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ON LEPTOGENESIS WITH HIERARCHICAL NEUTRINO MASSES*

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We discuss baryogenesis via leptogenesis in seesaw models with the heaviest right-chiral neutrino effectively decoupled from the seesaw mechanism and propose a natural bottom-up parametrization, which leads to a simple formula for the CP asymmetry in the decays of the lightest right-chiral neutrino. We show that for successful leptogenesis there is a lower bound on the mass of the lightest right-chiral neutrino. If this neutrino is to be produced thermally after inflation, the bound on its mass can be translated into a lower bound on the reheating temperature of the Universe, which can be in conflict with the upper bound required to avoid the gravitino problem. We also present possible ways of circumventing this difficulty.

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1. Introduction

Neutrino physics has become an exciting field of theoretical research. The discovery that the neutrinos have small masses and large mixings [1] posed a difficult question about a theoretical framework, in which these experimental results can be naturally embedded. Note that oscillation experiments give Δm_ν^2 but not the overall neutrino mass scale. The most severe direct experimental constraint for the overall scale comes from tritium beta decay (see *e.g.* [2] for a review of the experimental results). Even better, although indirect, bound can be obtained from the observations of the cosmic microwave background (CMB), which indicates that $\sum m_\nu \leq 0.7\,\mathrm{eV}$ [3], which favours the hierarchical spectrum of the neutrino masses. The fact that this mass scale is much lower than the masses of the other elementary

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particles, which can be elegantly explained by the seesaw mechanism [4], which assumes the existence of right-chiral neutrino fields N_i, which are singlets of the Standard Model gauge group and can, therefore, have explicit Majorana mass terms in the Lagrangian, as well as the Yukawa couplings, *i.e.* $\mathcal{L} \supset -\frac{1}{2}(M_{\rm R})_{ij} N_i N_j - (Y_\nu)_{ij} N_i^c l_j H$. For energies much lower than the mass M of the lightest right-chiral neutrino, it is then convenient to use the effective theory, in which the masses of the left-chiral neutrinos are given by an effective operator obtained by integrating out the heavy states $\mathcal{L}_{\rm eff} \supset \frac{C_{ij}}{M}\,(l_i H)\,(l_j H))$ and:

$$C = Y_\nu^T D_{\rm R}^{-1} Y_\nu\,, \tag{1}$$

where $D_{\rm R}$ is a mass matrix of the right-chiral neutrinos normalized to M

$$D_{\rm R}^{-1} = {\rm diag}(1, x^2, y^2) \qquad x, y < 1\,. \tag{2}$$

The eigenvalues ζ_i^2 of C are related to the masses left-chiral neutrinos by the formula:

$$\zeta_i^2 = \frac{2 M m_{\nu_i}}{v^2}\,, \tag{3}$$

where v is the vacuum expectation value of the Higgs field. The advantage of such a parametrization of the light neutrino masses is that the quantities ζ_i set a natural scale for the neutrino Yukawa couplings. Above the scale M the right-chiral neutrinos are treated as active states with masses M, M/x^2, M/y^2, respectively. One can expect that the masses of these particles are related to the Grand Unification scale.

However, it has been realized that hierarchical values of the neutrino masses and large neutrino mixing angles are not natural in the context of the seesaw mechanism (see *e.g.* [5]). This suggests the neutrino Yukawa matrix Y_ν having a special structure, following from horizontal symmetries [6] or dominance of one or at most two right-chiral neutrinos [7]. In the latter class of models, the decoupling of the heaviest right-chiral neutrino from the seesaw mechanism appears the most natural. The decoupling hypothesis is independently supported by the observation that in supersymmetric models the sneutrino-driven inflation can occur only if one of the right-chiral neutrinos effectively decouples from the seesaw mechanism [8].

An exciting by-product of the seesaw mechanism is the possibility of explaining the observed baryon number (B) asymmetry of the Universe [9], which is confirmed by the Big Bang Nucleosynthesis [10] and the recent measurements of the anisotropies in the CMB [3, 11]. The value of the baryon asymmetry deduced from the CMB anisotropies by the WMAP collaboration [3] is (3σ range):

$$0.8 \times 10^{-10} \leq Y_B \leq 10^{-10}\,. \tag{4}$$

The seesaw model predicts that, when the Universe cools down, the right-chiral neutrinos fall out of thermal equilibrium and decay, violating the lepton number (L). If there is enough CP violation in these decays, a net lepton number asymmetry may be produced in the Universe, since all the Sakharov conditions are fulfilled [12]. This asymmetry results in a B asymmetry due to $B+L$ violating sphaleron transitions [13]. Such a scenario, first proposed in [14], has been called baryogenesis via leptogenesis.

The main difficulty in the description of leptogenesis is the fact that the number of the parameters in the neutrino sector exceeds the number of the observables. Since the matrix C encodes information about the masses, mixing angles and the CP properties of the light neutrinos, the relation (1) is a set of 6 complex equations for 9 complex neutrino Yukawa couplings and 3 real masses of the right-chiral neutrinos, which cannot, therefore, be determined from the low-energy data. This difficulty can be overcome by specifying some 'textures' in the neutrino Yukawa matrix and the mass matrix of the right-chiral neutrinos which naturally lead to the predictions consistent with the experimental data (see [5] for a review of the solutions), including the observed baryon asymmetry of the Universe (see *e.g.* [15]). Another strategy is to describe the high-energy parameters, such as the CP asymmetries in the decays of the right-chiral neutrinos, in terms of the low-energy observables, so that the remaining freedom in the former is minimal (see *e.g.* [16, 17]; a variant of this strategy in supersymmetric models utilizes the constraints from the RG-induced lepton flavour violation, see *e.g.* [8, 18, 19]). This allows to derive certain bounds on the parameters relevant for leptogenesis [17, 20].

The purpose of this contribution is to propose a parametrization of the neutrino Yukawa couplings in models with the heaviest right-chiral neutrino decoupled and to discuss the CP asymmetries in the decays of the lightest right-chiral neutrino, which give rise to the baryon asymmetry of the Universe. For the extended discussion and details of the calculation the reader is referred to [21].

2. The setup

Let us assume a certain pattern of the light neutrino masses m_{ν_1}, m_{ν_2}, m_{ν_3} satisfying the experimental constraints at the low scale. We neglect possible threshold corrections to the neutrino masses [22]. For the hierarchical and inversely hierarchical pattern of the neutrino masses the renormalization group corrections only result in multiplying the masses by a common factor $\mathcal{O}(1)$ (see *e.g.* [23] and references therein).

Since our aim is to unravel the relation (1), it is convenient to choose the electroweak basis for the lepton doublets in which the matrix C is diagonal.

The passage to the basis in which the Yukawa matrix of the charged leptons is diagonal is given by the neutrino mixing matrix U, whose entries can be determined experimentally. Let us denote the matrix elements in Y_ν by $Y_{ij}, i,j = 1\ldots 3$. The solution of (1) gives *e.g.* Y_{11}, Y_{12}, Y_{13}, Y_{21}, Y_{23} and Y_{31} in terms of ζ_i, x, y, Y_{22}, Y_{32}, Y_{33}. We do not write these formulae here because of their length. We only note that even for given ζ_i, x, y there is a big (6-parameter, since Y_{ij} are complex numbers) ambiguity in the determination of the entries in the Yukawa matrix for the neutrinos. This is the same freedom which was noticed in [18] and described in terms of a complex orthogonal matrix Ω.

The CP asymmetry in the decays of the lightest right-chiral neutrino reads [24, 25]:

$$\varepsilon_i = \frac{1}{8\pi\left(Y_\nu Y_\nu^\dagger\right)_{ii}} \sum_{j\neq i} Im\left(Y_\nu Y_\nu^\dagger\right)^2_{ji} f\left(\frac{M_j^2}{M_i^2}\right), \tag{5}$$

where

$$f(\xi) = \begin{cases} \sqrt{\xi}\left((1+\xi)\ln(1+1/\xi) + \frac{2-\xi}{1-\xi}\right) & \text{non-SUSY case}\,, \\ \sqrt{\xi}\left(\ln(1+1/\xi) + \frac{2}{\xi-1}\right) & \text{SUSY case}\,. \end{cases} \tag{6}$$

For $\xi \gg 1$ these two functions differ only by a factor of 2 in the leading order. It was found by Davidson and Ibarra that for hierarchical masses of the right-chiral neutrinos there is an upper bound on the CP asymmetry [20]:

$$|\varepsilon_1| < \varepsilon_1^{\max} = \frac{3}{16\pi}\zeta_3^2\,. \tag{7}$$

Then the full numerical solution of the Boltzmann equations [26] in the non-supersymmetric case can be parametrized as [27]:

$$\frac{Y_B}{9\times 10^{-11}} \approx -\left(\frac{M}{2\times 10^{10}\,\mathrm{GeV}}\right)^2 \times \frac{4\times 10^{-6}}{(Y_\nu Y_\nu^\dagger)_{11}} \times \frac{\varepsilon_1}{\varepsilon_1^{\max}}\,. \tag{8}$$

The prediction for Y_B in the supersymmetric case is roughly the same as in the non-supersymmetric case, since the number of the degrees of freedom that wash out the asymmetry is compensated by an increase in $|\varepsilon_1|$ [28].

Note that the choice of the flavour basis for the left-chiral leptons is irrelevant for the calculation of the lepton asymmetries. Indeed, a rotation of the left-chiral leptons with the use of the matrix U results in the transformation: $Y_\nu \to Y_\nu U^\dagger$ which does not affect the quantity $Y_\nu Y_\nu^\dagger$ which enters Eqs. (5) and (8).

3. Solution for $\nu_{\rm L}$ and $\nu_{\rm R}$ hierarchical

Let us consider the case of hierarchical masses of right-chiral neutrinos, *i.e.* $y^2 \ll x^2 < 1$. Following [18], the Yukawa couplings of the neutrinos can be parametrized as:

$$Y_{ij} = \left(D_{\rm R}^{1/2}\right)_{ii} \Omega_{ij}\zeta_j \,, \tag{9}$$

where Ω is an orthogonal matrix. If the heaviest right-chiral neutrino decouples from the seesaw mechanism, its contribution to the relation (1) is negligible, which corresponds to the limit $y \to 0$. This limit can be easily calculated for (9) and $i = 1, 2$:

$$Y_{11} \;=\; 0 \,, \tag{10}$$

$$Y_{12} \;=\; -\sqrt{\zeta_2^2 - x^2 Y_{22}^2} \,, \tag{11}$$

$$Y_{13} \;=\; -\frac{\zeta_3}{\zeta_2} x Y_{22} \,, \tag{12}$$

$$Y_{21} \;=\; 0 \,, \tag{13}$$

$$Y_{23} \;=\; -\frac{\zeta_3}{\zeta_2 x}\sqrt{\zeta_2^2 - x^2 Y_{22}^2} \,. \tag{14}$$

Since $(Y_\nu)_{3j} = y^{-1}\Omega_{3j}\zeta_j$, a consistent passage to the limit $y \to 0$ is possible only for $\Omega_{32}, \Omega_{33} \to 0$ and $\zeta_1 \to 0$ (*i.e.* the lightest neutrino is massless). The latter results is obvious, as for $y \to 0$ the r.h.s. of Eq. (1) is a rank 2 matrix and C must have a zero eigenvalue.

The CP asymmetry in the decays of N_1 reads:

$$\varepsilon_1 \approx \left\{ \begin{array}{ll} \frac{3}{16\pi}(\zeta_3^2 - \zeta_2^2)\sin(2{\rm arg}Y_{22}^*) & \text{for } x|Y_{22}| \gg \zeta_2 \\ \frac{3\zeta_3^4}{16\pi\zeta_2^4} x^2 |Y_{22}|^2 \sin(2{\rm arg}Y_{22}^*) & \text{for } x|Y_{22}| \ll \zeta_2 \end{array} \right. \tag{15}$$

while the strength of the wash-out of the generated lepton asymmetry is given by [26]:

$$\left(Y_\nu Y_\nu^\dagger\right)_{11} = \left\{ \begin{array}{ll} \frac{x^2|Y_{22}|^2}{\zeta_2^2}(\zeta_3^2 + \zeta_2^2) & \text{for } x|Y_{22}| \gg \zeta_2 \\ \zeta_2^2 & \text{for } x|Y_{22}| \ll \zeta_2 \end{array} \right. . \tag{16}$$

In the derivation of these formulae only the leading powers of x were used. Both ε_1 and $(Y_\nu Y_\nu^\dagger)_{11}$ generically depend on the product xY_{22}. Therefore, we shall keep $x = 0.3$ fixed and discuss the results only for varying Y_{22}. The results of the full calculation of ε_1 and $(Y_\nu Y_\nu^\dagger)_{11}$ are presented in figure 1. They were obtained for $M = 2 \times 10^{10}$ GeV, $x = 0.3$, $m_{\nu_3} = 52$ meV, $m_{\nu_2} = 8.5$ meV and $\arg Y_{22} = \pi/4$. We also adopted realistic values of $y = 10^{-3}$ and $m_{\nu_1} = 10^{-3}$ meV. The moduli of Y_{32} and Y_{33} varied from $0.3|Y_{22}|$ to $3|Y_{22}|$ and their phases were also randomized.

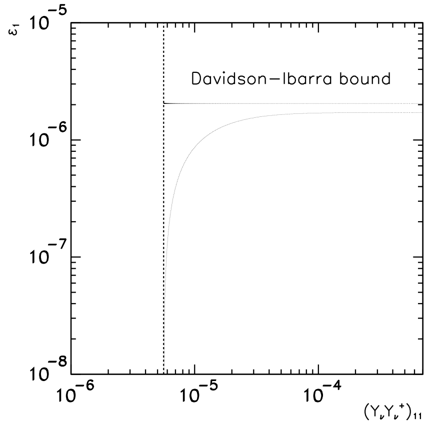

Fig. 1. The results of a full calculation of the CP asymmetry $|\varepsilon_1|$ *versus* the wash-out strength $(Y_\nu Y_\nu^\dagger)_{11}$. Details are given in the text.

4. Discussion

As it can be seen in figure 2, for $x|Y_{22}| \gg \zeta_2$ the resulting baryon asymmetry is suppressed due to a very strong wash-out, whereas for $x|Y_{22}| \ll \zeta_2$ there is a suppression of ε_1, which results in smaller values of the generated baryon asymmetry. The minima of different curves in figure 2 correspond to the same minimal value of $(Y_\nu Y_\nu^\dagger)_{11}$. As a consequence, there exists a lower bound on the mass of the lightest right-chiral neutrino which was estimated as [21]:

$$M > 2 \times 10^{11}\,\mathrm{GeV}\,. \tag{17}$$

Due to the abovementioned suppression of ε_1 for small $x|Y_{22}|$, the bound (17) is two orders of magnitude bigger than the one obtained recently in [17]. Such heavy right-chiral neutrino could be produced thermally in significant abundance after inflation, only if it was relativistic at the time of the reheating of the Universe, *i.e.* $T_{\mathrm{RH}} > M$. This creates a problem, because in supersymmetric theories, overpoduction of gravitinos that destroys nucleosynthesis [29] may occur already for $T_{\mathrm{RH}} > 10^7$ GeV [30].

One possible way of circumventing this problem is to assume that the two right-chiral neutrinos are almost degenerate in masses, which enhances the CP asymmetries in the decays of both right-chiral neutrinos contributing to the lepton asymmetry [31] and allows lowering the reheating temperature [21, 32] if $1 - x^2 \sim 10^{-7}$. The natural idea that such a tiny splitting of the masses of the right-chiral neutrinos might result from the renormalization group (RG) evolution of the neutrino parameters with exact mass degeneracy at some higher (presumably the GUT) scale has been recently presented in [33] and the complete calculation of the relevant CP asymme-

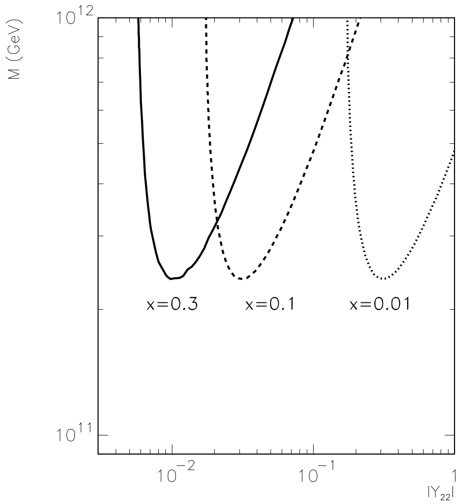

Fig. 2. Lower limits on M_1 for $x = 0.01$, 0.1 and 0.3 as functions of $|Y_{22}|$.

tries has been performed in [34]. It has been found that the enhancement of the CP asymmetries due to approximate mass degeneracy of the right-chiral neutrinos is partially compensated by other RG effects in the running of the neutrino Yukawa couplings. In spite of this compensation, thermal leptogenesis can be successful for moderate and large values of $\tan\beta$.

5. Conclusions

In conclusion, hierarchical masses and bi-large mixing pattern of light neutrinos are naturally explained in a broad class of theoretical seesaw models, in which the heaviest right-chiral neutrino decouples from the seesaw mechanism. Then, if the baryon number asymmetry of the Universe is to be generated by the decays of the lightest right-chiral neutrino, its mass and, consequently, the reheating temperature have to be higher than $\sim 2 \times 10^{11}$ GeV. Supersymmetric versions of such models face therefore the serious problem of gravitino overproduction if the right-chiral neutrinos are produced thermally.

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