REMARK ON THE CORE/HALO MODEL OF BOSE–EINSTEIN CORRELATIONS IN MULTIPLE PARTICLE PRODUCTION PROCESSES*

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The core/halo model describes the Bose–Einstein correlations in multihadron production taking into account the effects of long-lived resonances. The model contains the combinatorial coefficients α_j which were originally calculated from a recurrence relation. We show that α_j is the integer closest to the number $j!/e$.

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Bose–Einstein correlations in multiple particle production processes are much studied in order to get information about the interaction regions and about the hadronization processes. For detailed reviews see *e.g.* [1–3]. The starting point is, usually, the factorizeable approximation (see *e.g.* [4, 5]), where the n-particle density matrix for n indistinguishable particles is:

$$\rho(\boldsymbol{p}_1,\dots,\boldsymbol{p}_n;\boldsymbol{p}'_1,\dots,\boldsymbol{p}'_n) = \frac{1}{n!}\sum_{P,Q}\prod_{j=1}^{n}\rho(\boldsymbol{p}_{j_P};\boldsymbol{p}'_{j_Q})\,, \tag{1}$$

$\rho(\boldsymbol{p};\boldsymbol{p}')$ is some single particle density matrix and the summation is over all the permutations of the indices of $\boldsymbol{p}$ and $\boldsymbol{p}'$. The corresponding unsym-

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metrized density matrix is:

$$\rho^U(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n;\boldsymbol{p}'_1,\ldots,\boldsymbol{p}'_n)=\prod_{j=1}^{n}\rho(\boldsymbol{p}_j;\boldsymbol{p}'_j)\,. \tag{2}$$

The quantities usually presented are the n-body correlation functions

$$C_n(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)=\frac{\rho(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n;\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)}{\rho^U(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n;\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)}\,. \tag{3}$$

As seen from the definitions $C_n(\boldsymbol{p},\ldots,\boldsymbol{p})=n!$. There is a number of difficulties to check this prediction. On the experimental side, a pair of momenta, say $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$, can be reliably measured only if $Q_{ij}=\sqrt{-(p_i-p_j)^2}$ exceeds some $Q_{\min}$. At present it is difficult to go with $Q_{\min}$ below some (5–10) MeV (see *e.g.* [6] and references given there). This is important, because the very small Q region is expected to contain narrow peaks due to long-lived resonances [7]. There are other difficulties in the small Q region: Coulomb corrections, misidentified particles, perhaps effects of coherence. A model which assumes that these other factors can be either corrected for or neglected is the core/halo model[1] [9, 10]. According to this model for a group of identical particles close to each other in momentum space, but not so close as not to be resolved experimentally, the single particle density matrices $\rho(\boldsymbol{p}_{j_P};\boldsymbol{p}'_{j_Q})$ reach for $j_Q\neq j_P$ a common limit $f(\boldsymbol{p})\rho(\boldsymbol{p};\boldsymbol{p})$, where p is some average momentum of the particles in the group. Of course for $j_Q=j_P$ the matrix element in the numerator cancels with the corresponding matrix element in the denominator. Thus the correlation function extrapolated from the region accessible experimentally to the point $\boldsymbol{p}_1=\ldots,=\boldsymbol{p}_n$ is

$$C_n^{\rm extr}(\boldsymbol{p},\ldots,\boldsymbol{p})=\sum_{j=0}^{n}\left(\begin{array}{c}n\\ j\end{array}\right)\alpha_j f(\boldsymbol{p})^j\,, \tag{4}$$

where α_j is the number of permutations of j elements where no element keeps its place.

Let us note the identity

$$k!=\sum_{j=0}^{k}\left(\begin{array}{c}k\\ j\end{array}\right)\alpha_j\,, \tag{5}$$

following from the remark that every permutation of k elements can be characterized by the number j of elements which changed their places and

[1] Let us note, however an attempt to include partial coherence into this model [8].

that the number of choices of these elements is $\left(\begin{array}{c} k \\ j \end{array}\right)$. The formula can be used to calculate the coefficient α_j when all the coefficients α_k with indices $k < j$ are known [10]. In this note we derive a simpler formula for the coefficients α_n. Several derivations of this result can be found in mathematical textbooks. Here we use the idea of the proof from Ref. [11].

Let us multiply both sides of (5) by $(-1)^k \left(\begin{array}{c} n \\ k \end{array}\right)$ and sum over k from zero to n. On the left-hand side we get

$$n! \sum_{k=0}^{n} \frac{(-1)^k}{(n-k)!} = n!(-1)^n \sum_{k=0}^{n} \frac{(-1)^k}{k!} \tag{6}$$

and on the right-hand side

$$\begin{aligned} &\sum_{k=0}^{n} (-1)^k \left(\begin{array}{c} n \\ k \end{array}\right) \sum_{j=0}^{n} \left(\begin{array}{c} k \\ j \end{array}\right) \alpha_j \\ &= \sum_{j=0}^{n} \alpha_j (-1)^j \left(\begin{array}{c} n \\ j \end{array}\right) \sum_{k=j}^{n} \left(\begin{array}{c} n-j \\ k-j \end{array}\right) (-1)^{k-j} = (-1)^n \alpha_n \,, \end{aligned} \tag{7}$$

where the second equality follows from the remark that the sum over k yields 1 for $j = n$ and $(1-1)^{n-j} = 0$ for $j < n$. Comparing the two sides one finds

$$\alpha_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} \,. \tag{8}$$

The coefficient of $n!$ tends to e^{-1} when n increases. It is, however, an alternating series with monotonically decreasing, non-zero terms. For such series the sum of the first n elements approximates the limit with an error less than the absolute value of the first rejected term. Thus

$$\left| \alpha_n - \frac{n!}{e} \right| < \frac{1}{n+1} \tag{9}$$

and α_n, for $n > 0$, can be calculated as the integer closest to $n!/e$.

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