

SU(3) BREAKING IN DECAYS OF EXOTIC BARYONS

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Within the chiral soliton model the SU(3) breaking collective Hamiltonian introduces representation mixing in the baryonic wave functions. We calculate $\mathcal{O}(m_s)$ effects of this mixing on the decay widths of decuplet and antidecuplet baryons. We find importance of the 27-plet admixture in the Θ^+ and $\Xi_{\overline{10}}$ decays. The role of the $1/N_c$ nonleading terms in $\mathcal{O}(m_s)$ transition matrix elements is discussed.

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1. Introduction

There is almost no doubt today that the lightest member of the exotic antidecuplet has been discovered [1]. Most probably also the heaviest members of $\overline{10}$ were seen by NA49 experiment at CERN [2]. These states were predicted within the chiral soliton models [3–6]. Early estimates of *both* Θ^+ and $\Xi_{\overline{10}}$ masses from the second order mass formulae obtained in the Skyrme model are in a surprising agreement with present experimental findings [4]. Later, the masses, as well as the decay widths of the exotic states were computed within the chiral quark soliton model [5]. There, however, a freedom in relating the exotic-nonexotic splittings and the splittings within the exotic multiplets to the value of the pion–nucleon sigma term, $\Sigma_{\pi N}$, whose experimental value has varied over the years from 45 to almost 77 MeV [7], made the prediction of $\Xi_{\overline{10}}$ higher [5] than the 1860 MeV reported in [2]. That the chiral models can easily accommodate lighter $\Xi_{\overline{10}}$ masses is clear from earlier studies [4, 8, 9] and was emphasized recently in Ref. [10].

One of the most striking predictions of the seminal paper by Diakonov, Petrov and Polyakov [5] was the narrow width of antidecuplet states. Despite some misprints in this paper (see *e.g.* [10, 11]) and the model dependent corrections, the narrow width is one of the key features of the chiral model predictions which is in line with recent experimental findings. The small width appears due to the cancellation of the coupling constants multiplying 3 different group-theoretical structures entering the decay operator [5]. That this cancellation is consistent with the N_c counting, despite the fact that two out of the three above mentioned constants scale differently with N_c was shown in Ref. [12].

In Ref. [5] also the m_s corrections to the decay widths coming from the representation mixing in the baryonic wave functions, caused by the SU(3) breaking effective Hamiltonian, were estimated. However, two approximations have been used: firstly only the mixing with lowest possible representations was considered, boiling down to neglecting the SU(3)-flavor 27-plet; and secondly, these corrections were calculated only for the term leading in N_c . In Ref. [11] the second simplification was partially abandoned and in Ref. [9] the 27-plet contributions were evaluated, however, only for the leading term. It is the purpose of this work to discuss the m_s corrections to the decay widths without the above mentioned simplifications. Some of the results presented here were already discussed in Ref. [10].

The corrections due to the representation mixing constitute only a part of the full m_s correction which, however, are fully under control if the mass spectra are known. There exists another set of m_s corrections in the decay operator itself. The group theoretical structure of these terms is known [13], however, numerical analysis is not straightforward, since there are 3 new unknown constants which appear at this order. In the following we concentrate only on the mixing corrections. Therefore their magnitude can only serve as an estimate of a theoretical uncertainty introduced by m_s corrections.

Our findings can be summarized as follows: 27-plet admixtures are very important for the Θ^+ and $\Xi_{\overline{10}}$ decays, whereas for $\Sigma_{\overline{10}}$ decays they are only moderate and for $N_{\overline{10}}$ decays they can be safely neglected. Effects of the terms nonleading in N_c are important for Θ^+ decay: in fact they change the character of the correction from enhancement found in [5] to suppression discussed already in [10].

The paper is organized as follows: in Sect. 2 we introduce model parameters and discuss the magnitude of the representation mixing. In Sect. 3 we calculate decay corrections to the decuplet and antidecuplet decay widths. We explicitly display their dependence on the model parameter ρ and pion-nucleon sigma term, and examine the importance of the 27-plet. Conclusions are presented in Sect. 4.

2. Representation mixing

In a recent paper [10] it has been shown that the set of parameters of the symmetry breaking Hamiltonian

$$\hat{H}' = \alpha D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} D_{8i}^{(8)} \hat{S}_i \tag{1}$$

(where $D_{88}^{(8)}$ are SU(3) Wigner matrices, Y is hypercharge and \hat{S}_i is the collective spin operator [14]) which reproduces well the nonexotic spectra, as well as the measured mass of the $\Theta^+(1540)$ can be parameterized as follows¹:

$$\alpha = 336.4 - 12.9 \Sigma_{\pi N}, \quad \beta = -336.4 + 4.3 \Sigma_{\pi N}, \quad \gamma = -475.94 + 8.6 \Sigma_{\pi N}. \tag{2}$$

Moreover, the inertia parameters which describe the representation splittings

$$\Delta M_{10-8} = \frac{3}{2I_1}, \quad \Delta M_{\overline{10}-8} = \frac{3}{2I_2} \tag{3}$$

take the following values (in MeV)

$$\frac{1}{I_2} = 152.4, \quad \frac{1}{I_1} = 608.7 - 2.9 \Sigma_{\pi N}. \tag{4}$$

If, furthermore, one imposes additional constraint that $M_{\Xi_{10}^-} = 1860$ MeV, then $\Sigma_{\pi N} = 73$ MeV [10] (see also [16]) in agreement with recent experimental estimates [7]. The dependence of model parameters on $\Sigma_{\pi N}$ is plotted in Fig. 1(a).

Hamiltonian (1) introduces mixing between different representations [10, 13]:

$$\begin{aligned} |B_8\rangle &= |8_{1/2}, B\rangle + c_{10}^B |\overline{10}_{1/2}, B\rangle + c_{27}^B |27_{1/2}, B\rangle, \\ |B_{10}\rangle &= |10_{3/2}, B\rangle + a_{27}^B |27_{3/2}, B\rangle + a_{35}^B |35_{3/2}, B\rangle, \\ |B_{\overline{10}}\rangle &= |\overline{10}_{1/2}, B\rangle + d_8^B |8_{1/2}, B\rangle + d_{27}^B |27_{1/2}, B\rangle + d_{35}^B |\overline{35}_{1/2}, B\rangle, \end{aligned} \tag{5}$$

where $|B_{\mathcal{R}}\rangle$ denotes the state which reduces to the SU(3) representation \mathcal{R} in the formal limit $m_s \rightarrow 0$. The m_s dependent (through the linear m_s

¹ We use here $m_s/(m_u + m_d) = 12.9$ [15].

dependence of α, β and γ) coefficients in Eq. (5) read:

$$\begin{aligned}
 c_{\overline{10}}^B &= c_{\overline{10}} \begin{bmatrix} \sqrt{5} \\ 0 \\ \sqrt{5} \\ 0 \end{bmatrix}, & c_{27}^B &= c_{27} \begin{bmatrix} \sqrt{6} \\ 3 \\ 2 \\ \sqrt{6} \end{bmatrix}, \\
 a_{27}^B &= a_{27} \begin{bmatrix} \sqrt{15/2} \\ 2 \\ \sqrt{3/2} \\ 0 \end{bmatrix}, & a_{35}^B &= a_{35} \begin{bmatrix} 5/\sqrt{14} \\ 2\sqrt{5/7} \\ 3\sqrt{5/14} \\ 2\sqrt{5/7} \end{bmatrix}, \\
 d_8^B &= d_8 \begin{bmatrix} 0 \\ \sqrt{5} \\ \sqrt{5} \\ 0 \end{bmatrix}, & d_{27}^B &= d_{27} \begin{bmatrix} 0 \\ \sqrt{3/10} \\ 2/\sqrt{5} \\ \sqrt{3/2} \end{bmatrix}, & d_{\overline{35}}^B &= d_{\overline{35}} \begin{bmatrix} 1/\sqrt{7} \\ 3/(2\sqrt{14}) \\ 1/\sqrt{7} \\ \sqrt{5/56} \end{bmatrix}
 \end{aligned} \tag{6}$$

in the basis $[N, \Lambda, \Sigma, \Xi], [\Delta, \Sigma^*, \Xi^*, \Omega]$ and $[\Theta^+, N_{\overline{10}}, \Sigma_{\overline{10}}, \Xi_{\overline{10}}]$, respectively, and

$$\begin{aligned}
 c_{\overline{10}} &= -\frac{I_2}{15} \left(\alpha + \frac{1}{2}\gamma \right), & c_{27} &= -\frac{I_2}{25} \left(\alpha - \frac{1}{6}\gamma \right), \\
 a_{27} &= -\frac{I_2}{8} \left(\alpha + \frac{5}{6}\gamma \right), & a_{35} &= -\frac{I_2}{24} \left(\alpha - \frac{1}{2}\gamma \right), \\
 d_8 &= \frac{I_2}{15} \left(\alpha + \frac{1}{2}\gamma \right), & d_{27} &= -\frac{I_2}{8} \left(\alpha - \frac{7}{6}\gamma \right), & d_{\overline{35}} &= -\frac{I_2}{4} \left(\alpha + \frac{1}{6}\gamma \right).
 \end{aligned} \tag{7}$$

In Fig. 1(b) we plot the value of mixing coefficients (6) for the Σ particle². We see from Fig. 1(b) that previously neglected [5, 11] mixing with 27 both for 10 and $\overline{10}$ are potentially important (the latter one only for not too small $\Sigma_{\pi N}$). Whether the entire correction to the decay widths due to the admixture of $\mathcal{R} = 27$ remains large, depends on the values of the pertinent transition matrix elements which will be calculated in the next section.

² Note that Σ mixes in all the above representations, and therefore is useful for the sake of illustration.

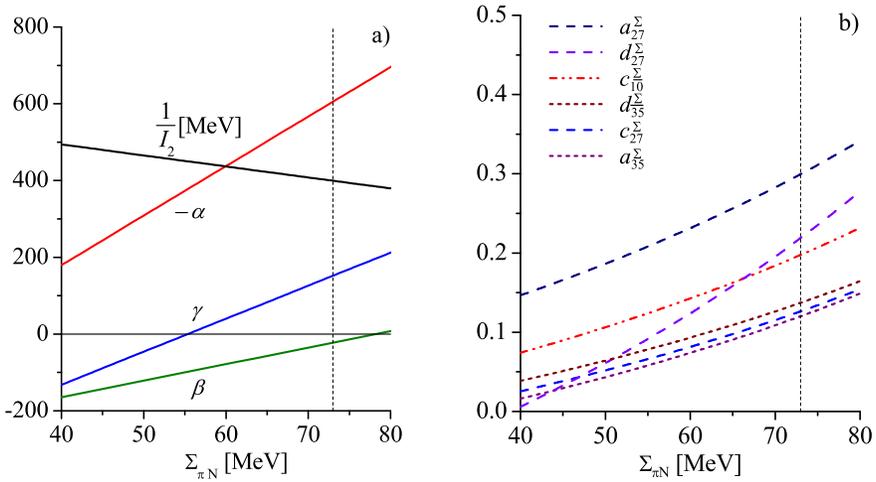


Fig. 1. Parameters of the splitting Hamiltonian (1) and $1/I_2$ (a) and (b) mixing coefficients (6) for Σ as functions of pion–nucleon sigma term $\Sigma_{\pi N}$.

3. Decay widths

In this section we shall calculate matrix elements which enter into the formula for the decay width for $B \rightarrow B' + \varphi$:

$$\Gamma_{B \rightarrow B' + \varphi} = \frac{1}{8\pi} \frac{p}{M M'} \overline{\mathcal{M}^2} = \frac{1}{8\pi} \frac{p^3}{M M'} \overline{\mathcal{A}^2} \quad (8)$$

up to linear order in m_s . The “bar” over the amplitude squared denotes averaging over initial and summing over final spin (and, if explicitly indicated, over isospin). Anticipating linear momentum dependence of the decay amplitude \mathcal{M} we have introduced reduced amplitude \mathcal{A} which does not depend on the kinematics, *i.e.* on the meson momentum p

$$p = \frac{\sqrt{(M^2 - (M' - m_\varphi)^2)(M^2 - (M' + m_\varphi)^2)}}{2M}. \quad (9)$$

For the discussion of the validity of (8) see [10]. In order to match former normalization [5] we shall define the decay amplitude as

$$\begin{aligned} \mathcal{M}_{B \rightarrow B' + \varphi} &= \langle B' | \hat{O}_\varphi^{(8)} | B \rangle \\ &= 3 \langle B' | G_0 D_{\varphi i} - G_1 d_{ibc} D_{\varphi b}^{(8)} \hat{S}_c - \frac{G_2}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{S}_i | B \rangle \times p_i, \end{aligned} \quad (10)$$

where the sum over repeated indices is assumed: $i = 1, 2, 3$ and $b, c = 4, \dots, 7$.

Coupling constants $G_{0,1,2}$ can be related to the elements of the axial current operators yielding relations [5, 13]:

$$\frac{9 F}{5 D} = \frac{G_0 + \frac{1}{2}G_1 + \frac{1}{2}G_2}{G_0 + \frac{1}{2}G_1 - \frac{1}{6}G_2} \tag{11}$$

and to

$$G_2 = \frac{2M_N}{3F_\pi} \Delta\Sigma, \tag{12}$$

where $\Delta\Sigma = 0.3 \pm 0.1$ is the “spin content of the proton” and $F_\pi = 93$ MeV. Note that formally constants G_0 and $G_{1,2}$ are of different order in N_c , however, as has been shown in [12] additional N_c dependence comes from the SU(3) Clebsch–Gordan coefficients [17].

In the following we shall use

$$G_1 = \rho G_0, \quad G_2 = \varepsilon G_0. \tag{13}$$

Equation (11) introduces relation between ρ and ε :

$$\varepsilon = \frac{9\frac{F}{D} - 5}{9\frac{F}{D} + 5}(\rho + 2). \tag{14}$$

Throughout this paper we fix $F/D = 0.59$ following Ref. [10].

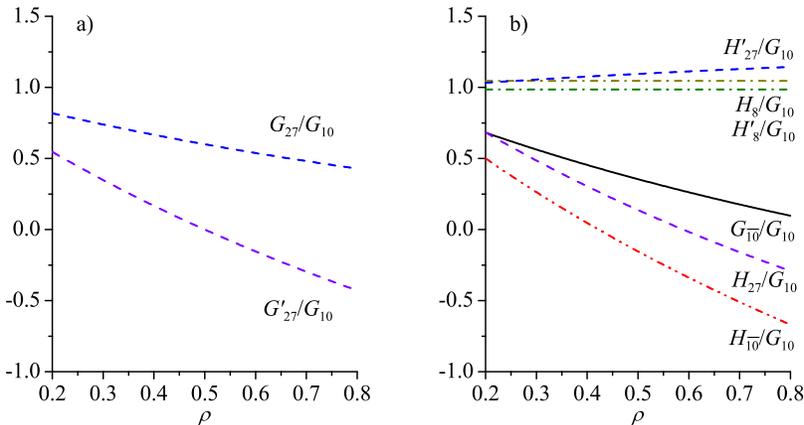


Fig. 2. Ratios of the effective couplings (a) (17) and (b) (24) to G_{10} as functions of the parameter ρ defined in Eq. (13).

3.1. Decuplet decays

Now let us consider matrix elements of the decay operator (10) between states (5). Decuplet can only decay to octet, and we have

$$\begin{aligned} \langle B'_8 | \hat{O}_\varphi^{(8)} | B_{10} \rangle &= \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | 10_{3/2}, B \rangle \\ &+ a_{27}^B \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | 27_{3/2}, B \rangle \\ &+ c_{27}^{B'} \langle 27_{1/2}, B' | \hat{O}_\varphi^{(8)} | 10_{3/2}, B \rangle . \end{aligned} \quad (15)$$

It is convenient to choose $\vec{p} = (0, 0, p)$. Then the matrix elements for $S_3 = S'_3 = 1/2$ read:

$$\begin{aligned} \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | 10_{3/2}, B \rangle &= 3 \frac{2}{\sqrt{15}} G_{10} \left(\begin{array}{cc|c} 8 & 8 & 10 \\ \varphi & B' & B \end{array} \right) \times p, \\ \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | 27_{3/2}, B \rangle &= 3 \frac{2\sqrt{2}}{9} G_{27} \left(\begin{array}{cc|c} 8 & 8 & 27 \\ \varphi & B' & B \end{array} \right) \times p, \\ \langle 27_{1/2}, B' | \hat{O}_\varphi^{(8)} | 10_{3/2}, B \rangle &= 3 \frac{1}{\sqrt{15}} G'_{27} \left(\begin{array}{cc|c} 8 & 27 & 10 \\ \varphi & B' & B \end{array} \right) \times p, \end{aligned} \quad (16)$$

where

$$G_{10} = G_0 + \frac{1}{2} G_1, \quad G_{27} = G_0 - \frac{1}{2} G_1, \quad G'_{27} = G_0 - 2G_1. \quad (17)$$

We see here that the transition matrix elements are generally different and depend on representations. For the decuplet decays we find that

$$G_{10} > G_{27} > G'_{27} \sim 0. \quad (18)$$

Hence, we do not expect very large modifications of decuplet decays widths. Most of the effect will come from 27 admixture in the initial 10 state rather than in the final octet state. However, the admixture coefficient a_{27} is relatively large, as can be seen from Fig. 1(b), and the enhancement factor due to the representation mixing will be of the order (for $\rho = 0.5$ and $\Sigma_{\pi N} = 73$ MeV)

$$R_{B \rightarrow B'+\varphi}^{(\text{mix})} \simeq 1 + 2 a_{27}^B \frac{G_{27}}{G_{10}} \times C_{B \rightarrow B'+\varphi} \sim 1 + 0.4 \times C_{B \rightarrow B'+\varphi}, \quad (19)$$

where $C_{B \rightarrow B'+\varphi} < 1$ is the Clebsch–Gordan factor for a given decay. In the table below we list possible decay modes and the relevant matrix elements. It should be stressed that for the consistency of the m_s expansion we should not literary square the matrix elements, but rather — as in Eq. (19) — keep only G_{10}^2 and the mixed term $2G_{10}(c G_{27} + c' G'_{27})$, neglecting $(c G_{27} + c' G'_{27})^2$.

Decay	Matrix element $\overline{\mathcal{A}}^2$
$\Delta \rightarrow N + \pi$	$\frac{3}{5} [G_{10} + \frac{5}{3}a_{27}G_{27} + \frac{1}{3}c_{27}G'_{27}]^2$
$\Sigma_{10} \rightarrow \Lambda + \pi$	$\frac{3}{10} [G_{10} + \frac{4}{3}a_{27}G_{27} + \frac{2}{3}c_{27}G'_{27}]^2$
$\Sigma_{10} \rightarrow \Sigma + \pi$	$\frac{3}{15} [G_{10} + c_{27}G'_{27}]^2$
$\Xi_{10} \rightarrow \Xi + \pi$	$\frac{3}{10} [G_{10} + \frac{7}{9}\sqrt{\frac{5}{6}}a_{27}G_{27} + \frac{3}{5}\sqrt{\frac{5}{6}}c_{27}G'_{27}]^2$

(20)

In Fig. 3 the correction factors

$$R^{(\text{mix})} = 1 + 2 \frac{\sqrt{\overline{\mathcal{A}}^2} - \sqrt{\mathcal{A}^2}|_{m_s=0}}{\sqrt{\mathcal{A}^2}|_{m_s=0}} \tag{21}$$

are plotted as functions of ρ and $\Sigma_{\pi N}$ for the decuplet decays displayed in Eq. (20). We see that they are moderately large for Δ and $\Sigma^* \rightarrow \Lambda + \pi$. For $\Sigma^* \rightarrow \Sigma + \pi$ the correction is small since it proceeds only through the admixture of 27 in the final state Σ , *i.e.* is given entirely in terms of G'_{27} ³.

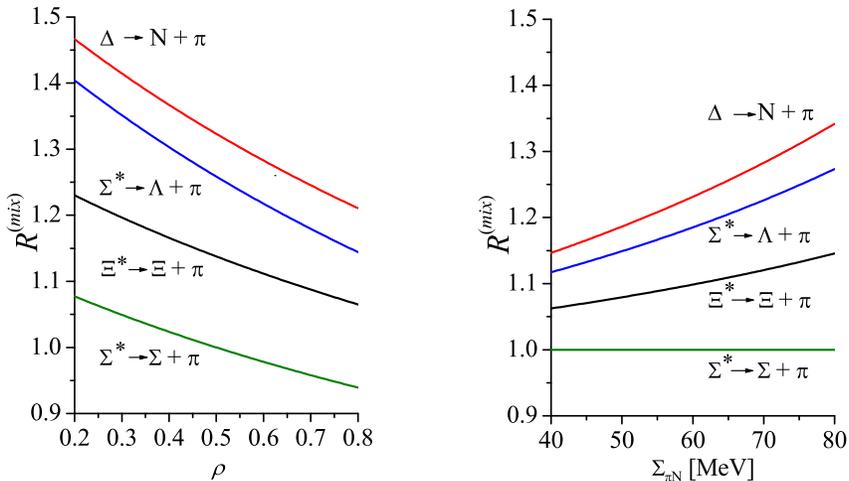


Fig. 3. Enhancement factors $R^{(\text{mix})}$ for the decuplet decays displayed in Eq. (20) as functions of ρ (for $\Sigma_{\pi N} = 73$ MeV) and $\Sigma_{\pi N}$ (for $\rho = 0.5$).

³ Note that the relevant SU(3) isoscalar factor vanishes for the admixture of 27 in the initial decuplet Σ^* .

3.2. Antidecuplet decays

Antidecuplet can directly decay only to octet:

$$\begin{aligned}
 \langle B'_8 | \hat{O}_\varphi^{(8)} | B_{\overline{10}} \rangle &= \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | \overline{10}_{1/2}, B \rangle \\
 &+ c_{10}^{B'} \langle \overline{10}_{1/2}, B' | \hat{O}_\varphi^{(8)} | \overline{10}_{1/2}, B \rangle \\
 &+ c_{27}^{B'} \langle 27_{1/2}, B' | \hat{O}_\varphi^{(8)} | \overline{10}_{1/2}, B \rangle \\
 &+ d_8^B \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | 8_{1/2}, B \rangle \\
 &+ d_{27}^B \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | 27_{1/2}, B \rangle.
 \end{aligned} \tag{22}$$

In Ref. [5] only the terms proportional to $c_{10}^{B'}$ and $d_8^B = -c_{10}^{B'}$ were considered. Moreover the assumption was made that all transition elements were equal to $G_{\overline{10}}$ defined below. Although this is true in the leading order in the “explicit” N_c counting⁴, the full expressions for these matrix elements are substantially different from $G_{\overline{10}}$ [10]. In Ref. [9] the admixtures of 27 were considered with, however, only the leading part for the transition elements. The comparison with [9] can be easily made by choosing $G_1 = G_2 = 0$, $G_0 = G_{\overline{10}}$ in the equations below.

In order to evaluate the transition matrix elements entering Eq. (22) it is convenient to choose $\vec{p} = (0, 0, p)$. Then the matrix elements for $S_3 = S'_3 = 1/2$ read:

$$\begin{aligned}
 \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | \overline{10}_{1/2}, B \rangle &= -3 \frac{1}{\sqrt{15}} G_{\overline{10}} \left(\begin{array}{cc|c} 8 & 8 & \overline{10} \\ \varphi & B' & B \end{array} \right) \times p, \\
 \langle \overline{10}_{1/2}, B' | \hat{O}_\varphi^{(8)} | \overline{10}_{1/2}, B \rangle &= 3 \frac{1}{2\sqrt{6}} H_{\overline{10}} \left(\begin{array}{cc|c} 8 & \overline{10} & \overline{10} \\ \varphi & B' & B \end{array} \right) \times p, \\
 \langle 27_{1/2}, B' | \hat{O}_\varphi^{(8)} | \overline{10}_{1/2}, B \rangle &= 3 \frac{7}{4\sqrt{15}} H'_{27} \left(\begin{array}{cc|c} 8 & 27 & \overline{10} \\ \varphi & B' & B \end{array} \right) \times p, \\
 \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | 8_{1/2}, B \rangle &= \\
 \frac{3}{2} \left[\frac{H_8}{\sqrt{3}} \left(\begin{array}{cc|c} 8 & 8 & 8_1 \\ \varphi & B' & B \end{array} \right) - \sqrt{\frac{3}{5}} H'_8 \left(\begin{array}{cc|c} 8 & 8 & 8_2 \\ \varphi & B' & B \end{array} \right) \right] \times p, \\
 \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | 27_{1/2}, B \rangle &= -3 \frac{1}{9} \sqrt{\frac{2}{5}} H_{27} \left(\begin{array}{cc|c} 8 & 8 & 27 \\ \varphi & B' & B \end{array} \right) \times p,
 \end{aligned} \tag{23}$$

where we have introduced the following constants⁵ [10, 11, 18]:

⁴ There is also “implicit” N_c dependence coming from the $SU(3)_{\text{flavor}}$ Clebsch–Gordan coefficients calculated for an arbitrary N_c [12, 17].

⁵ Note that our H'_{27} is identical to $H'_{\overline{10}}$ from Ref. [10].

$$\begin{aligned}
 G_{\overline{10}} &= G_0 - G_1 - \frac{1}{2}G_2, & H_{\overline{10}} &= G_0 - \frac{5}{2}G_1 + \frac{1}{2}G_2, \\
 H_{27} &= G_0 - 2G_1 + \frac{3}{2}G_2, & H'_{27} &= G_0 + \frac{11}{14}G_1 + \frac{3}{14}G_2, \\
 H_8 &= G_0 + \frac{1}{2}G_1 + \frac{1}{2}G_2, & H'_8 &= G_0 + \frac{1}{2}G_1 - \frac{1}{6}G_2.
 \end{aligned}
 \tag{24}$$

In Fig. 2(b) we plot the ratios of the transition constants (24) to G_{10} . Already here we see the potential source of trouble: the leading term in (22) is governed by $G_{\overline{10}}$ which is substantially smaller than G_{10} . This is the primary source of the suppression for the antidecuplet decay widths [5,10]. However, in view of the smallness of $G_{\overline{10}}$, the coefficients H'_{27} , H_8 and H'_8 , which come from the admixtures of 27 in the final octet state and of 8 in the initial antidecuplet state, pose a challenge to the validity of the m_s expansion, since the relevant mixing coefficients c_{27} and $d_8 = -c_{\overline{10}}$ are not enough suppressed. Therefore we might expect here relatively large corrections, their magnitude depending on the relative magnitude of different terms entering the expression for the decay amplitude \mathcal{A} .

Let us first list formulae for the decay widths of the (possibly) observed states Θ^+ and $\Xi_{\overline{10}}^+$:

Decay	Matrix element $\overline{\mathcal{A}^2}$
$\Theta^+ \rightarrow \begin{cases} n + K^+ \\ p + K^0 \end{cases}$	$\frac{3}{10} [G_{\overline{10}} + \frac{5}{4}c_{\overline{10}} H_{\overline{10}} - \frac{7}{4}c_{27} H'_{27}]^2$
$\Xi_{\overline{10}} \rightarrow \Xi + \pi$	$\frac{3}{10} [G_{\overline{10}} + \frac{7}{6}c_{27} H'_{27} + \frac{1}{3}d_{27} H_{27}]^2$
$\Xi_{\overline{10}} \rightarrow \Sigma + K$	$\frac{3}{10} [G_{\overline{10}} - \frac{5}{4}c_{\overline{10}} H_{\overline{10}} + \frac{7}{12}c_{27} H'_{27} - \frac{1}{3}d_{27} H_{27}]^2$

In Eq. (25) for $\Xi_{\overline{10}}$ we list expressions for the averaged decay widths (like $\Xi_{\overline{10}} \rightarrow \Xi + \pi$) that, however, for this particular case equal to the width of the specific decays of $\Xi_{\overline{10}}^{--}$ (like $\Xi_{\overline{10}}^{--} \rightarrow \Xi^- + \pi^-$). The relevant correction factors $R^{(mix)}$ are plotted in Fig. 4. We see that Θ^+ decay gets strongly suppressed, while $\Xi_{\overline{10}}$ decays get enhanced [10]. The strong suppression of Γ_{Θ^+} indicates that for $\rho \gtrsim 0.3$ and/or $\Sigma_{\pi N} \gtrsim 50$ MeV one cannot neglect the square of the admixtures which start to dominate. The numerical analysis of the impact of these and other factors on the physical decay widths was done in [10]. Let us remark here that for Θ^+ most of the negative correction comes from the 27 admixture, because the $\overline{10}$ admixture in the final nucleon is proportional to $H_{\overline{10}}$ which is small and changes sign around $\rho \sim 0.4$.

Therefore initial calculation of the m_s correction to this decay [5], which showed enhancement rather than suppression, indicates how important are the terms nonleading in N_c as well as the corrections due to the flavor 27-plet. The importance of the 27 admixtures is also visible in the case of $\Xi_{10}^- \rightarrow \Xi + \pi$ where all other admixtures are absent.

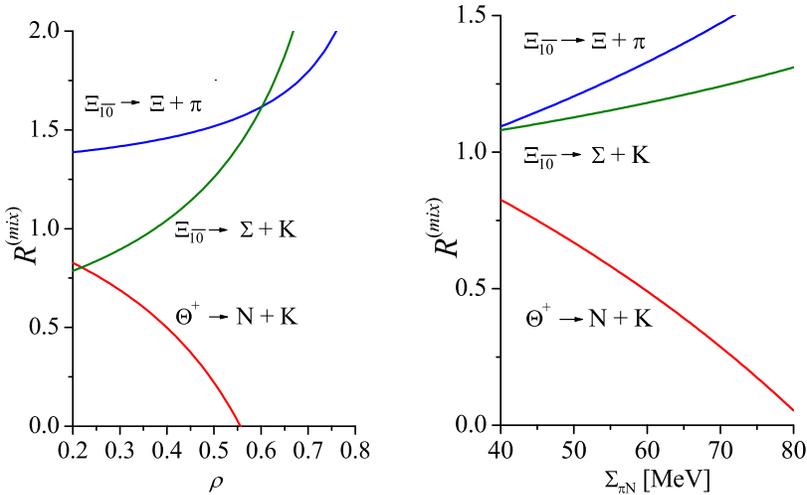


Fig. 4. Correction coefficients $R^{(mix)}$ for Θ^+ and Ξ_{10}^- decays as functions of ρ (for $\Sigma_{\pi N} = 73$ MeV) and $\Sigma_{\pi N}$ (for $\rho = 0.5$).

Next, let us list results for the nucleon-like states

Decay	Matrix element $\overline{\mathcal{A}}^2$
$N_{10}^- \rightarrow N + \pi$	$\frac{3}{20} [G_{10} + c_{10} (\frac{5}{4}H_{10} - \frac{5}{2}H_8 - \frac{9}{2}H'_8) + c_{27}\frac{49}{12}H'_{27} + \frac{1}{15}d_{27}H_{27}]^2$
$N_{10}^- \rightarrow N + \eta$	$\frac{3}{20} [G_{10} - c_{10} (\frac{5}{4}H_{10} - \frac{5}{2}H_8 + \frac{3}{2}H'_8) - \frac{7}{4}c_{27}H'_{27} - \frac{1}{5}d_{27}H_{27}]^2$
$N_{10}^- \rightarrow \Lambda + K$	$\frac{3}{20} [G_{10} + c_{10} (\frac{5}{2}H_8 + \frac{3}{2}H'_8) - \frac{7}{2}c_{27}H'_{27} + \frac{1}{5}d_{27}H_{27}]^2$
$N_{10}^- \rightarrow \Sigma + K$	$\frac{3}{20} [G_{10} + c_{10} (\frac{5}{2}H_{10} - \frac{5}{2}H_8 + \frac{9}{2}H'_8) - \frac{7}{3}c_{27}H'_{27} - \frac{1}{15}d_{27}H_{27}]^2$

(26)

which were discussed in some detail in Ref. [11]. Our formulae agree with the ones of Ref. [11] provided we neglect 27 admixtures and set $H_8 = H'_8$ which in view of Fig. 2(b) is legitimate. At the end of this section we shall discuss

the relative magnitude of the 27 admixtures with respect to the other ones. Note that for our set of parameters (2) for $\Sigma_{\pi N} = 73$ MeV $N_{\overline{10}}$ (1646 MeV) is below the threshold for $\Sigma + K$.

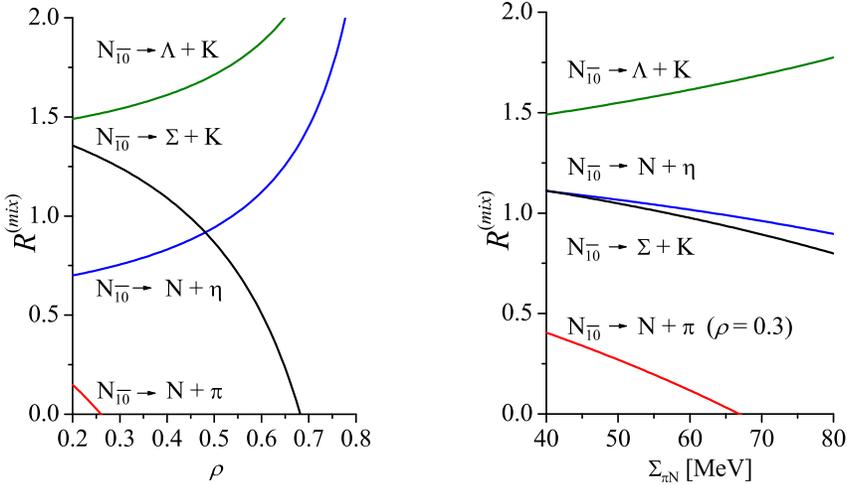


Fig. 5. Correction coefficients $R^{(mix)}$ for $N_{\overline{10}}$ decays as functions of ρ (for $\Sigma_{\pi N} = 73$ MeV) and $\Sigma_{\pi N}$ (for $\rho = 0.5$ and $\rho = 0.3$).

For $\Sigma_{\overline{10}}$ decays we get:

Decay	Matrix element $\overline{\mathcal{A}}^2$
$\Sigma_{\overline{10}} \rightarrow N + K$	$\frac{1}{10} [G_{\overline{10}} - c_{\overline{10}} (\frac{5}{2}H_{\overline{10}} - \frac{5}{2}H_8 + \frac{9}{2}H'_8) + \frac{7}{6}c_{27}H'_{27} - \frac{4}{15}H_{27}]^2$
$\Sigma_{\overline{10}} \rightarrow \Sigma + \pi$	$\frac{1}{10} [G_{\overline{10}} + c_{\overline{10}} (\frac{5}{2}H_{\overline{10}} - 5H_8) + \frac{7}{2}c_{27}H'_{27}]^2$
$\Sigma_{\overline{10}} \rightarrow \Sigma + \eta$	$\frac{3}{20} [G_{\overline{10}} + 3c_{\overline{10}}H'_8 - \frac{7}{3}c_{27}H'_{27} - \frac{4}{15}d_{27}H_{27}]^2$
$\Sigma_{\overline{10}} \rightarrow \Lambda + \eta$	$\frac{3}{20} [G_{\overline{10}} - 3c_{\overline{10}}H'_8 + \frac{7}{2}c_{27}H'_{27} + \frac{4}{15}d_{27}H_{27}]^2$
$\Sigma_{\overline{10}} \rightarrow \Xi + K$	$\frac{3}{30} [G_{\overline{10}} + c_{\overline{10}} (\frac{5}{2}H_8 + \frac{9}{2}H'_8) - \frac{14}{3}c_{27}H'_{27} + \frac{4}{15}d_{27}H_{27}]^2$

(27)

and the pertinent correction factors are plotted in Fig. 6. Note that for our set of parameters (2) for $\Sigma_{\pi N} = 73$ MeV $\Sigma_{\overline{10}}$ (1754 MeV) is below the threshold for $\Xi + K$.

Finally let us discuss the importance of the 27 admixtures both in the initial and final state. For decuplet decays this is the only possible m_s correction, however, the final state admixture is proportional to G'_{27} which

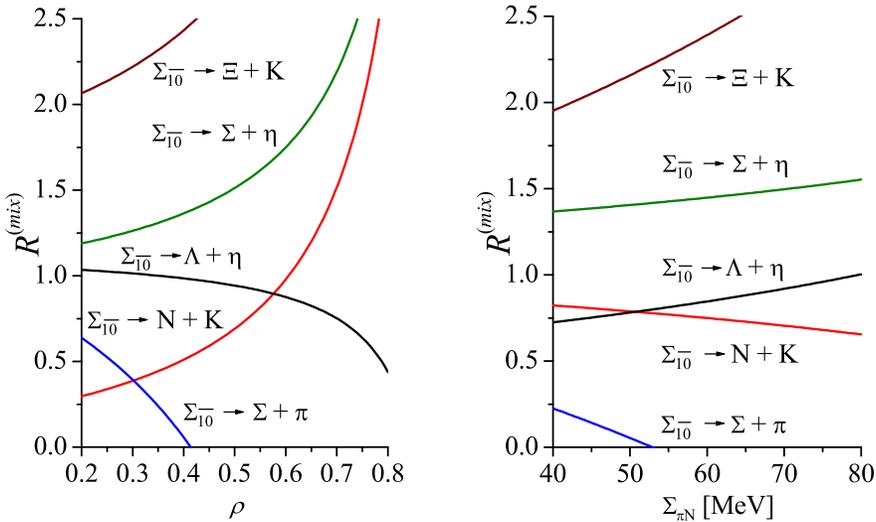


Fig. 6. Correction coefficients $R^{(mix)}$ for $\Sigma_{\overline{10}}$ decays as functions of ρ (for $\Sigma_{\pi N} = 73$ MeV) and $\Sigma_{\pi N}$ (for $\rho = 0.5$).

small. The magnitude of these corrections can be read off from Fig. 3. For antidecuplet decays only $\Xi_{\overline{10}} \rightarrow \Xi + \pi$ is entirely governed by $\mathcal{R} = 27 m_s$ corrections, while for the other decays there is a subtle interplay of 27 and $\overline{10} + 8$, however, in view of Fig. 2, the final state 27 admixture is by far more important than the one in the initial state. In Eq. (28) below we present the ratios of the $\overline{10}$ and 8 admixtures to 27 for $\rho = 0.5$ and $\Sigma_{\pi N} = 73$ MeV:

Decay	Ratio ($\overline{10} + 8$) to 27
$\Theta^+ \rightarrow N + K$	0.14
$\Xi_{\overline{10}} \rightarrow \Sigma + K$	0.58
$N_{\overline{10}} \rightarrow N + \pi$	-2.17
$N_{\overline{10}} \rightarrow N + \eta$	-0.89
$N_{\overline{10}} \rightarrow \Lambda + K$	-1.48
$N_{\overline{10}} \rightarrow \Sigma + K$	-0.83
$\Sigma_{\overline{10}} \rightarrow N + K$	-1.70
$\Sigma_{\overline{10}} \rightarrow \Sigma + \pi$	-1.98
$\Sigma_{\overline{10}} \rightarrow \Sigma + \eta$	-1.48
$\Sigma_{\overline{10}} \rightarrow \Lambda + \eta$	-1.01
$\Sigma_{\overline{10}} \rightarrow \Xi + K$	-1.91

(28)

An interesting pattern emerges: Θ^+ and $\Xi_{\overline{10}}$ decays are dominated by $\mathcal{R} = 27 m_s$ corrections, for $N_{\overline{10}}$ decays effects of 27 admixtures are in all but one case negligible, whereas for $\Sigma_{\overline{10}}$ decays $\overline{10} + 8$ and 27 pieces are comparable, with 27 admixture being at most 2 times smaller and of the opposite sign.

4. Summary

The representation mixing introduced by the symmetry breaking Hamiltonian (1) depends on the value of the $\Sigma_{\pi N}$ term. For $\Sigma_{\pi N} = 73$ MeV, the value suggested by recent experimental analysis [7], as well as by the chiral soliton model fits [10, 16], this mixing becomes large and introduces large $\mathcal{O}(m_s)$ corrections to the decay widths. In the case of the decuplet decays large means at most 35%, however, in the antidecuplet case these corrections may reach even a factor 3 or more. This is because the leading term, given by the transition matrix element $G_{\overline{10}}$ of Eq. (24), is small for the typical values of the coupling constants $G_{0,1,2}$ entering the decay operator (8). This poses a challenge to the SU(3) perturbation expansion in m_s . As far as two exotic states recently observed experimentally [1, 2] are concerned, we find strong suppression of the Θ^+ decay width and the enhancement for $\Xi_{\overline{10}}$. Before, however, a definitive conclusion may be drawn, the $\mathcal{O}(m_s)$ corrections to the decay operator (8) must be examined.

A key issue is whether, knowing the decay width of Δ , one can predict the decay width of Θ^+ . Other decays of the antidecuplet are in fact related to Θ^+ by phase space factors, for which we have to know the masses of the decaying particles, and the SU(3) Clebsch–Gordan coefficients and the $\mathcal{O}(m_s)$ correction factors calculated in this paper. In order to estimate $\Gamma_{\Theta^+ \rightarrow n+K^+}$ let us observe that

$$\left. \frac{\Gamma_{\Theta^+ \rightarrow n+K^+}}{\Gamma_{\Delta}} \right|_{m_s=0} = \frac{1}{2} \frac{M_{\Delta}}{M_{\Theta^+}} \left(\frac{p_K}{p_{\pi}} \right)^3 \left(\frac{G_{\overline{10}}}{G_{10}} \right)^2 = 0.67 \left(\frac{G_{\overline{10}}}{G_{10}} \right)^2, \quad (29)$$

where $G_{\overline{10}}/G_{10}$ is plotted in Fig. 2(b). For $\rho \sim 0.5$ we get $\Gamma_{\Theta^+ \rightarrow n+K^+}/\Gamma_{\Delta} \sim 0.1$, a fairly large suppression⁶. Next, the $\mathcal{O}(m_s)$ corrections increase Δ width by a factor of 1.3 and suppress Γ_{Θ^+} by a factor of 0.2, so that the relative suppression coming from the SU(3) breaking is 0.15. As said above this factor must be considered with care, however, the suppression mechanism is here evident and one may safely conclude that

$$\frac{\Gamma_{\Theta^+}}{\Gamma_{\Delta}} \ll 0.1 \quad (30)$$

⁶ Note that we discuss here one of the two possible decay modes of Θ^+ , rather than the total width, which would make this ratio 0.2.

indicating that the total width Γ_{Θ^+} is of the order of a few MeV. Further suppression factors were discussed in the original paper of Diakonov, Petrov and Polyakov [5] and more recently in [10].

In order to estimate the widths of the other members of antidecuplet we first calculate the ratios of a given decay width to the one of Θ^+ and then the $\mathcal{O}(m_s)$ correction factors (for $\rho = 0.5$ and $\Sigma_{\pi N} = 73$ MeV). The results are listed in (31) below:

Decay X	$R_\Gamma = \Gamma/\Gamma_{\Theta^+ \rightarrow n+K^+}$	$R_{\min} = R^{(\text{mix})}$	$R_{\max} = R^{(\text{mix})}/R_{\Theta^+ \rightarrow n+K^+}^{(\text{mix})}$
$\Theta^+ \rightarrow n + K^+$	1	0.20	1
$\Xi_{10}^- \rightarrow \Xi + \pi$	2.67	1.54	7.7
$\Xi_{10}^- \rightarrow \Sigma + K$	1.57	1.27	6.4
$N_{10}^- \rightarrow N + \pi$	3.82	?	?
$N_{10}^- \rightarrow N + \eta$	0.97	0.92	4.6
$N_{10}^- \rightarrow \Lambda + K$	0.08	1.66	8.3
$\Sigma_{10}^- \rightarrow N + K$	1.74	0.70	3.5
$\Sigma_{10}^- \rightarrow \Sigma + \pi$	3.47	?	?
$\Sigma_{10}^- \rightarrow \Sigma + \eta$	2.10	1.48	7.4
$\Sigma_{10}^- \rightarrow \Lambda + \eta$	0.34	0.99	4.9

(31)

The question marks in Eq. (31) indicate that the $R^{(\text{mix})}$ factors are negative so that our results are not reliable. Since also the large suppression factor for Θ^+ is not fully reliable, one could argue that the real correction factor lies somewhere between R_{\min} and R_{\max} defined in (31). Our estimates for the decay width in the antidecuplet take therefore the following form

$$\Gamma_X = \Gamma_{\Theta^+ \rightarrow n+K^+} \times R_\Gamma \times \begin{cases} R_{\max} \\ R_{\min} \end{cases} \tag{32}$$

and for the entries with question marks one can only conclude that

$$\Gamma_X < \Gamma_{\Theta^+ \rightarrow n+K^+} \times R_\Gamma. \tag{33}$$

Hence for 2 MeV total decay width of Θ^+ we get for example $\Gamma_{\Xi_{10}^- \rightarrow \Xi + \pi} \sim 4 \div 20.6$ MeV and $\Gamma_{\Xi_{10}^- \rightarrow \Sigma + K} \sim 2 \div 10$ MeV. This is more than the value quoted in Ref. [11], however, these authors did not take into account the 27 admixture which is quite important in this case. Also for $N_{10}^- \rightarrow N + \pi$ we find much stronger suppression than [11] due to the fact that for our set of parameters c_{10} is larger and $H_{10} < 0$ rather than small and positive.

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