# DEFICIT IN SMALL RETURNS ON INEFFICIENT MARKET

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The proposition of micro dynamics, called amplified imitation, is given to explain the property, called deficit in small returns, found in the return distributions of stocks traded on Polish stock market. The micro dynamics is verified in the Cont–Bouchaud model of a stock market.

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## 1. Introduction

Properties of returns  $r_{\Delta}$  defined as changes in a price p(t) of some financial asset over a time interval  $\Delta$ , have been carefully studied by econophysicists for the last decade. See [1–3] for the introduction and bibliography. The basic effort of investigations is aimed on the identification of some universal features in a time series of a financial asset. These features may give us a better understanding of the underlying mechanism that drives the dynamics of the stock market. The main features that have been found are: (i) the absence of short time correlations in a series of returns and a persistence of correlation when a series of absolute value of returns is investigated, (ii) the wings of distributions of returns are characterized by the power law decay with the exponent value about 3. These properties are called stylized-facts because they are present independently of the kind of the financial asset: stock, money or derivative, and independently of the geographic location of a market: Tokyo, New York or London.

However, in the studies of Polish stock market — a local, emergent and inefficient market, we find at least three characteristics that are different from the ones described above. (A) The wings of distributions of returns decay faster. The exponent value of this decay is about 4 [4]. (B) The nonlinear short-time correlations in daily returns investigated by Artificial Insymmetrized Patterns [5] indicate the Gaussian origin of the noise. The series from the mature markets exhibit self-similarity to the Lévy noise. (C) The Polish market crash in April 2000 can be named anti-bubble crash. Before that crash the Polish stock market had been a growing market since the Russian crash which happened in August of 1998. However, in contrast to the before-crash ordinary log-periodic price development, see [6], the price of Polish stocks developed in the log-periodic style as the market would suffer from the Russian crash. That is, the log-periodic oscillations were initiated by the Russian crash. This price dependence ended with the New Technology crash in April 2000. Such the after-crash behavior observed before the crash is called the anti-bubble crash, [7].

In the following paper we wish to present yet another feature that we found in our studies of the Polish stock market, namely, the deficit in small returns. Let us explain this property with the help of Fig. 1. This figure presents the distributions of returns calculated for the time series of the main index of the Polish stock exchange, called WIG (the upper panel of Fig. 1), as well as the distribution of returns for the collection of several carefully chosen stocks (the bottom panel of Fig. 1). It appears that the zero-return peaks are accompanied with valleys on the left or right side. If we consider the distribution of returns of individual stocks then these valleys, specially on the left side of the zero-return peak, become more evident. In Section 2 we show examples of this behavior. The phenomenon is observed not only in stocks of companies with low capital and low liquidity or on the verge of bankruptcy; the returns of liquid stocks from the top of the volume also exhibit the described property.

To explain this feature we propose a possible micro dynamics, called *amplified imitation*. The motivation for our proposition goes as follows.

The large markets like London City or New York Stock Exchange have developed firm structures and regulations due to their experience stretching more than a century. In effect, the complete information is available to all investors at the same moment of time. The fundamental assumptions of Fama stock market theory [8] hold. It is always underlined that investors are exceptional people because of their strong personalities, [9]. It is even said that the investors are overconfident about their own wisdom and their own understanding of a market. Therefore, they (the investors) seem to have no reason to search around for hints and advise. However, sharing the same information, opinion and often similar analysis tools, investors without any intention, unaware of it, make the same market buy/sell decisions. This behavior, called herding behavior, results in large price movements. It is believed that this causes the observed thick tails in the distribution curve of the returns [10].



Fig. 1. Upper panel: distribution of daily normalized returns of the Warsaw Stock Exchange index WIG. Bottom panel: distribution of daily normalized returns of many stocks selected from the Warsaw Stock Exchange and classified as irregular around the peak of the zero-return. The size of the histogram bin in case of the collection of stocks is twice larger than in case of WIG. Due to this operation we see two small valleys around the zero-return in case of WIG data also.

On poorly developed markets, such as emergent and local markets, the investors deal with assets of little liquidity. For example, on Polish stock market the majority of stocks (about 2/3 out of 235) are with less than 20 transactions per session (data from 2002 [11]). In addition, the investors encounter in their activity a strong political interference. As a result, the

investors often decide to find a "well-informed" agent, form a team and trust his/her decisions. It seems that intensional imitation, what means copying some other investor's decisions, must take place in this situation. We have learned about this strategy in discussions with Polish investors. Keeping in mind the properties of the investors described in the previous part of the article, one can have doubts about our investors stating the truth. Assuming, however, that this mechanism is plausible, we consider the intentional imitation in the percolation model of a stock market, Section 3. For our study of the imitation we choose the model proposed by Cont and Bouchaud [12] modified to a lattice version by Stauffer *et al.* [13].

The intentional imitation mechanism can be easily applied to other market models as, for example, Lux–Marchesi [15] or Levy–Levy–Solomon [16] or others (see [17] for market models review). The preliminary results of this proposition are presented in [14]. Here, by presenting the empirical evidence that the deficit in small returns is present on the real market, we give a strong support for the micro dynamics which leads to the similar effect.

## 2. Warsaw stock market study

In 2002 year there were 235 stocks on the main Warsaw stock exchange. We examine 75 time series, each collected for 7 years. All of them started in January 1997 and ended in December 2003. In these series we search for the property: deficit in small returns. The normalized returns are calculated as follows:

$$r_{\Delta} = \frac{\log p(t + \Delta) - \log p(t)}{\sigma_{\Delta}},$$

where p(t) is a price of a stock and  $\sigma_{\Delta}$  is the standard deviation error of the series. When preparing the histogram our crucial care is to equalize the needed accuracy and the market regulation on the minimal price change. Because of the minimal price change rule we observe the separation between the zero-return and any non-zero values of less than 0.064. Therefore we decide that setting 0.20 as the value for the size of a histogram bin is reasonable. At average this bin's size is 5 times greater than the separation value.

In the subsequent figures the distributions of normalized returns are plotted. Fig. 2 collects data which represents small firms: low capital and low liquidity stocks, Fig. 3 presents the main stocks of the Warsaw Stock Exchange, however, only those which give series of at least five years. Below we provide some description of the plotted stocks and companies which they represent. We give subsequently: (i) the DFA exponent to test the existence of long-term correlation (see [19] for the method description), (ii) the position of the stock on the volume list to present the role played on the market



Fig. 2. Time series of low liquidity stocks of small companies: Indykpol, Bauma, Apator and Forte. Black curves represent prices, gray curves denote volumes in the log-scale and area filled curves are distributions of returns.

by the stock, (iii) average number of transactions which take place during one session together with the number of sell-buy calls to give the information about liquidity of a stock, (iv) the position of a stock on the 2002-year return list together with the value of the 2002-year return. All data come from [11].

In Fig. 2 we present the following stocks:

**Indykpol** — the company that operates on turkey meat: (i) DFA gives: 0.28, (ii) 206th on the volume ranking list, (iii)  $\approx 2$  transactions per session (17 calls), (iv) 108th on the return ranking list: -30% Euro.

**Bauma** — the company which distributes products for construction works: (*i*) DFA gives: 0.37, (*ii*) 168th on the volume ranking list, (*iii*)  $\approx$  1 transactions per session (14 calls), (*iv*) 141st on the return ranking list: -47% Euro.

**Apator** — the company which distributes electricity metering systems: (i) DFA gives: 0.41, (ii) 59th on the volume ranking list, (iii)  $\approx 11$  transactions per session (52 calls), (iv) 7th on the return ranking list: 75% Euro.

**Forte** — the furniture enterprise: (i) DFA gives: 0.43, (ii) 139th on the volume ranking list, (iii)  $\approx 5$  transactions per session (29 calls), (iv) 15th on the return ranking list: 40% Euro.

Comparing the distribution of individual stocks presented in Fig. 2 with the averaged results plotted in Fig. 1 we notice distinctions. For example, returns that are bigger than two standard deviation errors occur at rather large probability; the wings of distributions are wide. It suggests that these markets are of high volatility. Next, deep valleys around the zero-return peak are observed indicating the strong deficit in small returns. Moreover, since the corresponding DFA exponents are less than 0.43 then all time series exhibit the antypersistency. All stocks shown in Fig. 2 represent illiquid assets. The stocks of INDYKPOL and BAUMA are illiquid because the price of these stocks has been permanently going down for last years. On the other hand the price of APATOR or FORTE stock has grown up for the last three years. It means, these stocks are rather illiquid because they belong to the group of the most profitable stocks on the Polish market. Investors are making profit by keeping these stocks.

Fig. 3 is to show how the small-return probability looks like if the stocks of strongest companies of the Polish market are considered. Here we collect the following stocks:

**TP S.A.** — the telecommunication enterprise: (i) DFA gives 0.49 (ii) 1st on the volume ranking list, (iii)  $\approx$  1299 transactions per session (4999 calls), (iv) 87th on the return ranking list: -20% Euro.

**PeKaO S.A.** — the bank: (i) DFA gives 0.43 (ii) 2nd on the volume ranking list, (%) (iii)  $\approx 565$  transactions per session (1081 calls), (iv) 33rd on the return ranking list: 7.5% Euro.



Fig. 3. Time series of stocks of companies with largest capitalization: TP, PEKAO, KGHM, BPH. Black curves represent prices, gray curves denote volumes on the log-scale and area filled curves are distributions of returns.

**KGHM** — the producer of copper and silver: (i) DFA gives 0.47 (ii) 4th on the volume ranking list, (iii)  $\approx 445$  transactions per session (1001 calls), (iv) 59th on the return ranking list: -9% Euro.

**BPH** — the bank: (i) DFA gives 0.46 (ii) 5th on the volume ranking list, (iii)  $\approx 180$  transactions per session (420 calls), (iv) 43rd on the return ranking list: 5% Euro.

The stocks presented in Fig. 3 are rather not profitable if the investor's strategy is to keep the stocks. However, it is easy to trade these stocks. Therefore the profit from these stocks comes from speculation. The deficit in small returns is less evident than in case of stocks of Fig. 2 but it is still noticeable.

#### 3. The model of amplified imitation

In the Stauffer *et al.* representation [13] of the Cont–Bouchaud proposition [12] one deals with a number of market agents which are distributed on a square lattice of a linear size L. Each agent is assigned to a site of this lattice at random with the probability p. A site can be occupied by one investor or left empty. The nearest-neighbor occupied sites form a cluster which then is interpreted as a coalition of agents or equivalently as a large market investor. The larger is the cluster, the stronger investor it represents. The strength of an investor is measured by its impact on the market. The coalition impact is measured by the cluster size. Hence, the cluster structure formed by occupied neighboring sites constitutes a set of heterogeneous investors. This set lasts for a limited period of time after which a new arrangement of stock market participants is introduced.

The market dynamics denotes that all traders from a coalition take the same steps: they buy or sell with the probability a, or just wait with a probability 1 - 2a. At each time step, the new stock price is set. The change in price is proportional to the total sum of demand and sell orders from all active investors. The parameter a is called *activity*. It can be seen as temperature of the market. If the activity is low,  $a < \frac{1}{L^2}$  then on average one cluster acts during a given market session. In effect of the construction of investors by clusters, the histogram of price changes reflects properties of the well-known cluster size distribution. In particular, if p is close to the percolation threshold then the power-law decay of wings of the distribution is observed due to the critical properties of the underlining percolating system [18].

Now let us introduce the following modification to the basic model: Assume that all coalitions which are larger than 2 are divided into two following classes of investors. Let class A consist of investors which are said to be 'well-informed' investors. It means that these investors act independently and at random. They buy or sell with the probability a and do nothing with the probability 1 - 2a. Let class B contain investors which are said to be intentionally imitating other investors. Each investor from class B chooses at random some other investor to imitate. If the choice of an investor to imitate is restricted to the investors of class A then we call the model: a model A. If this choice is not restricted then we call the model : a model AB. The introduced relation between coalitions might be represented by a directed graph. The nodes of the graph are investors and the edges represent the relation of imitation, see Fig. 4. An investor of class B is said to imitate some other investor if, in the subsequent time step, the investor of class B makes the same decision as the related to him investor.



Fig. 4. Directed graphs which present the imitation relation between investors. Grey nodes denote investors of class B, black nodes investors of class A. Each class B investor is related to some other investor. The directed edges show how the decision of one investor moves to other investors. Left panel: model A, right panel: model AB. Notice, one time step difference between a decision of the A and the corresponding class B investor in case of model A and the avalanche of decisions initiated by some class A investor.

The partition of coalitions into classes A and B is done at the probability b. The assignment of the imitation relation lasts for the same period of time as the cluster structure. The small investors (clusters of size 1 and 2) do not take part in the imitation micro dynamics. The activity of these investors is exactly the same as in the CB-model. These clusters may be interpreted as noise traders and they form a bath.

The simulations are performed on the square lattice of the linear size L = 200. The variety of investors are made from the cluster structure obtained at the percolation threshold. Namely, on the square lattice, the probability to assign an agent to a lattice site is set to  $p_c = 0.592746$ . The activity of a market is fixed at a = 0.001. The new arrangement of clusters

is done every 200 steps. One run consists of a million such arrangements. Experiments were repeated several times. In [14] there are presented examples of time series arisen from the both models. As it is expected at low value of b, that is when b < 0.1, we observe bursts of synchronized activity and periods of quiet. The volatility is twice bigger than in case of b > 0.1. There is no short-time correlation.

Here, Fig. 5 presents the probability distribution of returns obtained from the model when different values of b are considered. If there are a few imitative groups on a lattice then a large difference is observed in the distribution of returns from the original CB model. The difference means deficit in small returns and redundancy of large price movements. In case of model A we find valleys around the zero-return peak. However the described difference vanishes when b is approaching 0.1. b = 0.1 corresponds to the



Fig. 5. Distribution of normalized returns for the two models considered. Upper panel: model A, bottom panel: model AB. Huge returns are observed if b < 0.01. Two symmetric deeps around the zero-return peak are present in case of the model A.

fact that 10% of all investors run independently. For example, in case of the lattice with the linear size L = 200 it means that about 40 independent investors influences the price in the same way as investors from the unmodified model of a stock market.

The basic property of distributions of returns does not depend on the lattice size. Simulations with L = 100 and L = 300 do not give any noticeable difference in distributions of returns.

## 4. Closing

We have proposed a micro dynamics to explain the property: deficit in small returns, which we found in the distribution of daily returns of stocks traded on Polish stock market. We call this micro dynamics the amplified imitation to empahsize the concept of intentional herding behavior among investors. This micro dynamics may be considered as a strategy: *Follow the leader*.

Our simulations show that the mechanism of replicating other investor's decisions could be responsible for the observed properties of distributions, especially in case of decisions to sell. The persistent presence of the valley on the left side of the zero-return peak in the distributions of returns in all stock studied suggests that selling decisions are undertaken synchronously. Furthermore, the depth of these valleys suggests that the model A of imitation is better suited to the Polish stock market than the model AB. However, the results of our simulations should be considered qualitatively only. One should have in mind the general difference between distributions found in the real markets and those simulated ones. We can only speculate that on the Warsaw stock market, there are only a few independent investors, precisely identified by others as 'well informed' and they often initiate the decision to sell. The values of Detrended Fluctuation Analysis exponent additionally justify our conjecture about mutual influences that exist among investors.

It appears that the deficit in small returns can be also reconstructed by considering amplified imitation in the form called *follow the trend* [20]. The concept of follow the trend imitation is based on the strong influence of the last time step price change to the decision of an investor. Again, the synchronized decisions of many investors effect in a noticeable deficit in small returns.

Here we have considered a mechanism of the amplified imitation in the frame of Cont–Bouchaud model. The same mechanism applied to other models should lead to qualitatively similar results. This work is supported by Polish Ministry of Scientific Research and Information Technology : PB/1472/P03/2003/25. The simulations have been performed partially in TASK — Academic Computer Center in Gdańsk.

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