Vol. 35 (2004) ACTA PHYSICA POLONICA B No 5

TWO LIGHT STERILE NEUTRINOS THAT MIX MAXIMALLY WITH EACH OTHER AND MODERATELY WITH THREE ACTIVE NEUTRINOS*

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(Received February 24, 2004)

Since the 3+1 neutrino models with one light sterile neutrino turn out to be not very effective, at least two light sterile neutrinos may be needed to reconcile the solar and atmospheric neutrino experiments with the LSND result, *if* this is confirmed by the ongoing MiniBooNE experiment (and when the CPT invariance is assumed to hold for neutrino oscillations). We present an attractive 3+2 neutrino model, where two light sterile neutrinos mix maximally with each other, in analogy to the observed maximal mixing of muon and tauon active neutrinos. But, while the mixing of ν_e and $(\nu_\mu - \nu_\tau)/\sqrt{2}$ is observed as large (though not maximal), the mixing of ν_e with the corresponding combination of two light sterile neutrinos is expected to be only moderate because of the reported smallness of LSND oscillation amplitude. The presented model turns out, however, not to be *more* effective in explaining the hypothetic LSND result than the simplest 3+1 neutrino model. On the other hand, in the considered 3+2 model, the deviations from conventional oscillations of three active neutrinos appear to be *minimal* within a larger class of 3+2 models.

PACS numbers: 12.15.Ff, 14.60.Pq, 12.15.Hh

1. Introduction

As is well known, the neutrino experiments with solar ν_e's [1], atmospheric ν_μ's [2] and long-baseline reactor $\bar{\nu}_e$'s [3] are very well described by oscillations of three active neutrinos ν_e , ν_μ , ν_τ, where the mass-squared splittings of the related neutrino mass states ν_1, ν_2, ν_3 are estimated to be $\Delta m^2_{\rm sol} \equiv \Delta m^2_{21} \sim 7 \times 10^{-5}$ eV^2 and $\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} \sim 2 \times 10^{-3}$ eV^2 [4].

* Work supported in part by the Polish State Committee for Scientific Research (KBN), grant 2 P03B 129 24 (2003–2004).

The neutrino mixing matrix $U^{(3)} = \left(U^{(3)}_{\alpha i}\right)$ $(\alpha = e, \mu, \tau$ and $i = 1, 2, 3)$, appearing in the unitary transformation

$$\nu_\alpha = \sum_i U^{(3)}_{\alpha i}\, \nu_i \;, \tag{1}$$

is experimentally consistent with the global bilarge form

$$U^{(3)} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}} s_{12} & \frac{1}{\sqrt{2}} c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} s_{12} & -\frac{1}{\sqrt{2}} c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix} \;, \tag{2}$$

where $\theta_{12} \sim 33^\circ$ and $\theta_{23} = 45^\circ$, while $U^{(3)}_{e3} = s_{13} \exp(-i\delta)$ is neglected in accordance with the negative result of Chooz experiment with short-baseline reactor $\bar{\nu}_e$'s [5] (the experimental upper bound is estimated at $s_{13} < 0.03$). However, the signal of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations reported by LSND experiment with short-baseline accelerator $\bar{\nu}_\mu$'s [6] requires a third neutrino mass-squared splitting, say, $\Delta m^2_{\rm LSND} \sim 1\ {\rm eV}^2$ which cannot be justified by the use of only three neutrinos (unless the CPT invariance of neutrino oscillations is seriously violated, leading to considerable mass splittings of neutrinos and antineutrinos [7]; in the present note the CPT invariance is assumed to hold for neutrino oscillations). The LSND result will be tested soon in the ongoing MiniBooNE experiment [8]. If this test confirms the LSND result, we will need the light sterile neutrinos in addition to three active neutrinos to introduce extra mass splittings (and, at the same time, not to change significantly the solar, atmospheric and reactor neutrino oscillations).

While the 3+1 neutrino models with one light sterile neutrino are considered to be strongly disfavored by present data [9], the 3+2 neutrino schemes with two light sterile neutrinos may provide a much better description of current neutrino oscillations including the LSND effect (for a discussion on the compatibility of all short-baseline neutrino experiments in 3+1 and 3+2 models *cf.* Ref [10]).

The necessary existence in Nature of exactly two light sterile neutrinos was argued some years ago [11] on the ground of a new series of *generalized (Kähler-like) Dirac equations* which could describe three and only three generations of SM-active leptons and quarks, and two and only two generations of single SM-passive light neutrinos (light sterile neutrinos). The condition for it was an *intrinsic (Pauli-type) exclusion principle* that, when it was holding, cut off the series of the corresponding generalized Dirac fields to one triad of SM (15+1)-plets and one couple of SM singlets, respectively (all of spin 1/2). The subjects of this exclusion principle were the sets of *additional* Dirac bispinor indices appearing for the introduced generalized Dirac fields and treated as undistinguishable physical degrees of freedom obeying the Fermi statistics. However, in Refs. [11] it was wrongly presumed that two light sterile neutrinos mixed largely with two active neutrinos ν_e and ν_μ, what now must be specifically corrected, of course.

The cosmological problems of light sterile neutrinos will not be discussed in this note.

2. Overall neutrino mixing matrix

In the present note, we will conjecture that two light sterile neutrinos, call them ν_s and $\nu_{s'}$, mix maximally with each other, but only moderately with three active neutrinos ν_e, ν_μ, ν_τ. More precisely, we will assume the overall 5×5 neutrino mixing matrix $U^{(5)} = \left(U^{(5)}_{\alpha i}\right)$ $(\alpha = e, \mu, \tau, s, s'$ and $i = 1, 2, 3, 4, 5)$ in the form

$$
\begin{aligned}
U^{(5)} &= U^{(5)}(12)U^{(5)}(14)\\
&= \begin{pmatrix}
c_{12}c_{14} & s_{12} & 0 & c_{12}s_{14} & 0\\
-\frac{1}{\sqrt{2}}s_{12}c_{14} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}s_{12}s_{14} & 0\\
\frac{1}{\sqrt{2}}s_{12}c_{14} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}s_{12}s_{14} & 0\\
-\frac{1}{\sqrt{2}}s_{14} & 0 & 0 & \frac{1}{\sqrt{2}}c_{14} & \frac{1}{\sqrt{2}}\\
\frac{1}{\sqrt{2}}s_{14} & 0 & 0 & -\frac{1}{\sqrt{2}}c_{14} & \frac{1}{\sqrt{2}}
\end{pmatrix},
\end{aligned}
\tag{3}
$$

where

$$
U^{(5)}(12) = \begin{pmatrix}
c_{12} & s_{12} & 0 & 0 & 0\\
-\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & 0 & 0\\
\frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & 0 & 0\\
0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 1
\end{pmatrix},
$$

$$
U^{(5)}(14) = \begin{pmatrix}
c_{14} & 0 & 0 & s_{14} & 0\\
0 & 1 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0\\
-\frac{1}{\sqrt{2}}s_{14} & 0 & 0 & \frac{1}{\sqrt{2}}c_{14} & \frac{1}{\sqrt{2}}\\
\frac{1}{\sqrt{2}}s_{14} & 0 & 0 & -\frac{1}{\sqrt{2}}c_{14} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\tag{4}
$$

The first matrix factor in Eq. (3) arises from the bilarge form (2) of active-neutrino mixing matrix by its trivial 5×5 extension, while the second is an analogue of the first, when $e, \mu, \tau \leftrightarrow e, s, s'$ and $1, 2, 3 \leftrightarrow 1, 4, 5$. Thus, the cosine c_{14} and sine s_{14} are analogues of cosine c_{12} and sine s_{12}, though the angle θ_{14} is expected to be smaller than the large angle $\theta_{12} \sim 33^\circ$. Also $c_{45} = 1/\sqrt{2} = s_{45}$ with the maximal angle $\theta_{45} = 45^\circ$ are analogues of $c_{23} = 1/\sqrt{2} = s_{23}$ with the maximal $\theta_{23} = 45^\circ$. Finally, an analogue of $s_{13} = 0$ is $s_{15} = 0$. Both kinds of conditions are necessary for the maximal mixing of ν_μ with ν_τ and ν_s with $\nu_{s'}$.

The overall 5×5 neutrino mixing matrix (3) leads to the following unitary transformation $\nu_i = \sum_\alpha U^{(5)*}_{\alpha i}\, \nu_\alpha$ inverse to $\nu_\alpha = \sum_i U^{(5)}_{\alpha i}\, \nu_i$:

$$
\begin{aligned}
\nu_1 &= c_{14}\left(c_{12}\nu_e - s_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}}\right) - s_{14}\frac{\nu_s - \nu_{s'}}{\sqrt{2}}\,,\\
\nu_2 &= s_{12}\nu_e + c_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}}\,,\\
\nu_3 &= \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}\,,\\
\nu_4 &= s_{14}\left(c_{12}\nu_e - s_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}}\right) + c_{14}\frac{\nu_s - \nu_{s'}}{\sqrt{2}}\,,\\
\nu_5 &= \frac{\nu_s + \nu_{s'}}{\sqrt{2}}\,.
\end{aligned}
\tag{5}
$$

This displays explicitly the maximal mixing of ν_μ and ν_τ as well as of ν_s and $\nu_{s'}$, because these neutrinos appear in Eq. (5) through the combinations $(\nu_\mu \mp \nu_\tau)/\sqrt{2}$ as well as $(\nu_s \mp \nu_{s'})/\sqrt{2}$, where $(\nu_\mu + \nu_\tau)/\sqrt{2}$ as well as $(\nu_s + \nu_{s'})/\sqrt{2}$ are decoupled from other flavor neutrinos (do not mix with them).

In the flavor representation, where the charged-lepton mass matrix is diagonal, the active-neutrino mixing matrix (2) is at the same time the diagonalizing matrix for the active-neutrino effective Majorana mass matrix. In this flavor representation, the overall 5×5 effective neutrino mass matrix $M^{(5)} = \left(M^{(5)}_{\alpha\beta}\right)$ $(\alpha, \beta = e, \mu, \tau, s, s')$ can be calculated from the formula

$$M^{(5)}_{\alpha\beta} = \sum_i U^{(5)}_{\alpha i}\, m_i\, U^{(5)*}_{\beta i}\,, \tag{6}$$

where the matrix elements $U^{(5)}_{\alpha i}$ are given in Eq. (3). The form (6) is inverse to the diagonalization formula

$$\sum_{\alpha\,\beta} U^{(5)*}_{\alpha i}\, M^{(5)}_{\alpha\beta}\, U^{(5)}_{\beta j} = m_i \delta_{ij}\,. \tag{7}$$

3. Overall neutrino oscillations

We will use the $\nu_\alpha \to \nu_\beta$ neutrino oscillation probabilities (in the vacuum)

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\beta\alpha} - 4\sum_{j>i} U^{(5)*}_{\beta j} U^{(5)}_{\alpha j} U^{(5)}_{\beta i} U^{(5)*}_{\alpha i} \sin^2 x_{ji} \tag{8}$$

($\alpha, \beta = e, \mu, \tau, s, s'$ and $i, j = 1, 2, 3, 4, 5$), where

$$x_{ji} \equiv 1.27 \frac{\Delta m^2_{ji} L}{E}\,, \quad \Delta m^2_{ji} \equiv m^2_j - m^2_i \tag{9}$$

(Δm^2_{ji}, L and E are measured in eV^2, km and GeV, respectively). Here, CP violation is neglected *i.e.*, $U^{(5)*}_{\alpha i} = U^{(5)}_{\alpha i}$ (or, more generally, the quartic products in Eq. (8) are real). For $U^{(5)}_{\alpha i}$ we will make use of the matrix elements of $U^{(5)}$ as given in Eq. (3).

The formula (8) applied respectively to the $\bar\nu_\mu \to \bar\nu_e$, $\nu_e \to \nu_e$ and $\nu_\mu \to \nu_\mu$ oscillations leads to the probabilities (in the vacuum)

$$P(\bar\nu_\mu \to \bar\nu_e) \simeq 2c^2_{12}s^2_{12}c^2_{14}\sin^2 x_{21} + 2c^2_{12}s^2_{12}s^4_{14}\sin^2 x_{41}\,, \tag{10}$$

$$P(\nu_e \to \nu_e) \simeq 1 - 4c^2_{12}s^2_{12}s^2_{14}\sin^2 x_{21} - 4c^2_{12}s^2_{14}\left(1 - c^2_{12}s^2_{14}\right)\sin^2 x_{41} \tag{11}$$

and

$$\begin{aligned} P(\nu_\mu \to \nu_\mu) \simeq\; & 1 - c^2_{12}s^2_{12}c^2_{14}\sin^2 x_{21} - \left(1 - s^2_{12}s^2_{14}\right)\sin^2 x_{31} \\ & -2s^2_{12}s^2_{14}\left(1 - \frac{1}{2}s^2_{12}s^2_{14}\right)\sin^2 x_{41}\,, \end{aligned} \tag{12}$$

when $x_{31} \simeq x_{32}$ and $x_{41} \simeq x_{42} \simeq x_{43}$ (notice that here, x_{5i} and so, m^2_5 are absent). When, in addition, $x_{21} \ll |x_{31}| \ll x_{41}$ (*i.e.*, $m^2_1 < m^2_2 \ll m^2_3 \ll m^2_4$ or $m^2_3 \ll m^2_1 \simeq m^2_2 \ll m^2_4$) with $(x_{41})_{\mathrm{LSND}} = O(\pi/2)$, $(x_{21})_{\mathrm{sol}} = O(\pi/2)$, $(x_{31})_{\mathrm{Chooz}} \simeq (x_{31})_{\mathrm{atm}} = O(\pi/2)$ and $(x_{31})_{\mathrm{atm}} = O(\pi/2)$ for the LSND effect, for the solar ν_e's, for the Chooz reactor $\bar\nu_e$'s and for the atmospheric ν_μ's, respectively, we get from Eqs. (10), (11) and (12) the probabilities (in the vacuum)

$$P(\bar\nu_\mu \to \bar\nu_e)_{\mathrm{LSND}} \simeq 2c^2_{12}s^2_{12}s^4_{14}\sin^2(x_{41})_{\mathrm{LSND}}\,, \tag{13}$$

$$P(\nu_e \to \nu_e)_{\mathrm{sol}} \simeq 1 - 4c^2_{12}s^2_{12}c^2_{14}\sin^2(x_{21})_{\mathrm{sol}} - 2c^2_{12}s^2_{14}\left(1 - c^2_{12}s^2_{14}\right)\,, \tag{14}$$

$$P(\bar\nu_e \to \bar\nu_e)_{\mathrm{Chooz}} \simeq 1 - 2c^2_{12}s^2_{14}\left(1 - c^2_{12}s^2_{14}\right) \tag{15}$$

and

$$P(\nu_\mu \to \nu_\mu)_{\mathrm{atm}} \simeq 1 - \left(1 - s^2_{12}s^2_{14}\right)\sin^2(x_{31})_{\mathrm{atm}} - s^2_{12}s^2_{14}\left(1 - \frac{1}{2}s^2_{12}s^2_{14}\right)\,. \tag{16}$$

Of course, for solar ν_e's the MSW matter effect is significant, leading to the accepted LMA solar solution.

For the LSND effect of the order

$$P(\bar{\nu}_\mu \to \bar{\nu}_e)_{\rm LSND} \sim 10^{-3} \sin^2(x_{41})_{\rm LSND} \tag{17}$$

and of the mass scale, say, $\Delta m^2_{41} \sim 1$ eV^2 we obtain the estimation

$$s^2_{14} \sim \left(\frac{10^{-3}}{2c^2_{12}s^2_{12}}\right)^{1/2} \sim 0.049 \tag{18}$$

and so, $\theta_{14} \sim 13°$, when $\theta_{12} \sim 33°$. This implies the following estimates:

$$\begin{aligned} P(\nu_e \to \nu_e)_{\rm sol} \;\; &\sim \; 1 - (0.83 - 0.041)\sin^2(x_{21})_{\rm sol} - 0.066\,, & (19)\\ P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm Chooz} \;\; &\sim \; 1 - 0.066\,, & (20) \end{aligned}$$

and

$$P(\nu_\mu \to \nu_\mu)_{\rm atm} \sim 1 - (1 - 0.015)\sin^2(x_{31})_{\rm atm} - 0.014\,. \tag{21}$$

Here, $4c^2_{12}s^2_{12} \sim 0.83$ ($c^2_{12} \sim 0.70$ and $s^2_{12} \sim 0.30$).

It can be noticed from Eq. (11) for $\nu_e \to \nu_e$ or $\bar{\nu}_e \to \bar{\nu}_e$ oscillations that the third mass-squared splitting $\Delta m^2_{\rm LSND} \equiv \Delta m^2_{41} \sim 1$ eV^2, characteristic for the reported LSND effect (Eq. (13)), may be manifested in principle also for $\nu_e \to \nu_e$ or $\bar{\nu}_e \to \bar{\nu}_e$ oscillations in any other experiment at an energy E and a baseline L, where $(x_{41})_{\rm other} \simeq (x_{41})_{\rm LSND} = O(\pi/2)$ (*i.e.*, $(L/E)_{\rm other} \simeq (L/E)_{\rm LSND}$). In this case,

$$\begin{aligned} P(\nu_e \to \nu_e)_{\rm other} = P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm other} &\simeq 1 - 4c^2_{12}s^2_{14}\left(1 - c^2_{12}s^2_{14}\right)\sin^2(x_{41})_{\rm other} \\ &\sim 1 - 0.13\sin^2(x_{41})_{\rm other}\,, \end{aligned} \tag{22}$$

when $\theta_{12} \sim 33°$ and $s^2_{14} \sim 0.049$. Any such experiment might play for the LSND accelerator effect a somewhat similar role as that played by the Chooz experiment for the SuperKamiokande atmospheric experiment, where $(x_{31})_{\rm Chooz} \simeq (x_{31})_{\rm atm} = O(\pi/2)$ (*i.e.*, $(L/E)_{\rm Chooz} \simeq (L/E)_{\rm atm}$). An analogical remark on Δm^2_{41} may pertain also to Eq. (12) for $\nu_\mu \to \nu_\mu$ oscillations.

4. Conclusions

When waiting for the test of LSND effect by the MiniBooNE experiment that may confirm or refute the LSND result, we presented in this note a 3+2 neutrino model, where two light sterile neutrinos mix maximally with each other and only moderately with three active neutrinos. The way of mixing is described by the 5×5 mixing matrix (3).

Then, the LSND effect of the order (17) implies the estimates (19), (20) and (21) for the solar anomaly, Chooz negative result and atmospheric anomaly, respectively.

Note that in the conventional three-neutrino scheme (including, in general, $s_{13} \neq 0$) the result corresponding to Eq. (15) is

$$P(\bar{\nu}_e \to \bar{\nu}_e)_{\text{Chooz}} \simeq 1 - 4c_{13}^2 s_{13}^2 \sin^2(x_{31})_{\text{Chooz}}\,, \tag{23}$$

when $x_{31} \simeq x_{32}$. From the negative result of Chooz experiment $4c_{13}^2 s_{13}^2 < 0.12$ as $s_{13}^2 < 0.03$ (here, $(x_{31})_{\text{Chooz}} \simeq (x_{31})_{\text{atm}} = O(\pi/2)$).

As is shown in Appendix A, the simplest 3+1 model with one light sterile neutrino leads to *the same* estimates (19), (20) and (21) as the 3+2 model with two maximally mixing light sterile neutrinos, if the LSND effect is of the order (17). In fact, the oscillations (A.7)–(A.9) and (A.10) are *identical* with those given in Eqs. (13)–(15) and (16).

Thus, the attractive 3+2 neutrino model with maximal mixing of two sterile neutrinos, presented in this note, is not *more* effective in explaining the hypothetic LSND result than the simplest 3+1 neutrino model. On the other hand, as is indicated in Appendix B, in the 3+2 model with maximal mixing of two sterile neutrinos where $s_{15} = 0$ (but $s_{14} \neq 0$), the deviations from conventional oscillations of three active neutrinos (where $s_{14} = 0$ and $s_{15} = 0$) are *minimal* within a larger class of 3+2 models allowing for $s_{15} \neq 0$ (in addition to $s_{14} \neq 0$).

Appendix A

Comparing with the simplest 3+1 neutrino model

Consider the simplest 3+1 neutrino model with one sterile neutrino, described by the overall 4×4 mixing matrix bilarge in three active neutrinos:

$$\begin{aligned} U^{(4)} &= U^{(4)}(12)U^{(4)}(14) \\ &= \begin{pmatrix} c_{12}c_{14} & s_{12} & 0 & c_{12}s_{14} \\ -\frac{1}{\sqrt{2}}s_{12}c_{14} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}s_{12}s_{14} \\ \frac{1}{\sqrt{2}}s_{12}c_{14} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}s_{12}s_{14} \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix}, \end{aligned} \tag{A.1}$$

where

$$U^{(4)}(12) = \begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U^{(4)}(14) \;=\; \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix} . \tag{A.2}$$

The cosine c_{14} and sine s_{14} correspond to an unknown mixing angle θ_{14} that has to be estimated from the reported LSND result.

The form (A.1) of the mixing matrix $U^{(4)} = \left(U^{(4)}_{\alpha i}\right)$ ($\alpha = e, \mu, \tau, s$ and $i = 1, 2, 3, 4$) leads to the following unitary transformation $\nu_i = \sum_\alpha U^{(4)*}_{\alpha i}\, \nu_\alpha$ inverse to $\nu_\alpha = \sum_i U^{(4)}_{\alpha i}\, \nu_i$:

$$\begin{aligned} \nu_1 \;&=\; c_{14}\left(c_{12}\nu_e - s_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}}\right) - s_{14}\nu_s \,, \\ \nu_2 \;&=\; s_{12}\nu_e + c_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}} \,, \\ \nu_3 \;&=\; \frac{\nu_\mu + \nu_\tau}{\sqrt{2}} \,, \\ \nu_4 \;&=\; s_{14}\left(c_{12}\nu_e - s_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}}\right) + c_{14}\nu_s \,. \end{aligned} \tag{A.3}$$

This displays the maximal mixing of active neutrinos ν_μ and ν_τ which appear in the combinations $(\nu_\mu \mp \nu_\tau)/\sqrt{2}$, where $(\nu_\mu + \nu_\tau)/\sqrt{2}$ is decoupled from other flavor neutrinos, while the mixing of ν_e and $(\nu_\mu - \nu_\tau)/\sqrt{2}$ is large (though not maximal) having the mixing angle $\theta_{12} \sim 33^\circ$.

The mixing matrix (A.1) implies the neutrino oscillation probabilities (in the vacuum)

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) \;\simeq\; 2c_{12}^2 s_{12}^2 c_{14}^2 \sin^2 x_{21} + 2c_{12}^2 s_{12}^2 s_{14}^4 \sin^2 x_{41} \,, \tag{A.4}$$

$$P(\nu_e \to \nu_e) \;\simeq\; 1 - 4c_{12}^2 s_{12}^2 c_{14}^2 \sin^2 x_{21} - 4c_{12}^2 s_{14}^2 (1 - c_{12}^2 s_{14}^2) \sin^2 x_{41} \tag{A.5}$$

and

$$\begin{aligned} P(\nu_\mu \to \nu_\mu) \;\simeq\; & 1 - c_{12}^2 s_{12}^2 c_{14}^2 \sin^2 x_{21} - (1 - s_{12}^2 s_{14}^2) \sin^2 x_{31} \\ & -2s_{12}^2 s_{14}^2 (1 - \frac{1}{2} s_{12}^2 s_{14}^2) \sin^2 x_{41} \,, \end{aligned} \tag{A.6}$$

when $x_{31} \simeq x_{32}$ and $x_{41} \simeq x_{42} \simeq x_{43}$. Hence, when $x_{21} \ll |x_{31}| \ll x_{41}$ with $(x_{41})_{\rm LSND} = O(\pi/2)$, $(x_{21})_{\rm sol} = O(\pi/2)$, $(x_{31})_{\rm Chooz} = O(\pi/2)$ and $(x_{31})_{\rm atm} = O(\pi/2)$, respectively, for the LSND effect, for the solar ν_e's, for the Chooz reactor $\bar{\nu}_e$'s and for the atmospheric ν_μ's, one obtains the probabilities (in the vacuum)

$$P(\bar{\nu}_\mu \to \bar{\nu}_e)_{\rm LSND} \simeq 2c_{12}^2 s_{12}^2 s_{14}^4 \sin^2(x_{41})_{\rm LSND}\,, \tag{A.7}$$

$$P(\nu_e \to \nu_e)_{\rm sol} \simeq 1 - 4c_{12}^2 s_{12}^2 c_{14}^2 \sin^2(x_{21})_{\rm sol} - 2c_{12}^2 s_{14}^2 (1 - c_{12}^2 s_{14}^2)\,, \tag{A.8}$$

$$P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm Chooz} \simeq 1 - 2c_{12}^2 s_{14}^2 (1 - c_{12}^2 s_{14}^2) \tag{A.9}$$

and

$$P(\nu_\mu \to \nu_\mu)_{\rm atm} \simeq 1 - (1 - s_{12}^2 s_{14}^2) \sin^2(x_{31})_{\rm atm} - s_{12}^2 s_{14}^2 \left(1 - \frac{1}{2} s_{12}^2 s_{14}^2\right)\,. \tag{A.10}$$

From Eq. (A.7) the LSND effect of the order $P(\bar{\nu}_\mu \to \bar{\nu}_e)_{\rm LSND} \sim 10^{-3} \sin^2(x_{41})_{\rm LSND}$ and of the mass scale, say, $\Delta m_{41}^2 \sim 1$ eV^2 one gets the estimation

$$s_{14}^2 \sim \left(\frac{10^{-3}}{2c_{12}^2 s_{12}^2}\right)^{1/2} \sim 0.049\,, \tag{A.11}$$

when $\theta_{12} \sim 33^\circ$. This gives the following estimates:

$$P(\nu_e \to \nu_e)_{\rm sol} \sim (1 - 0.83 - 0.041) \sin^2(x_{21})_{\rm sol} - 0.066\,, \tag{A.12}$$

$$P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm Chooz} \sim 1 - 0.066 \tag{A.13}$$

and

$$P(\nu_\mu \to \nu_\mu)_{\rm atm} \sim 1 - (1 - 0.015) \sin^2(x_{31})_{\rm atm} - 0.014\,. \tag{A.14}$$

We can see that the oscillations (A.7)–(A.10) and their estimates (A.12)–(A.14) in the case of $s_{14}^2 \sim 0.049$, valid in the simplest 3+1 neutrino model, are *identical* with the oscillations (13)–(16) and their estimates (19)–(21) in the case of $s_{14}^2 \sim 0.049$, obtained in the 3+2 neutrino model with maximal mixing of two light sterile neutrinos. This identity is, of course, a consequence of the fact that $U_{\alpha i}^{(4)} = U_{\alpha i}^{(5)}$ and $U_{\alpha 5}^{(5)} = 0$ for $\alpha = e, \mu$ and $i = 1, 2, 3$, as it can be seen from Eqs. (A.1) and (3). And this is true also for $\alpha = \tau$, the oscillations involving ν_τ being identical in both cases.

Appendix B

Perturbing the maximal mixing of two sterile neutrinos

The mixing matrix (3) in the 3+2 model with two maximally mixing light sterile neutrinos requires the trivial value $s_{15} = 0$ corresponding to $\theta_{15} = 0$. Now, introduce the nontrivial mixing angle $\theta_{15} \neq 0$, replacing the factor matrix $U^{(5)}(14)$ in Eq. (3) by the more general form

$$U^{(5)}(14,15) = \begin{pmatrix} c_{14}c_{15} & 0 & 0 & s_{14}c_{15} & s_{15} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}}(s_{14}+c_{14}s_{15}) & 0 & 0 & \frac{1}{\sqrt{2}}(c_{14}-s_{14}s_{15}) & \frac{1}{\sqrt{2}}c_{15} \\ \frac{1}{\sqrt{2}}(s_{14}-c_{14}s_{15}) & 0 & 0 & -\frac{1}{\sqrt{2}}(c_{14}+s_{14}s_{15}) & \frac{1}{\sqrt{2}}c_{15} \end{pmatrix} \tag{B.1}$$

which is the 5×5 trivially extended canonical form of 3×3 real unitary matrix for $\alpha = e, s, s'$ and $i = 1, 4, 5$ with $c_{45} = 1/\sqrt{2} = s_{45}$. Then, the mixing matrix (3) transits into the new overall 5×5 neutrino mixing matrix

$$\begin{aligned} U^{(5)} &= U^{(5)}(12)U^{(5)}(14,15) \\ &= \begin{pmatrix} c_{12}c_{14}c_{15} & s_{12} & 0 & c_{12}s_{14}c_{15} & c_{12}s_{15} \\ -\frac{1}{\sqrt{2}}s_{12}c_{14}c_{15} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}s_{12}s_{14}c_{15} & -\frac{1}{\sqrt{2}}s_{12}s_{15} \\ \frac{1}{\sqrt{2}}s_{12}c_{14}c_{15} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}s_{12}s_{14}c_{15} & \frac{1}{\sqrt{2}}s_{12}s_{15} \\ -\frac{1}{\sqrt{2}}(s_{14}+c_{14}s_{15}) & 0 & 0 & \frac{1}{\sqrt{2}}(c_{14}-s_{14}s_{15}) & \frac{1}{\sqrt{2}}c_{15} \\ \frac{1}{\sqrt{2}}(s_{14}-c_{14}s_{15}) & 0 & 0 & -\frac{1}{\sqrt{2}}(c_{14}+s_{14}s_{15}) & \frac{1}{\sqrt{2}}c_{15} \end{pmatrix} . \end{aligned} \tag{B.2}$$

Of course, for $s_{15} = 0$ the mixing matrix (B.2) comes back to the form (3).

A consequence of the new mixing matrix is the following unitary transformation $\nu_i = \sum_\alpha U^{(5)*}_{\alpha i}\, \nu_\alpha$:

$$\begin{aligned} \nu_1 &= c_{14}\left[c_{15}\left(c_{12}\nu_e - s_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}}\right) - s_{15}\frac{\nu_s + \nu_{s'}}{\sqrt{2}}\right] - s_{14}\frac{\nu_s - \nu_{s'}}{\sqrt{2}} , \\ \nu_2 &= s_{12}\nu_e + c_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}} , \\ \nu_3 &= \frac{\nu_\mu + \nu_\tau}{\sqrt{2}} , \\ \nu_4 &= s_{14}\left[c_{15}\left(c_{12}\nu_e - s_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}}\right) - s_{15}\frac{\nu_s + \nu_{s'}}{\sqrt{2}}\right] + c_{14}\frac{\nu_s - \nu_{s'}}{\sqrt{2}} , \\ \nu_5 &= s_{15}\left(c_{12}\nu_e - s_{12}\frac{\nu_\mu - \nu_\tau}{\sqrt{2}}\right) + c_{15}\frac{\nu_s + \nu_{s'}}{\sqrt{2}} . \end{aligned} \tag{B.3}$$

We can see from Eqs. (B.3) that here, the maximal mixing of ν_μ and ν_τ is maintained, while the maximal mixing of ν_s nd $\nu_{s'}$ is perturbed if $s_{15} \neq 0$ (beside $s_{14} \neq 0$).

The new mixing matrix (B.2) leads to the neutrino oscillation probabilities (in the vacuum):

$$\begin{aligned} P(\bar{\nu}_\mu \to \bar{\nu}_e)_{\rm LSND} &\simeq 2c_{12}^2 s_{12}^2 s_{14}^2 (s_{14}^2 + s_{15}^2) \sin^2(x_{41})_{\rm LSND} + c_{12}^2 s_{12}^2 s_{15}^4 \,, && \text{(B.4)} \\ P(\nu_e \to \nu_e)_{\rm sol} &\simeq 1 - 4c_{12}^2 s_{12}^2 (1 - s_{14}^2 - s_{15}^2) \sin^2(x_{21})_{\rm sol} \\ &\quad -2c_{12}^2 (s_{14}^2 + s_{15}^2) \,, && \text{(B.5)} \\ P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm Chooz} &\simeq 1 - 2c_{12}^2 (s_{14}^2 + s_{15}^2) && \text{(B.6)} \end{aligned}$$

and

$$P(\nu_\mu \to \nu_\mu)_{\rm atm} \simeq 1 - \left[1 - s_{12}^2 (s_{14}^2 + s_{15}^2)\right] \sin^2(x_{31})_{\rm atm} - s_{12}^2 (s_{14}^2 + s_{15}^2) \,, \tag{B.7}$$

when $x_{31} \simeq x_{32}$, $x_{41} \simeq x_{42} \simeq x_{43}$, $x_{51} \simeq x_{52} \simeq x_{53} \simeq x_{54}$ and $x_{21} \ll |x_{31}| \ll x_{41} \ll x_{51}$ with $(x_{41})_{\rm LSND} = O(\pi/2)$, $(x_{21})_{\rm sol} = O(\pi/2)$ and $(x_{31})_{\rm Chooz} \simeq (x_{31})_{\rm atm} = O(\pi/2)$. Here, the respective higher powers of $s_{14}^2 \ll 1$ and $s_{15}^2 \ll 1$ are *neglected.*

From Eq. (B.4), for the LSND effect of the order $P(\bar{\nu}_\mu \to \bar{\nu}_e)_{\rm LSND} \sim 10^{-3} \sin^2(x_{41})_{\rm LSND}$ one obtains the estimation

$$\left(s_{14}^4 + s_{14}^2 s_{15}^2 + \frac{1}{2\sin^2(x_{41})_{\rm LSND}} s_{15}^4\right)^{1/2} \sim \left(\frac{10^{-3}}{2c_{12}^2 s_{12}^2}\right)^{1/2} \sim 0.049 \,, \tag{B.8}$$

when $\theta_{12} \sim 33^\circ$. Here, $\sin^2(x_{41})_{\rm LSND} \sim 1/2$ to 1. Thus, $s_{14}^2 + s_{15}^2 = (s_{14}^4 + 2s_{14}^2 s_{15}^2 + s_{15}^4)^{1/2} >$ (lhs of Eq. (B.8)) ~ 0.049 if $s_{15} \neq 0$, while $s_{14}^2 \sim 0.049$ if $s_{15} = 0$ (as is the case in the 3+2 model with maximal mixing of two sterile neutrinos). Hence, one can infer that the deviations from conventional oscillations of three active neutrinos (with $s_{14} = 0$ and $s_{15} = 0$), being proportional to $s_{14}^2 + s_{15}^2$ in Eqs. (B.5)–(B.7), get larger magnitudes in the case of $s_{15} \neq 0$ (and $s_{14} \neq 0$) than in the case of $s_{15} = 0$ (but $s_{14} \neq 0$), where two sterile neutrinos mix maximally (leading to the estimates (19)–(21)). Thus, when $s_{15} = 0$, the 3+2 neutrino models defined in Eq. (B.2) for various values of s_{15} become *minimal* (in the sense of the discussed deviations). Such a minimal character of the deviations from conventional neutrino oscillations is connected, therefore, with the maximal mixing of two sterile neutrinos, realized if $s_{15} = 0$ (but $s_{14} \neq 0$).

The oscillation probabilities (B.4)–(B.7) are valid obviously in the option of *hierarchical* sterile neutrinos, where $m_4^2 \ll m_5^2$ implying $x_{41} \ll x_{51}$. Then, our conclusion of minimal character of deviations from conventional neutrino oscillations works for $s_{15} = 0$ (but $s_{14} \neq 0$). It turns out that in the opposite option of *degenerate* sterile neutrinos, where $m_4^2 \simeq m_5^2$ leading to $x_{41} \simeq x_{51}$ and $x_{54} \simeq 0$, the new mixing matrix (B.2) provides exactly the oscillation probabilities of the form (13)–(16), where s_{14}^2 is replaced now by $s_{14}^2 + s_{15}^2 - s_{14}^2 s_{15}^2$, equal approximately to $s_{14}^2 + s_{15}^2$. In this degenerate option, the LSND effect of the order $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)_{\rm LSND} \sim 10^{-3} \sin^2(x_{41})_{\rm LSND}$ gives the estimation

$$s_{14}^2 + s_{15}^2 - s_{14}^2 s_{15}^2 \sim \left(\frac{10^{-3}}{2c_{12}^2 s_{12}^2}\right)^{1/2} \sim 0.049\,, \tag{B.9}$$

and the deviations from conventional oscillations of three active neutrinos are identical to those in the 3+2 model with two maximally mixing sterile neutrinos. They are equal to the previous minimal deviations appearing in the hierarchical option if $s_{15} = 0$ (but $s_{14} \neq 0$).

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