Vol. 35 (2004) *ACTA PHYSICA POLONICA B* No 4

GRANULAR SEGREGATION BY AN OSCILLATING RATCHET MECHANISM*

Akito Igarashi

Department of Applied Mathematics and Physics,
Kyoto University, Kyoto 606-8501 Japan

and Chuichi Horiuchi

Mitsubishi Electric Information Systems Corporation
4-13-23 Shibaura, Minato-ku, Tokyo 108-0023, Japan

(Received January 9, 2004)

We report on a method to segregate granular mixtures which consist of two kinds of particles by an oscillating "*ratchet*" mechanism. The segregation system has an asymmetrical sawtooth-shaped base which is vertically oscillating. Such a ratchet base produces a directional current of particles owing to its transport property. It is a counterintuitive and interesting phenomenon that a vertically vibrated base transports particles horizontally. This system is studied with numerical simulations, and it is found that we can apply such a system to segregation of mixtures of particles with different properties (radius or mass). Furthermore, we find out that an appropriate inclination of the ratchet-base makes the quality of segregation high.

PACS numbers: 45.70.Mg, 05.40.Jc, 02.70.Ns

1. Introduction

Ratchets can produce a directed current of particles without any external directional force. They have been studied as a model of molecular motors [1, 2], and applied to investigations into their transport properties. A ratchet system, or Brownian motor, contains a saw-tooth shaped asymmetrical potential which is switching off from time to time, and transports Brownian particles with fluctuational thermal noise (random force). By such

* Presented at the XVI Marian Smoluchowski Symposium on Statistical Physics, Zakopane, Poland, September 6–11, 2003.

a ratchet mechanism, we can develop a new device for transportation or rectification or separation. Motivated by such a possibility, we investigate a method of granular segregation with the ratchet mechanism.

We have usually segregated granular mixtures with a sieve. The difference of particle's size or shape is essential for segregation with the use of a sieve. That is, smaller particles drop down to a box below the sieve, and larger ones still remain in the sieve. We cannot, however, segregate particles which have the same size and shape by sieves. Even in such a case, we can segregate mixtures by the method reported in this paper with the use of an oscillatory ratchet mechanism if particles have some different physical coefficients such as mass or a restitution coefficient. It is a counterintuitive and interesting phenomenon that a vertically vibrated base transports particles horizontally. Such a method has been studied recently by several authors [3–7].

In our model we vertically vibrate a two-dimensional box with an asymmetric sawtooth-shaped base. Particles poured upon such a vibrated box display directional net motion. Since the direction of the motion fundamentally depends on properties of particles and the shape of the base, we can construct a novel granular sieve if the two kinds of particles move with much different speeds or to opposite directions respectively on the same ratchet base.

We perform simulations with an event-driven algorithm [8, 9]. Each collision is handled as one event in our simulation. Determining the next event is a core part in this event-driven algorithm since we can calculate analytically the motion of particle between two events (collisions). In this paper, with the numerical simulation, we mainly study motion of particles, and demonstrate that it is possible to segregate mixtures when we set the values of the parameters of the system properly.

2. Our segregation model

We consider particles which are modeled as hard disks colliding with a base of a two-dimensional box, jumping and moving in the box. A particle has following parameters; mass m, radius r, friction coefficient μ, restitution coefficient e, tangential restitution coefficient β, and moment of inertia α. The base of the box shapes an asymmetrical sawtooth whose inclination is θ, and the number of teeth is L. A tooth of the ratchet base has following parameters, which determine the shape of a tooth: height h, width w, and asymmetric ratio parameter $a \quad (0 \leq a \leq 1)$ (Fig. 1). If the asymmetric parameter a is 0.5, it is an isosceles triangle. If $a < 0.5$, its right side slope is gentler than the left side one. We pour granular mixtures into the system, (Fig. 2) and the base is oscillating vertically with the amplitude A and frequency f.

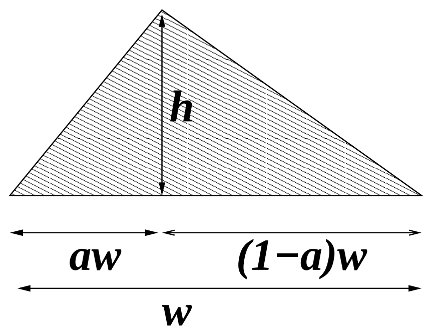


Fig. 1. The shape of a tooth of the ratchet base. h denotes the hight, w the width and a the asymmetric ratio parameter of a tooth.

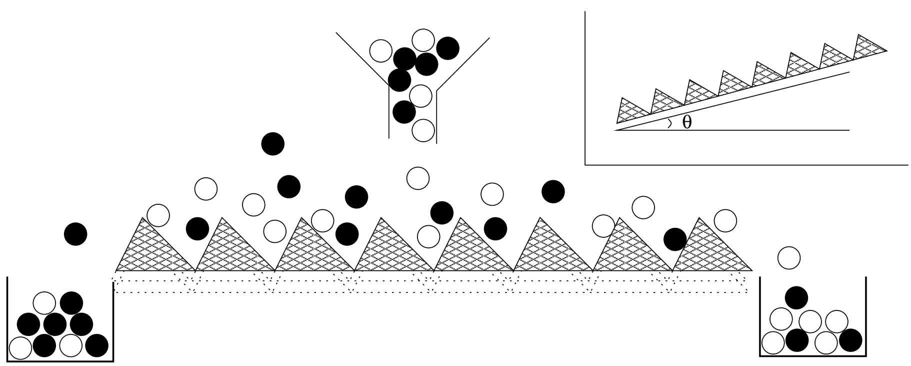


Fig. 2. Schematic figure of our segregation system. Mixtures consisting of two kinds of particles (white and black ones) are dropped to the ratchet base. If we choose parameters of the system appropriate, white and black particles mainly move to the left and right boxes, respectively. In particular, the inclination of the ratchet base, θ, is one of the most important of all the parameters.

The reason why particles have a directional net current is intuitively explained as follows: Since particles fall toward the base at a spatially uniform density, for $a < 0.5$ the particles fall to the long right slope more frequently than to the short left slope. Because of the shape of a tooth, the particles falling at the right gentle slope will go to the right and those falling at the left steep slope to the left after collisions with the ratchet base (see Fig. 1). Therefore, they tend to move to the right on the average.

We apply a simplified collision model based on the ideas of Luding, [8] where particles fundamentally follow Coulomb's friction law. A typical problem arising in event-driven granular simulations is called an inelastic collapse problem. To avoid this problem, we use a simple rule which restricts minimum relative velocities of particles after collisions [6].

3. Results

3.1. Single particle system

3.1.1. The directional current of particles

At first, we investigate a single particle system, that is, we ignore collisions between particles. Since a particle changes its velocity only when it collides with the base, the power source which produces a directional current is only the oscillation of the ratchet base.

Fig. 3 shows the result of the simulation for time evolution of distributions of particle's displacements, and we can see that the particle moves toward the right on the average, because the asymmetric parameter a is 0.3, that is, a right slope of the tooth is longer than a left one. We obtain that the ratio of the number of collisions with short slopes to that with long slopes is about 0.53 for the case of Fig. 3, which roughly coincides with the ratio of the length of the short and long slopes.

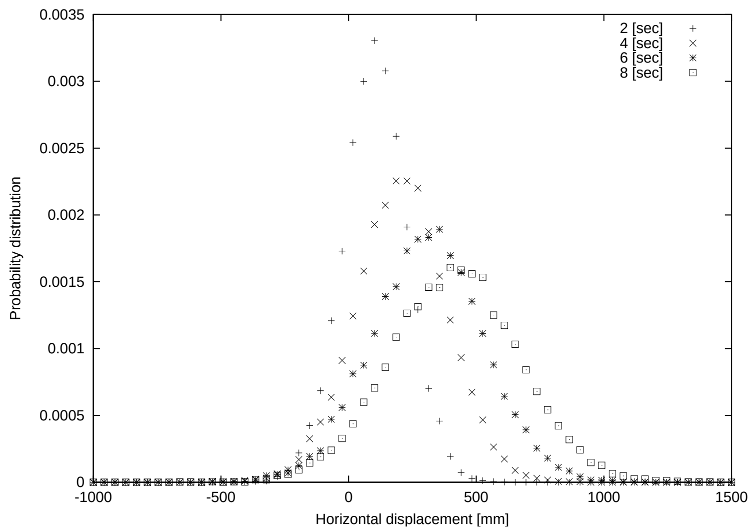
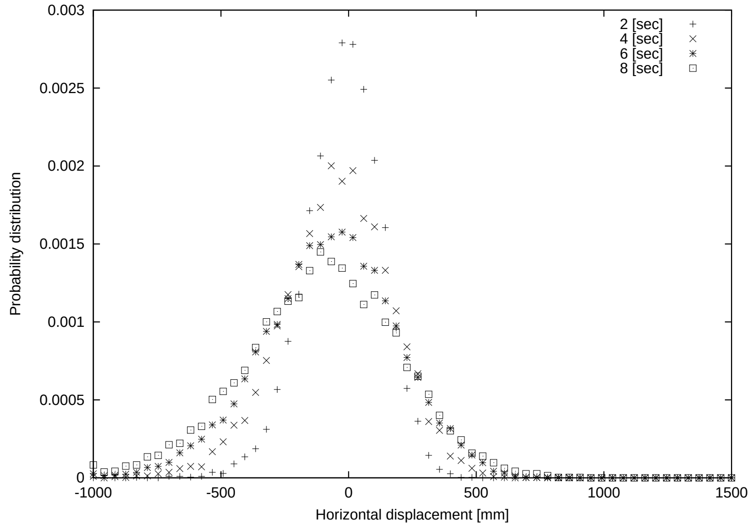


Fig. 3. Time evolution of distributions of particle's displacements. We set r=1mm, m=1g, e=0.6, μ=0.1, β=0.4, w=20mm, h=6mm, a=0.3, f=23Hz, A=2mm, and θ=0.

We can change the direction of current, inclining the base. Fig. 4 shows that the current changes its direction to the left on the average when we set the angle of inclination; $\theta = .209$ rad, even if we fixed a to be 0.3.



$\theta = 0.209$ rad

Fig. 4. Time evolution of distributions of the particle's displacements for $\theta = 0.209$ rad. Parameters are the same as in Fig. 3 except for θ. We observe current directed to the left on the average.

3.1.2. Relationship between other parameters and the average horizontal velocity $\langle V_x \rangle$

If we change parameters of particles or the ratchet-base, particles also change their average velocities.

In Fig. 5, we plot relations between the frequency f and the average horizontal velocity $\langle V_x \rangle$ for various a and conclude that the more asymmetric the tooth is, the faster velocity is obtained. In Fig. 7, we show $\langle V_x \rangle$ as a function of f for various θ with r = 1mm (A), and r = 4mm (B). The parameter θ intensely affects to the large particles as we can see in Fig. 7. The reason is explained with Fig. 6. If a radius of a particle is large enough, the net current of the particle depends on the inclination rather than its ratchet-shape because the ratchet-effect for large particles will become relatively ineffective as seen in Fig. 6. That is, while a large particle jumps to the right direction, it will be more easily caught on the slope in front than small one, and therefore the ratchet-effect is reduced.

Moreover, we can find suitable values of parameters for segregation if we change not only θ but other parameters. For example when we change a restitution coefficient e, the probability of particles jump to next valley will be changed. Therefore the net current will be also changed.

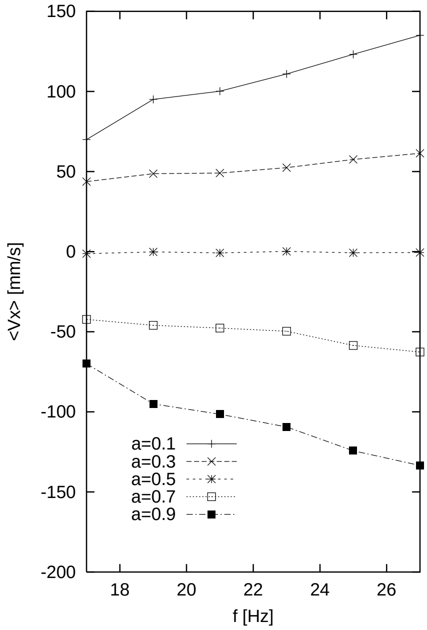


Fig. 5. The asymmetric parameter a determines the direction of the current. Parameters are the same as in Fig. 3 except for a.

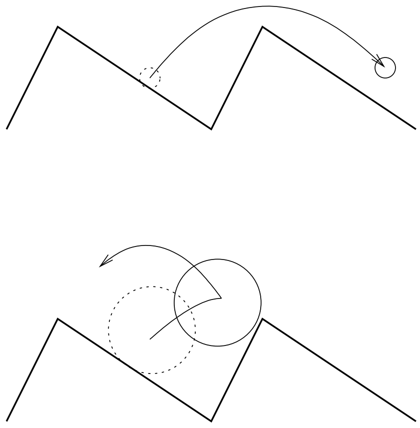
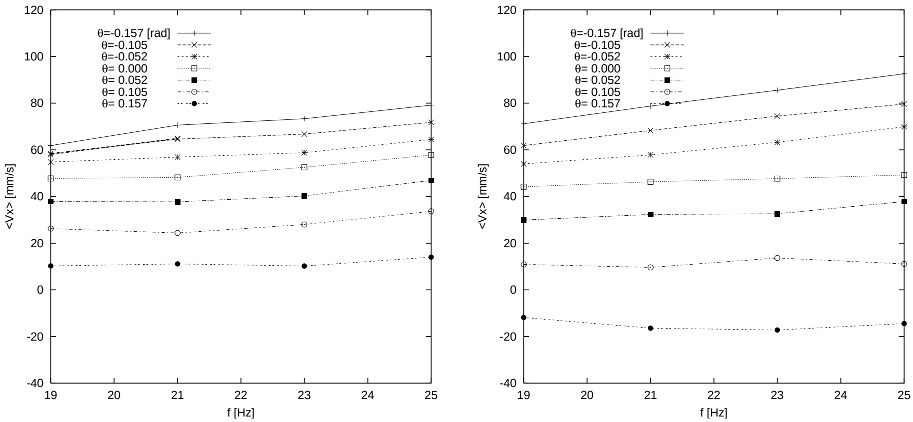


Fig. 6. After the collision with a gentle slope of the base, particles jump to the direction of the next valley. Although the small particle usually arrives at the next valley, the large particle sometimes collides with the steep slope of the next tooth and go back to the original valley. Therefore, the ratchet effect on motion of the large particle is reduced.

Because this is a result for single particle system, we may not easily apply this result to many particle systems because of collisions with other particles. In next section, we investigate systems with many particles in order to segregate mixtures.



(A) (B)

Fig. 7. These figures show $\langle V_x \rangle$ for various θ as a function of f. Parameters of the ratchet base are the same as in Fig. 3 except for θ and r. The radius of a particle is (A): $r = 1$mm, (B): $r = 4$mm. $\langle V_x \rangle$ is exceedingly influenced by the radius of particles.

3.2. System with many particles

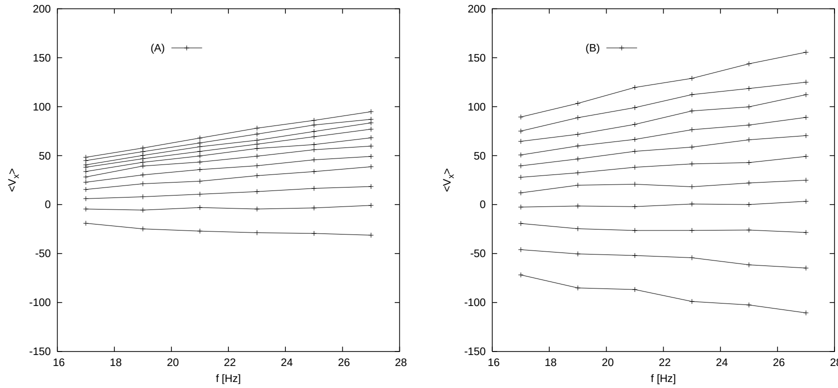
As mentioned in the last section, particles can be separated for appropriate parameters in the case where collisions between particles can be neglected. In this section we consider a system with N particles, where collisions with other particles cannot be ignored. We use mixtures consisting of two kinds of particles; particles (A) and particles (B), where its component ratio is 1:1. d, "*density*" of particles in the system, is defined by,

$$d = \frac{S}{W}, \tag{1}$$

where W is the horizontal length of the base, and S is the sum of diameters of all particles. Needless to say, d depends on N and its component ratio as well, and collisions between particles occur more times for larger d.

3.2.1. The dependence of particles' velocities on θ

From our simulation, it is shown in Fig. 8 that directional currents still remain as in the single particle case in Figs. 3 and 4 even if we take into account collisions with other particles.



(A) (B)

Fig. 8. The average velocity of each kind of particles for various θ in a mixture of particles (A) and (B), where (A): r = 1mm, m = 10g. (B): r = 2mm, m = 1g. Other parameters are the same as in Fig. 3 and d = 3.93. We plot lines for (A) and (B) when $\theta = -0.262$(Top lines), -0.209(second lines), $-0.157, -0.105, -0.05, 0, 0.105, 0.157, 0.209, 0.262$ rad. (Bottom lines). For example, the top lines of both figures represent $\langle V_x \rangle$ for each kind of particles when both (A) and (B) exist in the system simultaneously and $\theta = -0.262$ rad.

We set parameters $d = 3.93$ and use a periodic boundary condition. The particles (A) consists of ones for r = 1mm and m = 10g, and the particles (B) consists of ones for r = 2mm and m = 1g. Other parameters are the same as in Fig. 3 except for θ. If we choose particles with the same r and different m or the same m and different r as (A) and (B), the segregation quality of the mixture is not so good as the above (A) and (B).

For $\theta = 0$, particles (A) and particles (B) move, on the average, toward right direction together, but the particles (B) is faster than the particles (A). Hence we can segregate these particles by separating the faster particles and the slower particles. For $\theta = 0.052$ rad, since the motion of particles (A) and (B) have little difference, it is difficult to segregate. For $\theta = 0.105$ rad, the two kinds of particles move toward different directions respectively. Because the particles (A) go to the right and (B) go to the left on the average, we can segregate them if we place boxes on the right and left side ends of the system (Fig. 2).

Fig. 8 shows $\langle V_x \rangle$ as a function of f for both particles (A) and (B) in a mixture. Since the particles (B) consist of large ones, the angle θ affects more significantly them as we described in Section 3.1. The granular mixtures will be segregated when we set the angle of inclination to $\theta = 0.157$ rad. which displayed by the third lines from the bottom in Fig. 8, because the currents of two groups have opposite directions mutually.

3.2.2. Segregation quality

In this section we describe the segregation quality which is practically the most important variable. We define Q as the segregation quality, and P as the ratio of processed particles. Q is $0 \leq Q \leq 1$,

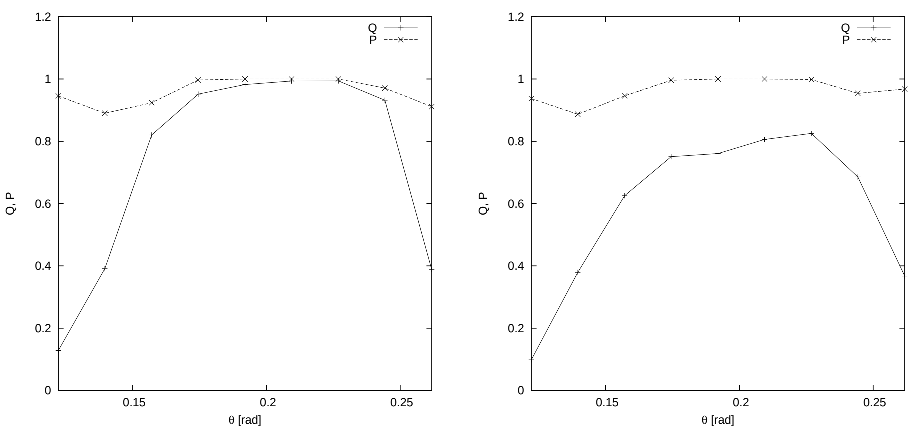
$$Q = \max\left(\frac{N_{\mathrm{al}}}{N_{\mathrm{a}}} + \frac{N_{\mathrm{br}}}{N_{\mathrm{b}}},\ \frac{N_{\mathrm{ar}}}{N_{\mathrm{a}}} + \frac{N_{\mathrm{bl}}}{N_{\mathrm{b}}}\right) - 1. \tag{2}$$

where N_{al} is a number of particles (A) dropped into the left box, N_{ar} is a number of particles (A) dropped into the right box, N_{bl} is a number of particles (B) dropped into the left box, N_{br} is a number of particles (B) dropped into the right box, and $N_{\mathrm{a,b}} = N_{\mathrm{al,bl}} + N_{\mathrm{ar,br}}$. If $Q = 1$, the mixture is perfectly segregated, and if $Q = 0$, the component ratio of the mixture remains as 1:1 after segregation. P is

$$P = \frac{N_{\mathrm{a}} + N_{\mathrm{b}}}{N}. \tag{3}$$

Of course, P and Q are defined only if $N_{\mathrm{a,b}} \neq 0$.

In Fig. 9, the values of parameters are set $a = 0.1$, and $f = 23$ and we use particles whose properties are ($r = 1$mm, $m = 10$g) and ($r = 5$mm, $m = 1$g), the other parameters are the same as in Fig. 3. Fig. 9 shows that we can segregate mixtures by this method and the quality of segregation become higher for low d if we adjust θ to an appropriate value. The more particles we pour into the system simultaneously (for large d case), the lower



$d = 4.08$ $d = 12.25$

Fig. 9. The segregation quality Q and processed ratio P are plotted as functions of θ, where we restrict the processing time for segregation to 100 sec.

segregation quality is obtained as seen in the right panel of Fig. 9. In such a case, we have to extend the system width and get d down in order to make the quality Q high. Therefore, in the high density case, the segregation quality is lowered by collisions between particles. The so-called Brazil-nut effects, [10] that is, in a vertically shaken container large particles tend to move upward, might be related to the lowering of the quality, because the large particles moving upward collide less frequently with the ratchet base.

4. Conclusion

In conclusion, we can segregate granular mixtures if we find appropriate values of parameters of the ratchet-base for which the currents of particles have much different values between two kinds of particles. Among the parameters, we, in particular, pay attention to the inclination of the ratchet base, θ, in this paper, and many other parameters still remain unexplored. If we adjust all parameters to appropriate values, it is not hard to predict that we obtain much better results for segregation. The single particle system is instrumental in conjecture of such appropriate values of parameters.

This work has been supported by a Grand-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

REFERENCES

[1] J. Prost, J.F. Chauwin, L. Peliti, A. Ajdari, *Phys. Rev. Lett.* **72**, 2652 (1994).
[2] A. Vilfan, E. Frey, F. Schwabl, *Europhys. Lett.* **45**, 283 (1999).
[3] Z. Farkas, F. Szalai, D.E. Wolf, T. Vicsek, *Phys. Rev.* **E65**, 022301 (2002).
[4] C. Keller, F. Marquardt, C. Bruder, *Phys. Rev.* **E65**, 041927 (2002).
[5] J.F. Wambaugh, C. Reichhardt, C.J. Olson, *Phys. Rev.* **E65**, 031308 (2002).
[6] Z. Farkas, P. Tegzes, A. Vukics, T. Vicsek, *Phys. Rev.* **E60**, 7022 (1999).
[7] Z. Farkas, T. Fülöp, *J. Phys. A: Math. Gen.* **34**, 3191 (2001).
[8] S. Luding, *Phys. Rev.* **E52**, 4442 (1995).
[9] S. McNamara, S. Luding, *Phys. Rev.* **E58**, 813 (1998).
[10] A. Rosato, K.M. Strandburg, F. Prinz, R.H. Swendson, *Phys. Rev. Lett.* **58**, 1038 (1987).