 1.7) but the resulting value of b_1 is more repulsive than the free πN value by at least two error bars, or three error bars if the more reasonable value of $\alpha = 1.25$ is adopted as representing the average behaviour of rms radii of neutron distributions. This enhanced repulsion is a clear indication of medium effects. Other medium effects in the potential are likely to be present in the phenomenological two-nucleon absorption terms B_0 and C_0 which also have real dispersive parts. These are more difficult to handle and we will not discuss these here except for noting that the real parts of B_0 and C_0 are expected not to exceed in absolute values the corresponding imaginary parts, respectively. The anomalous s -wave repulsion observed with the conventional potential is due to the combined action of the too repulsive b_1 (which enters also quadratically, see Eq. (6)) and of $\text{Re}B_0$ which is also found to be too repulsive compared to the above expectations (see also Section 6 below).

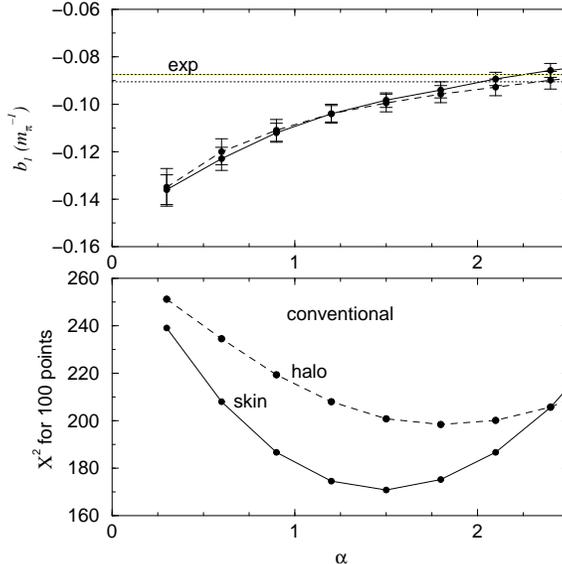


Fig. 1. Values of χ^2 for 100 data points (lower) and best-fit values of b_1 (upper) as function of the neutron density parameter α . ‘exp’ marks the experimental value for the free πN interaction.

4. Medium-modified pion decay constant

The in-medium s -wave interaction of pions have been discussed recently by Weise [13] in terms of partial restoration of chiral symmetry in dense matter. Since b_1 in free-space is well approximated in lowest chiral-expansion order by the Tomozawa–Weinberg expression [20]

$$b_1 = -\frac{\mu_{\pi N}}{8\pi f_\pi^2} = -0.08 m_\pi^{-1}, \tag{8}$$

then it can be argued that b_1 will be modified in pionic atoms if the pion decay constant f_π is modified in the medium. The square of this decay constant is given, in leading order, as a linear function of the nuclear density,

$$f_\pi^{*2} = f_\pi^2 - \frac{\sigma}{m_\pi^2} \rho \tag{9}$$

with σ the pion–nucleon sigma term. This leads to a density-dependent isovector amplitude such that b_1 becomes

$$b_1(\rho) = \frac{b_1(0)}{1 - 2.3\rho} \tag{10}$$

for $\sigma = 50$ MeV [21] and with ρ in units of fm^{-3} . This model was found [22, 23] to be very successful when tested against large data sets.

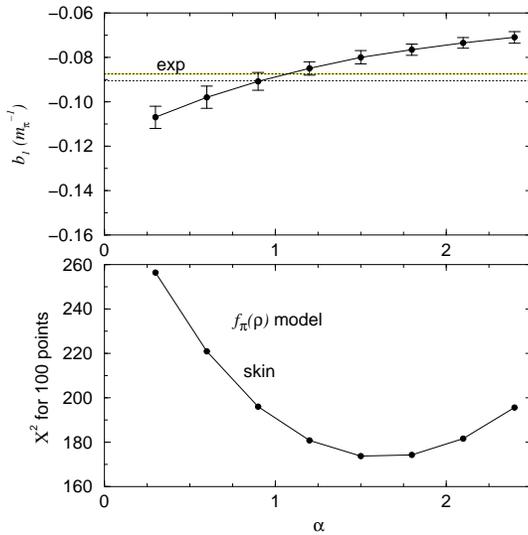


Fig. 2. Values of χ^2 for 100 data points (lower) and best-fit values of b_1 (upper) as function of the neutron density parameter α . ‘exp’ marks the experimental value for the free πN interaction.

Figure 2 shows results similar to those of Fig. 1 but with b_1 as given by Eq. (10). Only the results for the ‘skin’ shape of the neutron densities are plotted as the ‘halo’ shape always produces poorer fits to the data. It is seen from the figure that the quality of fits to the data is the same as for the conventional model, but the values of b_1 have shifted now such that for $\alpha \sim 1.25$ they agree with the free πN value, thus indicating that the density dependence of the decay constant could account for the medium modification of b_1 . Using the present model the parameter $\text{Re}B_0$ is found to be considerably less repulsive than in the conventional model, and within errors in agreement with expectations [22].

5. Energy-dependent amplitudes

Recently, Kolomeitsev, Kaiser and Weise [15] have suggested that pionic atom data could be reproduced using a pion optical potential underlain by chirally expanded πN amplitudes, retaining the energy dependence of the amplitudes $b_0(E)$ and $b_1(E)$ for zero-momentum ($\mathbf{q} = 0$) pions in nuclear matter in order to impose the minimal substitution requirement $E \rightarrow E - V_c$, where V_c is the Coulomb potential. This has the advantage of enabling one to use a systematic chiral expansion as an input [14], rather than singling out the leading order term Eq. (8) for b_1 .

The chiral expansion of the πN amplitudes for $\mathbf{q} = 0$ at the two-loop level is well approximated by the following expressions [14, 15]:

$$4\pi \left(1 + \frac{m_\pi}{M}\right) b_0(E) \approx \left(\frac{\sigma - \beta E^2}{f_\pi^2} + \frac{3g_A^2 m_\pi^3}{16\pi f_\pi^4}\right), \quad (11)$$

$$4\pi \left(1 + \frac{m_\pi}{M}\right) b_1(E) \approx -\frac{E}{2f_\pi^2} \left(1 + \frac{\gamma E^2}{(2\pi f_\pi)^2}\right), \quad (12)$$

g_A is the nucleon axial-vector coupling constant, $g_A = 1.27$, β and γ are tuned to reproduce the threshold values $b_0(m_\pi) \approx 0$ and $b_1(m_\pi) = -0.0885_{-0.0021}^{+0.0010} m_\pi^{-1}$ [8], respectively. For b_0 , in view of the accidental cancellations that lead to its near vanishing we limit our discussion to the f_π^{-2} term in Eq. (11), therefore choosing $\beta = \sigma m_\pi^{-2}$.

Implementing the minimal substitution requirement in the calculation of pionic atom observables, the constant parameters $b_{0,1}$ of the conventionally energy independent optical potential have been replaced in our calculation [24] by

$$b_{0,1}(r) = b_{0,1} - \delta_{0,1}(\text{Re}B + V_c(r)), \quad (13)$$

where $\delta_{0,1} = \partial b_{0,1}(E)/\partial E$ is the appropriate slope parameter at threshold, $\text{Re}B$ is the (real) binding energy of the corresponding pionic atom state and

$V_c(r)$ is the Coulomb potential. The constant fit parameters $b_{0,1}$ are then expected to agree with the corresponding free πN threshold amplitudes if the energy dependence is indeed responsible for the renormalized values found in conventional analyses.

In addition to the above ‘chiral’ energy dependence for *off-shell* $\mathbf{q} = 0$ pions we also present results for the empirically known *on-shell* πN amplitudes, when the pion energy E and its three-momentum \mathbf{q} are related by $E^2 = m_\pi^2 + \mathbf{q}^2$. This choice corresponds to the original suggestion by Ericson and Tauscher [25] to consider the effect of energy dependence in pionic atoms. Ericson subsequently [26] pointed out that, for strongly repulsive short-range NN correlations, the on-shell requirement follows naturally from the Agassi–Gal theorem [27] for scattering off non-overlapping nucleons. The corresponding πN amplitudes are denoted as ‘empirical’ and are taken from the SAID data base [28].

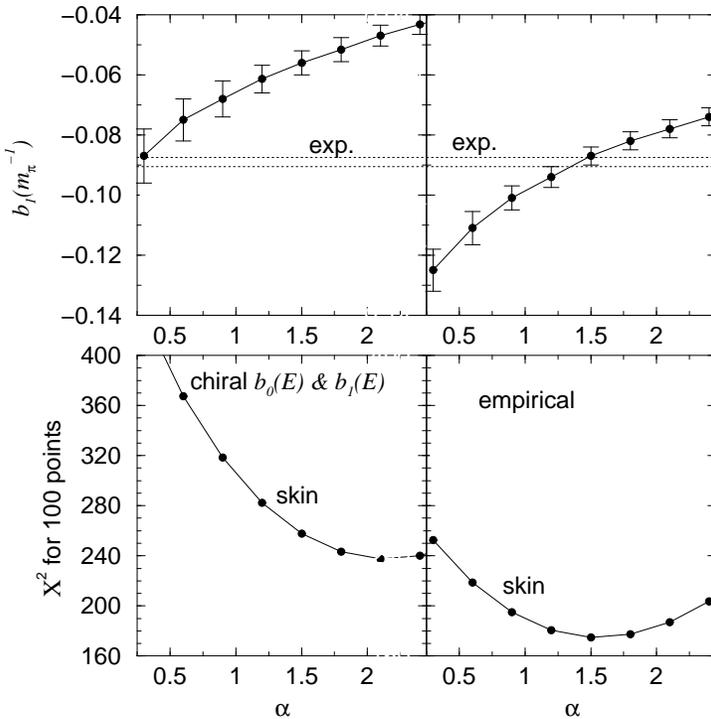


Fig. 3. Values of χ^2 for 100 data points (lower) and best-fit values of b_1 (upper) as function of the neutron density parameter α . ‘exp’ marks the experimental value for the free πN interaction. Left panels, for the chiral energy dependence of b_0 and b_1 ; right panels, for the empirical energy dependence.

Figure 3 shows results for the two kinds of energy-dependence mentioned above. The left panels show that implementing the chiral off-shell energy dependence leads to very significant deterioration in the fit to the data both for pionic atoms (lower part) and for the free πN interaction (upper part). Note also that the minimum in the χ^2 curve occurs at unacceptably large value of the parameter α . In contrast, the right hand side of the figure shows good agreement both for pionic atoms and for the free πN interaction, in accordance with the original suggestion made by Ericson and Tauscher [25]. Perhaps it is not too surprising that the off-shell $\mathbf{q} = 0$ nuclear matter approach is inadequate when the pion-nuclear optical potential that generates pionic-atom wavefunctions is being considered.

6. Statistical considerations and data sets

The results discussed so far have all been based on the ‘global 3’ data set of Ref. [19] which consists of 100 data points. Any conclusion regarding medium effects must obviously be linked to the uncertainties in the extracted parameter values, and these invariably depend on the size of the data sets used. In addition the uncertainties may depend critically on assumptions made in the analysis such as assigning fixed values to some parameters. These points are demonstrated in Table I.

The different columns of the table refer to various data sets, ranging from 120 points for nuclei between ^{12}C and ^{238}U to just 20 points where the deeply bound states provide half of the data. The top half of the Table is for unrestricted χ^2 fits whereas the lower part is for corresponding fits with the parameter $\text{Re}B_0$ set to zero. All the results in this table are for a conventional potential, *i.e.* without explicit medium effects such as density-dependence or energy-dependence of the amplitudes, and for neutron densities calculated for $\alpha = 1.5$ (Eq. (7)). From the top half of the table it is seen that the values of b_0 are essentially in agreement with the free πN value, within their rather large uncertainties and that the values of b_1 disagree with the corresponding free πN value only for the two large data sets since the uncertainties become too large for the two smaller data sets to reach such a conclusion. This distinction becomes even sharper when the uncertainty due to the neutron distributions is also included [19]. Another feature of the top half of the table is that the parameter $\text{Re}B_0$ is distinctly different from zero, although its uncertainty is not small, only when the large data bases are being used. For the two smaller data sets its value is consistent with zero.

Some recent papers insist on using the value of $\text{Re}B_0 = 0$ [2–4] and the lower part of the table addresses the consequences of such a constraint. The first consequence is that the uncertainty of the parameter b_0 becomes much smaller than before and its values, in all cases, deviate sharply from the free

TABLE I

Dependence of parameter values and uncertainties on the extent of data base and on assumptions made regarding $\text{Re}B_0$. Deterioration in the quality of fits is measured by $\frac{\Delta\chi^2}{\chi^2/F}$, the increase of χ^2 in units of χ^2 per degree of freedom. Values of r_n are for $\alpha = 1.5$.

Data	'global 2' ^{12}C to ^{238}U	'global 3' ^{20}Ne to ^{238}U	light $N = Z$ + light $N > Z$ 1s only	light $N = Z$ +'deep' 1s only
Points	120	100	22	20
χ^2	237	171	54	35
χ^2/F	2.1	1.8	3.0	2.2
$b_0(m_\pi^{-1})$	0.000(6)	-0.001(7)	-0.009(17)	-0.016(13)
$b_1(m_\pi^{-1})$	-0.101(3)	-0.098(3)	-0.095(13)	-0.094(7)
$\text{Re}B_0(m_\pi^{-4})$	-0.085(30)	-0.082(30)	-0.048(72)	-0.017(60)
$\text{Im}B_0(m_\pi^{-4})$	0.049(2)	0.052(2)	0.049(2)	0.051(2)
χ^2	259	190	56	35
χ^2/F	2.3	2.0	2.9	2.1
$\frac{\Delta\chi^2}{\chi^2/F}$	9.6	10.6	0.7	0
$b_0(m_\pi^{-1})$	-0.018(1)	-0.019(1)	-0.020(3)	-0.020(2)
$b_1(m_\pi^{-1})$	-0.102(3)	-0.099(3)	-0.093(12)	-0.094(7)
$\text{Re}B_0(m_\pi^{-4})$	0. (fixed)	0. (fixed)	0. (fixed)	0. (fixed)
$\text{Im}B_0(m_\pi^{-4})$	0.048(2)	0.051(2)	0.048(2)	0.050(2)

'deep' refers to deeply bound 1s states in $^{115,119,123}\text{Sn}$ and ^{205}Pb .

$b_0 = -0.0001_{-0.0021}^{+0.0009} m_\pi^{-1}$ and $b_1 = -0.0885_{-0.0021}^{+0.0010} m_\pi^{-1}$ for the free πN interaction.

πN value. This conclusion remains valid also for all the types of medium effects for b_1 considered in the present work. The second consequence is that for the large data sets the constraint $\text{Re}B_0 = 0$ indeed leads to a significant deterioration in the quality of fits, as evidenced by the increase of χ^2 . Recall that the natural unit for such increases is the value of χ^2/F , the χ^2 per degree of freedom. For the large data sets the increase is 10 such units, which is very significant. Note also that the values of b_1 do *not* depend on whether or not $\text{Re}B_0$ is set to zero. Similar results are obtained for the other models for b_1 discussed above, but with values of b_1 in agreement with what is displayed in the figures. It is concluded that analyses of reduced data sets with the constraint of $\text{Re}B_0 = 0$ must lead to unreliable results. In fact, analysing *only* the deeply bound states for $^{115,119,123}\text{Sn}$ and ^{205}Pb , we obtain excellent fits to the data with values of b_1 anywhere between -0.07 and $-0.13 m_\pi^{-1}$.

7. Summary

We have shown that global fits to large sets of data on strong interaction effects in pionic atoms in terms of a theoretically-motivated optical potential lead to very good description of the data ($\chi^2/N \sim 1.7$) with well-determined s -wave isovector amplitude that differs from the corresponding free πN amplitude by three standard deviations. This difference indicates modification of the interaction in the medium and it was shown that a chiral-motivated model where the pion decay constant is modified in the medium is capable of reproducing the medium effects. An alternative approach where the empirical on-shell energy dependence of the πN amplitude is included within the minimal substitution $E \rightarrow E - V_c$ was also shown to be successful. In contrast, the fully off-shell chiral energy dependence of the amplitudes fails badly. The importance of unrestricted and unbiased fits to large scale data sets has been demonstrated.

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