DERIVATION OF THE POST-POST-NEWTONIAN EQUATIONS OF MOTION FOR THE POINT PARTICLES BY THE EIH APPROXIMATION METHOD FROM THE EINSTEIN FIELD EQUATIONS WITH THE INFELD–PLEBANSKI STRESS-ENERGY TENSOR

Iwona Nowak

Institute of Physics, University of Rzeszów
Rejtana 16A, 35-310 Rzeszów, Poland

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The post-post-Newtonian equations of motion for the point particles using the EIH approximation method are derived. These equations are deduced from the gravitational field equations with the stress-energy tensor proposed by Infeld and Plebanski. The Infeld-coordinates and EIH-coordinates are used. Regularization of the metric tensor and its derivatives on the world lines of particles by means of the modified Dirac $\hat{\delta}$-function, which is contained in the stress-energy tensor, has been done.

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1. Introduction

The discoveries made in astrophysics [1, 2] as well as the last attempts made in several research labs (for instance, in LIGO and VIRGO) to detect gravitational waves, require theoretical description of the compact binary object dynamics. This one in its turn, requires the derivation of the post-post-post-Newtonian (3PN) equations of motion and the radiation corrections to them. 2.5PN equations of motion for the point particles were derived for the first time by Damour and Deruelle [3–6] by using the results of [7]. The method of derivation utilized the stress energy-tensor which involved the Dirac δ-function (this idea was first introduced by Infeld [8]), as well as the Hadamard "*partie finie*" regularization procedure. 2.5PN equations of motion were derived by Blanchet *et al.* [9] directly from the field equations, the method of derivation also included stress-energy tensor with Dirac δ-function and Hadamard "*partie finie*" regularization procedure. Later on this regularization procedure was discussed more thoroughly in [10]. It is worthy to mention that in [3–7,9] the harmonic coordinates were used. The harmonic coordinates were also used by Kopejkin [11], but this author derived the 2PN equations of motion by means of Fock method [12]. In this method the spherically-symmetric non-rotating bodies are treated as the three-dimensional space domains where the stress-energy tensor differs from zero, while the point particles are treated as the centers of mass of these bodies. In order to evaluate the metric tensor and its derivatives on the world lines of the point particles, the suitable equations describing the internal structure of the bodies, were used. The 2PN equations of motion obtained by Kopejkin are identical to those obtained in [3–7,9].

In [13] the 2PN equations of motion and the radiation correction to them were obtained by means of harmonic coordinates and surface integral approach [14–16]. Another method of derivation of 2PN equations of motion was given in [17]. The authors solved the field equations by means of retarded integral over the past null cone of the field point (chosen to be within the near zone in order to obtain the equations of motion, and to be within the far zone in order to evaluate the gravitational radiation).

In order to obtain the 2PN equations of motion, is also possible to use ADM Hamiltonian-method [18, 19]. This Hamiltonian-method was developed a the 2PN level in early works [20,21] but completely understood in [22]. The equations of motion obtained by the ADM method differ from those ones obtained in [3–7,9,11,13,17] because of different coordinate conditions.

In this work the 2PN equations of motion are derived from the gravitational field equations by means of Einstein, Infeld, Hoffmann (EIH) approximation method [14–16] (see also [23]) with the stress-energy tensor in the Infeld–Plebanski form [24, 25]. We used here the Infeld coordinates [25] as well as EIH coordinates [16] which enable to write down the field equations in the form of Poisson equations. The modified Dirac δ-function which is contained in Infeld–Plebanski stress-energy tensor allows to regularize the metric tensor and its derivatives on world lines of the particles. 2PN equations of motion obtained in this work differ from those ones obtained in [3–7,9,11,13,17,22].

2. Mathematical background

We start from the field equations of the form

$$R_{\mu\nu} = -\frac{8\pi k}{c^2} T^*_{\mu\nu} \,, \tag{2.1}$$

where

$$T^*_{\mu\nu} = \left(g_{\mu\varrho} g_{\nu\sigma} - \tfrac{1}{2} g_{\mu\nu} g_{\varrho\sigma}\right) T^{\varrho\sigma} \,. \tag{2.2}$$

We consider the stress-energy tensor for the N-point particle system with the mass m_A and the coordinates ξ^a_A, $A=1,2,\dots,N$ in the Infeld–Plebanski form [25]

$$\sqrt{-g}T^{\mu\nu}=\sum_A m_A\hat{\delta}(\bar{x}-\bar{\xi}_A)\frac{d\xi^\mu_A}{dx^0}\frac{d\xi^\nu_A}{dx^0}(\frac{d\sigma_A}{dx^0})^{-1}\,, \tag{2.3}$$

where

$$d\sigma^2_A=\overset{A}{\overline{g_{\mu\nu}}}\,d\xi^\mu_A d\xi^\nu_A,\qquad \overset{A}{\overline{g_{\mu\nu}}}=\int\hat{\delta}(\bar{x}-\bar{\xi}_A)g_{\mu\nu}(d\bar{x})\,. \tag{2.4}$$

Here we use the following notations: k is the gravitational constant, c-the light velocity; Greek indices run from 0 to 3, Latin indices (if there are no additional remarks) run from 1 to 3, repetition of indices implies summation; the capital letters A,B,C take the values $1,2,\dots,N$; the derivatives with respect to x^μ are denoted by ∂_μ, with repect to x^a — by ∂_a, and with respect to ξ^a_A - by ∂^A_a; the dot above a function denotes the derivative with respect to time t; $\Delta\equiv\partial_a\partial_a$, $\Delta_A\equiv\partial^A_a\partial^A_a$, $\xi^0_A=x^0=ct$, $(\xi^a_A)=(\bar{\xi}_A)$, $|\bar{x}|=r$, $\bar{x}-\bar{\xi}_A=\bar{r}_A$, $|\bar{x}-\bar{\xi}_A|=r_A$, $\bar{\xi}_A-\bar{\xi}_B=\bar{r}_{AB}$, $|\bar{\xi}_A-\bar{\xi}_B|=r_{AB}$, $N^a_A=r^{-1}_A(x^a-\xi^a_A)$, $N^a_{AB}=r^{-1}_A(\xi^a_A-\xi^a_B)$, $V^a_A=\frac{d\xi^a_A}{dt}$, $W^a_A=\frac{d^2\xi^a_A}{dt^2}$; the metric signature $(+,-,-,-)$ is chosen. We denote, in general

$$\overset{A}{\overline{(\dots)}}\equiv\int(\dots)\hat{\delta}(\bar{r}_A)(d\bar{x}).$$

The Infeld–Plebanski $\hat{\delta}$-function satisfies the following conditions [26]:

(D1) $\hat{\delta}(\bar{x})$ can be formally differentiated up to any order.

(D2) $\hat{\delta}(\bar{x})=0$ if $\bar{x}\neq 0$: $\hat{\delta}(\bar{x})$ can be treated as a spherically symmetric function.

(D3) For an arbitrary region $V(\bar{x}_0)$ containing $\bar{x}_0$ as an internal point and for any arbitrary continuous function $f(\bar{x})$ of $\bar{x}$ in $V(\bar{x}_0)$, we have

$$\int\limits_{V(\bar{x}_0)}\hat{\delta}(\bar{x}-\bar{x}_0)f(\bar{x})(d\bar{x})=f(\bar{x}_0)\,.$$

(D4) For any neighbourhood $V(0)$ of the point $|\bar{x}|=0$ we have

$$\int\limits_{V(0)}\hat{\delta}(\bar{x})|\bar{x}|^{-p}(d\bar{x})=0,\qquad p=1,2,\dots,L\,,$$

where L is an arbitrary but fixed integer.

The first three conditions are characteristic for the ordinary Dirac δ-function. The fourth condition is a generalization of the third one for singular function $f(\bar{x}) = r^{-p}$.

The Ricci tensor is determined by the following expression:

$$\begin{aligned} R_{\alpha\beta} = & \tfrac{1}{2} g^{\mu\nu}(\partial_\mu\partial_\nu g_{\alpha\beta} + \partial_\alpha\partial_\beta g_{\mu\nu} - \partial_\alpha\partial_\mu g_{\beta\nu} - \partial_\beta\partial_\mu g_{\alpha\nu}) \\ & - g^{\mu\nu} g^{\sigma\varrho}(\Gamma_{\mu\alpha\sigma}\Gamma_{\nu\varrho\beta} - \Gamma_{\mu\alpha\beta}\Gamma_{\nu\varrho\sigma}) , \end{aligned} \tag{2.5}$$

where

$$\Gamma_{\alpha\beta\gamma} = \tfrac{1}{2}(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\beta\gamma}) . \tag{2.6}$$

From the integrability conditions for the field equations (2.1) one can obtain the equations of motion for the point particles [25]

$$\frac{d^2\xi_A^n}{(dx^0)^2} + \left[\overset{A}{\Gamma^n_{\mu\nu}} - \frac{d\xi_A^n}{dx^0} \overset{A}{\Gamma^0_{\mu\nu}} \right] \frac{d\xi_A^\mu}{dx^0}\frac{d\xi_A^\nu}{dx^0} = 0 , \tag{2.7}$$

with

$$\Gamma^\alpha_{\mu\nu} = \tfrac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}) . \tag{2.8}$$

In order to solve the field equations (2.1) by means of the EIH successive approximation method [25], the following postulates have to be assumed:

(a) The metric tensor can be expanded in a power series with respect to c^{-1}:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{\mu\nu} = \sum_{i=1}^{\infty} c^{-i} \underset{(i)}{h_{\mu\nu}} . \tag{2.9}$$

(b) The derivative with respect to time $t = c^{-1}x^0$ and the derivatives with respect to space coordinates x^a do not change the approximation order of the expansion coefficients of $g_{\mu\nu}$

$$\underset{(i)}{h_{\mu\nu}} \sim \frac{\partial}{\partial t} \underset{(i)}{h_{\mu\nu}} \sim \frac{\partial}{\partial x^a} \underset{(i)}{h_{\mu\nu}} . \tag{2.10}$$

In our solution, only even powers of inverse light velocity for g_{00}, g_{mn} and only odd powers for g_{0n} are taken into account:

$$\begin{aligned} g_{00} &= 1 + c^{-2} \underset{(2)}{h_{00}} + c^{-4} \underset{(4)}{h_{00}} + c^{-6} \underset{(6)}{h_{00}} + O(c^{-7}) , \\ g_{0n} &= c^{-3} \underset{(3)}{h_{0n}} + c^{-5} \underset{(5)}{h_{0n}} + O(c^{-6}) , \\ g_{mn} &= -\delta_{mn} + c^{-2} \underset{(2)}{h_{mn}} + c^{-4} \underset{(4)}{h_{mn}} + O(c^{-5}) . \end{aligned} \tag{2.11}$$

Such a choice of the powers in the expansions characterizes the standing wave to which we can restrict our consideration deriving the 2PN equations of motion. In the 2PN approximation the equations of motion do not depend on the radiation term $\underset{(5)}{h_{00}}$ (see also [27]).

The approximate field equations one can write in the Poisson form with the compact-supported sources using the identities

$$\partial_a(fg) \equiv f\partial_a g + g\partial_a f\,, \tag{2.12}$$

$$\partial_a\partial_b(fg) \equiv f\partial_a\partial_b g + g\partial_a\partial_b f + \partial_a f\partial_b g + \partial_a g\partial_b f\,. \tag{2.13}$$

Taking into account the identities (2.12), (2.13) and the additional formulae:

$$\begin{aligned} \partial_a r_A^{-1} &= -r_A^{-2} N_A^a\,, \\ \partial_a\partial_b r_A^{-1} &= r_A^{-3}[3N_A^a N_A^b - \delta_{ab}] - \frac{4}{3}\pi\delta(\bar{r}_A)\delta_{ab} \end{aligned}$$

we can define the derivatives of the product of the singular functions by the products of their derivatives (see Appendix A, for details).

The 2PN equations of motion are derived here using the Infeld coordinate condition [25]:

$$\partial_m(\sqrt{-g}g^{mn}) = 0\,, \qquad \partial_\mu(\sqrt{-g}g^{\mu 0}) = 0\,, \tag{2.14}$$

as well as the EIH-coordinates [16]:

$$\partial^m\gamma_{mn} = 0, \qquad \partial^\mu\gamma_{\mu 0} = 0, \qquad \gamma_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu}\eta^{\alpha\beta}h_{\alpha\beta}\,. \tag{2.15}$$

3. The post-Newtonian approximation

Taking into account the formulae (2.2), (2.3), (2.5), (2.6), (2.10), (2.11) and (2.14), from the field equations (2.1) we get the following expressions:

$$\begin{aligned} \varDelta\ \underset{(2)}{h_{00}} &= 8\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)\,, \\ \varDelta\ \underset{(2)}{h_{mn}} &= 8\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)\delta_{mn}\,, \\ \varDelta\ \underset{(3)}{h_{0n}} &= -16\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)V_A^n\,. \end{aligned} \tag{3.1}$$

The solutions to Eqs. (3.1) take the form:

$$\underset{(2)}{h_{00}} = -2\varPhi\,, \qquad \underset{(2)}{h_{mn}} = -2\varPhi\delta_{mn}\,, \qquad \underset{(3)}{h_{0n}} = 4\varPhi_n\,, \tag{3.2}$$

where

$$\Phi = k\sum_A m_A r_A^{-1}, \qquad \Phi_n = k\sum_A m_A V_A^n r_A^{-1}\,. \tag{3.3}$$

Using (2.2), (2.3), (2.5), (2.6),(2.10), (2.11), (2.14) and the solution (3.2), from Eq. (2.1) in the post-Newtonian approximation we have

$$\Delta \underset{(4)}{h_{00}} + 2\ddot{\Phi} + 4\Phi\Delta\Phi - 4\partial_k\Phi\partial_k\Phi = 16\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)\left[\tfrac{3}{4}V_A^2 - 3\Phi + \tfrac{1}{2}\overset{A}{\overline{\Phi}}\right]. \tag{3.4}$$

From

$$\ddot{\Phi} = \tfrac{1}{2}\Delta\ddot{\chi}, \qquad \chi = k\sum_A m_A r_A, \tag{3.5}$$

$$\Delta\Phi = -4\pi k\sum_A m_A\hat{\delta}(\bar{r}_A).$$

and from the identity

$$\Delta\Phi^2 = 2\Phi\Delta\Phi + 2\partial_k\Phi\partial_k\Phi, \tag{3.6}$$

which follows from (2.12), (2.13) (see the formulas (A.5), (A.6) in Appendix A), we get

$$\Delta\left[\underset{(4)}{h_{00}} + \ddot{\chi} - 2\Phi^2\right] = 16\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)\left[\tfrac{3}{4}V_A^2 - \Phi + \tfrac{1}{2}\overset{A}{\overline{\Phi}}\right]. \tag{3.7}$$

Taking into account the first expression in (3.3), we have

$$\begin{aligned}\int \hat{\delta}(\bar{r'}_A)\Phi'|\bar{x}-\bar{x}'|^{-1}(d\bar{x}') &= k\sum_{\substack{B\\ B\neq A}} m_B\int \hat{\delta}(\bar{r'}_A)r_B'^{-1}|\bar{x}-\bar{x}'|^{-1}(d\bar{x}')\\ &+ km_A\int \hat{\delta}(\bar{r'}_A)r_A'^{-1}|\bar{x}-\bar{x}'|^{-1}(d\bar{x}')\,.\end{aligned} \tag{3.8}$$

The last integral in (3.8) can give a contribution to the solution of Eq. (3.7) for $\bar{x}' - \bar{\xi}_A = 0$. Now, we expand the expression $|\bar{x}-\bar{x}'|^{-1}$ in the power series with respect to $r_A' = |\bar{x'} - \bar{\xi}_A| \approx 0$

$$|\bar{x}-\bar{x}'|^{-1} = \sum_{m=0}^{\infty} P_m(N_A^k N_A'^k) r_A'^m r_A^{-(m+1)}, \qquad r_A' < r_A \tag{3.9}$$

where $P_m(N^k_A N'^k_A)$ are the Legendre polynomials. The property (D) of the function $\hat\delta$ yields [see (B.5) Appendix B]

$$\begin{aligned}\int \hat\delta(\bar r'_A) r'^{-1}_A |\bar x - \bar x'|^{-1}(d\bar x') &= \int \hat\delta(\bar r'_A) P_1(N^k_A N'^k_A) r^{-2}_A (d\bar x')\\ &= r^{-2}_A N^k_A \int \hat\delta(\bar r'_A) N'^k_A (d\bar r'_A) = 0\,, \end{aligned} \tag{3.10}$$

since $P_1(N^k_A N'^k_A) = N^k_A N'^k_A$. The first integral on the right-hand side of Eq. (3.8) can be easily calculated, because the integrand is continuous for $A \neq B$. Then, finally, from (3.8) and (3.10), we obtain

$$\int \hat\delta(\bar r'_A)\Phi' |\bar x - \bar x'|^{-1}(d\bar x') = \overset{A}{\overline{\Phi}}\, r^{-1}_A \tag{3.11}$$

where

$$\overset{A}{\overline{\Phi}} = k \sum_{\substack{B\\ B\neq A}} m_B r^{-1}_{AB}\,.$$

Taking into account the last result, we can integrate Eq. (3.7) and we get

$$\underset{(4)}{h_{00}} = 2\Phi^2 - \ddot\chi + k\sum_A m_A (2\,\overset{A}{\overline{\Phi}} - 3V^2_A) r^{-1}_A . \tag{3.12}$$

From the formulas (2.7), (2.8), (2.9) and (2.11) in the post-Newtonian approximation we have

$$\frac{d^2\xi^n_A}{dt^2} = \underset{(0)}{F^n_A} + c^{-2}\underset{(2)}{F^n_A}\,, \tag{3.13}$$

where

$$\underset{(0)}{F^n_A} = -\frac{1}{2}\,\overline{\partial_n \underset{(2)}{h_{00}}}^{A}\,, \tag{3.14}$$

$$\begin{aligned}\underset{(2)}{F^n_A} = &-\frac{1}{2}\overline{\partial_n \underset{(4)}{h_{00}}}^{A} - \frac{1}{2}\overline{\underset{(2)}{h_{nk}}\,\partial_k \underset{(2)}{h_{00}}}^{A} + \overline{\underset{(3)}{\dot h_{0n}}}^{A} + \frac{1}{2}\overline{\underset{(2)}{\dot h_{00}}}^{A} V^n_A\\ &+ \overline{\underset{(2)}{\dot h_{nk}}}^{A} V^k_A - \overline{\partial_n \underset{(3)}{h_{0k}}}^{A} V^k_A + \overline{\partial_k \underset{(3)}{h_{0n}}}^{A} V^k_A + \overline{\partial_k \underset{(2)}{h_{00}}}^{A} V^k_A V^n_A\\ &-\frac{1}{2}\overline{\partial_n \underset{(2)}{h_{mk}}}^{A} V^k_A V^m_A + \overline{\partial_m \underset{(2)}{h_{nk}}}^{A} V^m_A V^k_A\,.\end{aligned} \tag{3.15}$$

Inserting the solutions (3.2) and (3.12) into Eq. (3.15) we get the following integrals with the singular functions which, after integrating, yield

$$\begin{aligned}
&\int \hat{\delta}(\bar{r}_A) r_A^{-2} N_A^k (d\bar{x}) = 0\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^{-3} N_A^k (d\bar{x}) = 0\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^{-2} N_A^k N_A^s N_A^b (d\bar{x}) = 0\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^{-1} (\delta_{kb} - N_A^b N_A^k)(d\bar{x}) = 0\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^{-2} N_A^n r_B^{-1} (d\bar{x}) = 0, \qquad A \neq B\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^{-1} r_B^{-2} N_B^n (d\bar{x}) = 0, \qquad A \neq B\,.
\end{aligned} \tag{3.16}$$

The first four integrals vanish due to the property (D) of $\hat{\delta}$ [see (B.5) in Appendix B]. In order to evaluate the two last integrals, we used additionally the following forms of the expansion of r_B^{-1}, $r_B^{-2} N_B^n$, into the power series with respect to $r_A \approx 0$:

$$\begin{aligned}
r_B^{-1} &= r_{AB}^{-1} - r_{AB}^{-2} N_{AB}^k N_A^k r_A + \left\{\tfrac{3}{2} r_{AB}^{-3} N_{AB}^k N_{AB}^s N_A^k N_A^s - \tfrac{1}{2} r_{AB}^{-3}\right\} r_A^2 + O(r_A^3),\\
r_B^{-2} N_B^n &= r_{AB}^{-2} N_{AB}^n + \left\{ r_{AB}^{-3} N_A^n - 3 r_{AB}^{-3} N_{AB}^n N_{AB}^b N_A^b \right\} r_A + O(r_A^2)\,.
\end{aligned} \tag{3.17}$$

From the above expansion and the property (D) of $\hat{\delta}$, it follows that the last two integrals in (3.16) vanish. From (3.13)–(3.16) and (3.2), (3.12), we have

$$\begin{aligned}
\frac{d^2 \xi_A^n}{dt^2} \;=\; & k \sum_{\substack{B\\ B\neq A}} m_A \partial_n^A r_{AB}^{-1} + c^{-2} \Bigg\{ k \sum_{\substack{B\\ B\neq A}} m_B \Big[\tfrac{1}{2} V_B^m V_B^k \partial_m^A \partial_k^A \partial_n^A r_{AB}\\
&+ \left(4 V_B^n V_A^k - 4 V_A^n V_A^k - 4 V_B^n V_B^k + 3 V_A^n V_B^k\right) \partial_k^A r_{AB}^{-1}\\
&+ \left(V_A^2 - 4 V_A^k V_B^k + \tfrac{3}{2} V_B^2\right) \partial_n^A r_{AB}^{-1} - k(5 m_A + 4 m_B) r_{AB}^{-1} \partial_n^A r_{AB}^{-1} \Big]\\
&- \tfrac{1}{2} k^2 \sum_{\substack{B\\ B\neq A}} \sum_{\substack{C\\ C\neq A\\ C\neq B}} m_B m_C \left(\partial_k^B r_{BC}^{-1}\, \partial_k^B \partial_n^B r_{AB} - 8 r_{AB}^{-1} \partial_n^B r_{BC}^{-1} \right.\\
&\left. - 2 r_{BC}^{-1} \partial_n^B r_{AB}^{-1} - 8 r_{AB}^{-1} \partial_n^C r_{AC}^{-1} \right) \Bigg\}.
\end{aligned} \tag{3.18}$$

For the two bodies $N=2$, we get from (3.18) the following equations

$$\frac{d^2\xi_1^n}{dt^2} = km_2\partial_n^1 r_{12}^{-1} + c^{-2}\Big\{km_2\Big[\tfrac{1}{2}V_2^m V_2^k \partial_m^1\partial_k^1\partial_n^1 r_{12} + (4V_2^n V_1^k - 4V_1^n V_1^k$$
$$-4V_2^n V_2^k + 3V_1^n V_2^k)\partial_k^1 r_{12}^{-1} + (V_1^2 - 4V_1^k V_2^k + \tfrac{3}{2}V_2^2)\partial_n^1 r_{12}^{-1}$$
$$-k(5m_1+4m_2)r_{12}^{-1}\partial_n^1 r_{12}^{-1}\Big]\Big\}\,. \tag{3.19}$$

Substituting the index 1 by 2 in (3.19) we obtain the equations of motion for the second body. The equations (3.19) have been derived for the first time from the field equations by Einstein, Infeld and Hoffmann [14] using the coordinate conditions (2.15). These equations have been derived also by Infeld and Plebanski [24] under condition (2.14).

Here we developed the procedure of receiving the post-Newtonian approximation, since we would like to illustrate the technique of regularization by means of the Infeld–Plebanski $\hat{\delta}$-function and to point out the significant role of the identities (2.12), (2.13).

4. The solution of field equations in the 2PN approximation

Taking into account the solutions (3.2), (3.12) and the formulae (2.2), (2.3), (2.5), (2.6), (2.10), (2.11) and (2.14) from (2.1) we get

$$\Delta \underset{(4)}{h}_{mn} - 4\partial_m\Phi\partial_n\Phi + 4\partial_k\Phi\partial_k\Phi\delta_{mn} + 2\ddot{\Phi}\delta_{mn} + 4\Phi\Delta\Phi\delta_{mn} + 4\partial_n\dot{\Phi}_m + 4\partial_m t\Phi_n$$
$$= 16\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)\left[V_A^n V_A^m - \tfrac{1}{4}V_A^2\delta_{mn} - \tfrac{1}{2}\overset{A}{\Phi}\,\delta_{mn}\right]. \tag{4.1}$$

The second term on the left-hand side of Eq. (4.1) can be transformed by means of the formulae

$$\Delta \ln S_{AB} = r_A^{-1}r_B^{-1}, \qquad S_{AB} = r_A + r_B + r_{AB}, \qquad A \neq B. \tag{4.2}$$

$$\Delta \ln r_A = r_A^{-2}, \tag{4.3}$$

$$\partial_m\partial_n r_A^{-2} = 2\partial_m r^{A}_{-1}\partial_n r_A^{-1} + 2r_A^{-1}\partial_m\partial_n r_A^{-1}. \tag{4.4}$$

The last one results from (A.3) for $L=2$. From Eqs. (4.2)–(4.4), (A.9) and (A.13) and the first expression of (3.3), we obtain

$$\partial_m\Phi\partial_n\Phi = \Delta\Bigg[k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\partial_m^A\partial_n^B\ln S_{AB}$$
$$+\tfrac{1}{8}k^2\sum_A m_A^2(\partial_m\partial_n\ln r_A + r_A^{-2}\delta_{mn})\Bigg] + \tfrac{4}{3}\pi k^2\sum_A m_A^2\hat{\delta}(\bar{r}_A)r_A^{-1}\delta_{mn}\,. \tag{4.5}$$

The third term on the left-hand side of (4.1) one can transform by means of (3.6). In order to transform the fourth term on the left-hand side of Eq. (4.1), we use (3.5). We transform the two last terms on the left-hand side of (4.1) by means of the following formula

$$\Phi_n = \tfrac{1}{2}\Delta\chi_n, \qquad \chi_n = k\sum_A m_A r_A V_A^n\,. \tag{4.6}$$

Taking into account (4.5), (4.6) and

$$2r_A^{-1}\partial_m\partial_n r_A = \partial_m\partial_n \ln r_A + r_A^{-2}\delta_{mn},$$

from (4.1) we have

$$\begin{aligned}
&\Delta\Big[\underset{(4)}{h_{mn}} + 2\partial_n\dot{\chi}_m + 2\partial_m\dot{\chi}_n + \ddot{\chi}\delta_{mn} + 2\Phi^2\delta_{mn}\\
&-4k^2\sum_A\sum_{\substack{B\\ B\neq A}} m_A m_B \partial_m^A\partial_n^B \ln S_{AB} - k^2\sum_A m_A^2 r_A^{-1}\partial_m\partial_n r_A\Big]\\
&= 16\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)\Big[V_A^n V_A^m - \tfrac{1}{4}V_A^2\delta_{mn} - \tfrac{1}{2}\overset{A}{\Phi}\,\delta_{mn}\Big]\\
&+\tfrac{16}{3}\pi k^2\sum_A m_A^2\hat{\delta}(\bar{r}_A) r_A^{-1}\delta_{mn}\,.
\end{aligned} \tag{4.7}$$

Taking into account (3.10), we obtain the solution of Eq. (4.7) in the form

$$\begin{aligned}
\underset{(4)}{h_{mn}} =& -\ddot{\chi}\delta_{mn} - 2\Phi^2\delta_{mn} - 2\partial_n\dot{\chi}_m - 2\partial_m\dot{\chi}_n + 4k^2\sum_A\sum_{\substack{B\\ B\neq A}} m_A m_B\partial_m^A\partial_n^B \ln S_{AB}\\
&+k^2\sum_A m_A^2 r_A^{-1}\partial_m\partial_n r_A - 2k\sum_A m_A r_A^{-1}\Big[2V_A^m V_A^n - \Big(\overset{A}{\Phi} + \tfrac{1}{2}V_A^2\Big)\delta_{mn}\Big].
\end{aligned} \tag{4.8}$$

Using the solutions (3.2) and Eqs. (2.2), (2.3), (2.5), (2.6), (2.10), (2.11) and (2.14), from Eq. (2.1) we get

$$\begin{aligned}
&\Delta\,\underset{(5)}{h_{0n}} + 16\partial_k\Phi\partial_n\Phi_k + 12\dot{\Phi}\partial_n\Phi - 8\Phi\Delta\Phi_n\\
&= 16\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)\Big[2\Phi V_A^n + 2\Phi_n - \overset{A}{\Phi}\,V_A^n - \tfrac{1}{2}V_A^2 V_A^n\Big].
\end{aligned} \tag{4.9}$$

Just in a similar way as in the case of Eq. (4.5), we transform the second and the third terms on the left-hand side of (4.9)

$$\partial_k \Phi \partial_n \Phi_k = \Delta \left[k^2 \sum_A \sum_{\substack{B \\ B \neq A}} m_A m_B \partial_k^A \partial_n^B \ln S_{AB} V_B^k + \tfrac{1}{4} k^2 \sum_A m_A^2 r_A^{-1} \partial_k \partial_n r_A V_A^k \right]$$
$$+ \tfrac{4}{3} \pi k^2 \sum_A m_A^2 \hat{\delta}(\bar{r}_A) r_A^{-1} V_A^n, \tag{4.10}$$

$$\dot{\Phi} \partial_n \Phi = \Delta \Big[- k^2 \sum_A \sum_{\substack{B \\ B \neq A}} m_A m_B \partial_k^A \partial_n^B \ln S_{AB} V_A^k - \tfrac{1}{4} k^2 \sum_A m_A^2 r_A^{-1} \partial_k \partial_n r_A V_A^k \Big]$$
$$- \tfrac{4}{3} \pi k^2 \sum_A m_A^2 \hat{\delta}(\bar{r}_A) r_A^{-1} V_A^n \, . \tag{4.11}$$

Inserting Eqs. (4.10) and (4.11) into (4.9), we get

$$\Delta \Big[\underset{(5)}{h_{0n}} + 4k^2 \sum_A \sum_{\substack{B \\ B \neq A}} m_A m_B \partial_k^A \partial_n^B \ln S_{AB} \left(4V_B^k - 3V_A^k \right)$$
$$+ k^2 \sum_A m_A^2 r_A^{-1} \partial_m \partial_n r_A V_A^m \Big]$$
$$= 16\pi k \sum_A m_A \delta(\bar{r}_A) \Big[2\Phi_n - \overset{A}{\Phi} V_A^n - \tfrac{1}{2} V_A^2 V_A^n \Big] - \tfrac{16}{3} \pi k^2 \sum_A m_A^2 \delta(\bar{r}_A) r_A^{-1} V_A^n \, . \tag{4.12}$$

From (3.10), (3.11) and from the similar formulae for Φ_n we obtain the solution Eq. (4.12) in the form

$$\begin{aligned} \underset{(5)}{h_{0n}} = {} & 2k \sum_A m_A r_A^{-1} V_A^2 V_A^n - k^2 \sum_A m_A^2 r_A^{-1} \partial_k \partial_n r_A V_A^k \\ & - 4k^2 \sum_A \sum_{\substack{B \\ B \neq A}} m_A m_B \partial_k^A \partial_n^B \ln S_{AB} (4V_B^k - 3V_A^k) \\ & + 4k^2 \sum_A \sum_{\substack{B \\ B \neq A}} m_A m_B r_A^{-1} r_{AB}^{-1} (V_A^n - 2V_B^n) \, . \end{aligned} \tag{4.13}$$

More difficult task is to find the function $\underset{(6)}{h_{00}}$. Taking into account the solutions (3.2), (3.12), (4.8), (4.13), the formulae (2.2), (2.3), (2.5), (2.6),(2.10), (2.11) and (2.14), from (2.1) we have

$$
\begin{aligned}
&\Delta \underset{(6)}{h_{00}} + 12\dot{\Phi}\dot{\Phi} + 4\Phi\ddot{\Phi} - 16\partial_k\Phi_s\partial_s\Phi_k + 16\Phi_k\partial_k\dot{\Phi} - 4\partial_k\Phi\partial_k\ddot{\chi} \\
&+16\partial_k\Phi_s\partial_k\Phi_s + 8\Phi\partial_k\Phi\partial_k\Phi + 8\dot{\Phi}_k\partial_k\Phi + 8\partial_k\partial_s\Phi\partial_k\dot{\chi}_s + (\chi)^{\cdot\cdot\cdot\cdot} \\
&-12\Phi^2\Delta\Phi + 2\ddot{\chi}\Delta\Phi + 8k\sum_A m_A r_A^{-1}\partial_k\partial_s\Phi V_A^k V_A^s \\
&+4k\sum_A m_A\Big(2\overset{A}{\overline{\Phi}} - 3V_A^2\Big)\partial_k\Phi\partial_k r_A^{-1} + 4k^3\sum_A m_A^3 r_A^{-5} \\
&-\Big[k\sum_A m_A\Big(2\overset{A}{\overline{\Phi}} - 3V_A^2\Big)r_A^{-1}\Big]^{\cdot\cdot} - 2k\sum_A m_A\Big(2\overset{A}{\overline{\Phi}} - 3V_A^2\Big)\Phi\Delta r_A^{-1} \\
&-4k\sum_A m_A\left(\overset{A}{\overline{\Phi}} + \tfrac{1}{2}V_A^2\right)r_A^{-1}\Delta\Phi - k^3\sum_A\sum_{\substack{B\\B\neq A}} m_A^2 m_B\partial_k\partial_s r_B^{-1}\partial_k\partial_s\ln r_A \\
&-8k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\partial_k\partial_s\Phi\partial_k^A\partial_s^B\ln S_{AB} = 16\pi k\sum_A m_A\hat{\delta}(\bar{r}_A)\Big\{\tfrac{1}{4}(\overset{A}{\overline{\Phi}})^2 + 9\Phi^2 \\
&-3\Phi\overset{A}{\overline{\Phi}} - \tfrac{5}{2}\Phi V_A^2 + \tfrac{7}{4}\overset{A}{\overline{\Phi}}V_A^2 - \tfrac{3}{2}\ddot{\chi} + \tfrac{1}{4}\overset{A}{\overline{\ddot{\chi}}} - 2\overset{A}{\overline{\Phi_k}}V_A^k + 4\Phi_k V_A^k - \partial_k\dot{\chi}_k + \tfrac{7}{16}V_A^4 \\
&+km_A\Big(3\overset{A}{\overline{\Phi}} - \tfrac{5}{2}V_A^2\Big)r_A^{-1} + k\sum_{\substack{B\\B\neq A}} m_B\Big(3\overset{B}{\overline{\Phi}} - \tfrac{5}{2}V_B^2\Big)r_B^{-1} - \tfrac{1}{4}k\sum_{\substack{B\\B\neq A}} m_B\Big(2\overset{B}{\overline{\Phi}} - 3V_B^2\Big)r_{AB}^{-1} \\
&+\tfrac{1}{2}k^2 m_A^2 r_A^{-2} + k^2\sum_{\substack{B\\B\neq A}} m_A m_B\big(r_A^{-1}r_B^{-1} - r_A^{-1}r_{AB}^{-1} - r_B^{-1}r_{AB}^{-1}\big) \\
&+\tfrac{1}{4}k^2\sum_{\substack{B\\B\neq A}} m_B^2 r_B^{-2} + \tfrac{1}{2}k^2\sum_{\substack{B\\B\neq A}}\sum_{\substack{C\\C\neq A\\C\neq B}} m_C m_B\left(r_B^{-1}r_C^{-1} - r_B^{-1}r_{BC}^{-1} - r_C^{-1}r_{BC}^{-1}\right)\Big\}. \qquad (4.14)
\end{aligned}
$$

Using Eqs. (4.2)–(4.4) and (A.9), (A.13) we transform the terms on the left-hand side of Eq. (4.14) successively:
the second term

$$
\begin{aligned}
\dot{\Phi}\dot{\Phi} = \Delta\Big\{&k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\partial_s^A\partial_k^B\ln S_{AB}V_A^s V_B^k \\
&+\tfrac{1}{4}k^2\sum_A m_A^2 r_A^{-1}\partial_k\partial_s r_A V_A^k V_A^s\Big\} + \tfrac{4}{3}\pi k^2\sum_A m_A^2 r_A^{-1}\hat{\delta}(\bar{r}_A)V_A^2\,, \qquad (4.15)
\end{aligned}
$$

the third term

$$\Phi\ddot{\Phi} = \Delta\Big\{k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\left[\partial_s^B\partial_k^B \ln S_{AB}V_B^k V_B^s + \partial_k^B \ln S_{AB}W_B^k\right]$$
$$+k^2\sum_A m_A^2\left[\tfrac{3}{4}r_A^{-1}\partial_k\partial_s r_A V_A^k V_A^s - \tfrac{1}{2}r_A^{-2}V_A^2 - \tfrac{1}{2}\partial_k \ln r_A W_A^k\right]\Big\}$$
$$-\tfrac{4}{3}\pi k^2\sum_A m_A^2 r_A^{-1}\hat{\delta}(\bar{r}_A)V_A^2\,, \tag{4.16}$$

the fourth term

$$\partial_k\Phi_s\partial_s\Phi_k = \Delta\Big\{k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\partial_s^B\partial_k^A \ln S_{AB}V_A^s V_B^k$$
$$+\tfrac{1}{4}k^2\sum_A m_A^2 r_A^{-1}\partial_k\partial_s r_A V_A^k V_A^s\Big\} + \tfrac{4}{3}\pi k^2\sum_A m_A^2 r_A^{-1}\hat{\delta}(\bar{r}_A)V_A^2\,, \tag{4.17}$$

the fifth term

$$\Phi_k\partial_k\dot{\Phi} = \Delta\{-k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\partial_s^B\partial_k^B \ln S_{AB}V_A^k V_B^s$$
$$+k^2\sum_A m_A^2[-\tfrac{3}{4}r_A^{-1}\partial_k\partial_s r_A V_A^k V_A^s + \tfrac{1}{2}r_A^{-2}V_A^2]\} + \tfrac{4}{3}\pi k^2\sum_A m_A^2 r_A^{-1}\hat{\delta}(\bar{r}_A)V_A^2\,, \tag{4.18}$$

and the fourteenth term

$$k\sum_A m_A r_A^{-1}\partial_k\partial_s\Phi V_A^k V_A^s = \Delta\Big\{k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\partial_k^B\partial_s^B \ln S_{AB}V_A^k V_A^s$$
$$+k^2\sum_A m_A^2\left[\tfrac{3}{4}r_A^{-1}\partial_k\partial_s r_A V_A^k V_A^s - \tfrac{1}{2}r_A^{-2}V_A^2)\right]\Big\} - \tfrac{4}{3}\pi k^2\sum_A m_A^2 r_A^{-1}\hat{\delta}(\bar{r}_A)V_A^2\,. \tag{4.19}$$

We transform the terms sixth, seventh, eighth, ninth, tenth and fifteenth on the left-hand side of Eq. (4.14) by means of Eqs. (3.3), (4.6), (3.5) and (A.1)–(A.6). Then, we get

$$\partial_k\Phi\partial_k\ddot{\chi} = \tfrac{1}{2}\Delta(\Phi\ddot{\chi}) - \tfrac{1}{2}\ddot{\chi}\Delta\Phi - \tfrac{1}{2}\Phi\Delta\ddot{\chi} = \tfrac{1}{2}\Delta(\Phi\ddot{\chi}) - \tfrac{1}{2}\ddot{\chi}\Delta\Phi - \Phi\ddot{\Phi}\,,$$
$$\partial_k\Phi_s\partial_k\Phi_s = \tfrac{1}{2}\Delta\Phi_k^2 - \Phi_k\Delta\Phi_k\,,$$
$$\Phi\partial_k\Phi\partial_k\Phi = \tfrac{1}{6}\Delta\Phi^3 - \tfrac{1}{2}\Phi^2\Delta\Phi,$$

$$2\dot{\Phi}_k\partial_k\Phi + 2\partial_k\partial_s\Phi\partial_k\dot{\chi}_s = \Delta[\partial_k\Phi\dot{\chi}_k] - \dot{\chi}_k\partial_k\Delta\Phi\,,$$

$$k\sum_A m_A(2\overset{A}{\Phi} - 3V_A^2)\partial_k\Phi\partial_k r_A^{-1} = \tfrac{1}{2}\Delta\Big\{k\sum_A m_A(2\overset{A}{\Phi} - 3V_A^2)\Phi r_A^{-1}\Big\}$$

$$-\tfrac{1}{2}k\sum_A m_A\left(2\overset{A}{\Phi} - 3V_A^2\right)r_A^{-1}\Delta\Phi - \tfrac{1}{2}k\sum_A m_A\left(2\overset{A}{\Phi} - 3V_A^2\right)\Phi\Delta r_A^{-1}\,. \quad (4.20)$$

Above all from (3.5) for the seventeenth term on the left-hand side of Eq. (4.14) we have

$$\left[k\sum_A m_A\left(2\overset{A}{\Phi} - 3V_A^2\right)r_A^{-1}\right]^{\cdot\cdot} = \tfrac{1}{2}\Delta\left[k\sum_A m_A\left(2\overset{A}{\Phi} - 3V_A^2\right)r_A\right]^{\cdot\cdot}. \quad (4.21)$$

The terms eleventh and sixteenth can be written down in the form

$$\begin{aligned} (\chi)^{\cdot\cdot\cdot\cdot} &= \tfrac{1}{12}\Delta k\sum_A m_A(r_A^3)^{\cdot\cdot\cdot\cdot}\,, \\ k^3\sum_A m_A^3 r_A^{-5} &= \tfrac{1}{6}\Delta k^3\sum_A m_A^3 r_A^{-3} + 2\pi k^3\sum_A m_A^3 r_A^{-2}\hat{\delta}(\bar{r}_A)\,, \end{aligned} \quad (4.22)$$

where in the expressions above we used additionally the formulae:

$$\Delta r_A^3 = 12 r_A\,, \quad (4.23)$$

$$\Delta r_A^{-3} = 6 r_A^{-5} - 12\pi r_A^{-2}\delta(\bar{r}_A)\,, \quad (4.24)$$

where the last expression follows from (A.13) for $L = 3$.

In order to transform the expression

$$k^3\sum_A\sum_{\substack{B\\B\neq A}} m_A^2 m_B\partial_k\partial_s r_B^{-1}\partial_k\partial_s\ln r_A,$$

we take into account the formula (A.14) and the following property of δ-function

$$\delta(\bar{r}_A)\ln r_B = \delta(\bar{r}_A)\ln r_{AB}, \qquad A\neq B,$$

as well as the relation

$$N_A^k N_B^k = \tfrac{1}{2}\left(r_A^{-1}r_B + r_B^{-1}r_A - r_{AB}^2 r_A^{-1}r_B^{-1}\right)\,.$$

Then, we get

$$k^3 \sum_A \sum_{\substack{B \\ B\neq A}} m_A^2 m_B \partial_k \partial_s r_B^{-1} \partial_k \partial_s \ln r_A$$
$$= \tfrac{1}{2} \Delta \Big\{ k^3 \sum_A \sum_{\substack{B \\ B\neq A}} m_A^2 m_B \Big[\partial_k^A \partial_k^B (r_B^{-1} \ln r_A - r_B^{-1} \ln r_{AB})$$
$$- \tfrac{1}{2} r_B^{-1} r_A^{-2} + \tfrac{1}{2} r_{AB}^{-2} r_B^{-1} + \tfrac{1}{2} r_B r_A^{-2} r_{AB}^{-2} \Big] \Big\}$$
$$+ 2\pi k^3 \sum_A \sum_{\substack{B \\ B\neq A}} m_A^2 m_B \hat{\delta}(\bar{r}_A) r_A^{-1} \left(r_B r_{AB}^{-2} - r_B^{-1} \right) . \qquad (4.25)$$

The rest of terms in (4.14) except the twenty first, include the $\hat{\delta}$-function. Inserting (4.15)–(4.22) into (4.14), we get the final equation for $\underset{(6)}{h_{00}}$ in the form

$$\Delta \Big\{ \underset{(6)}{h_{00}} + 8\Phi_b^2 + \tfrac{4}{3}\Phi^3 - 2\Phi \ddot{\chi} + 4\partial_k \Phi \dot{\chi}_k + \tfrac{1}{12} k \sum_A m_A (r_A^3)^{\cdot\cdot\cdot\cdot}$$
$$+ \tfrac{2}{3} k^3 \sum_A m_A^3 r_A^{-3} - 4k^2 \sum_A m_A^2 \partial_k \ln r_A W_A^k - k^2 \sum_A m_A^2 r_A^{-1} \partial_k \partial_s r_A V_A^k V_A^s$$
$$+ 2k \sum_A m_A \Big(2 \overset{A}{\overline{\Phi}} - 3V_A^2 \Big) \Phi r_A^{-1} - \tfrac{1}{2} \Big[k \sum_A m_A \Big(2 \overset{A}{\overline{\Phi}} - 3V_A^2 \Big) r_A \Big]^{\cdot\cdot}$$
$$- \tfrac{1}{2} k^3 \sum_A \sum_{\substack{B \\ B\neq A}} m_A^2 m_B \Big[\partial_k^A \partial_k^B (r_B^{-1} \ln r_A - r_B^{-1} \ln r_{AB}) - \tfrac{1}{2} r_B^{-1} r_A^{-2} + \tfrac{1}{2} r_{AB}^{-2} r_B^{-1}$$
$$+ \tfrac{1}{2} r_B r_A^{-2} r_{AB}^{-2} \Big] - k^2 \sum_A \sum_{\substack{B \\ B\neq A}} m_A m_B \Big[\Big(16 V_A^s V_B^k - 12 V_A^k V_B^s \Big) \partial_k^A \partial_s^B \ln S_{AB}$$
$$+ (16 V_B^s V_A^k - 8 V_B^s V_B^k - 8 V_A^s V_A^k) \partial_k^B \partial_s^B \ln S_{AB} - 8 \partial_k^B \ln S_{AB} W_B^k \Big] \Big\}$$
$$= 16\pi k \sum_A m_A \hat{\delta}(\bar{r}_A) \Big\{ \tfrac{1}{4} (\overset{A}{\overline{\Phi}})^2 + 5\Phi^2 - 5\Phi \overset{A}{\overline{\Phi}} + \tfrac{1}{2} \Phi V_A^2 + \tfrac{7}{4} \overset{A}{\overline{\Phi}} V_A^2 - \tfrac{1}{2} \ddot{\chi}$$
$$+ \tfrac{1}{4} \overset{A}{\overline{\ddot{\chi}}} - 2 \overset{A}{\overline{\Phi_n}} V_A^n - \partial_k \dot{\chi}_k + \tfrac{7}{16} V_A^4 + k m_A \Big(\overset{A}{\overline{\Phi}} - \tfrac{3}{2} V_A^2 \Big) r_A^{-1} + \tfrac{1}{3} k m_A r_A^{-1} V_A^2$$
$$- \tfrac{1}{4} k \sum_{\substack{B \\ B\neq A}} m_B \Big(2 \overset{B}{\overline{\Phi}} - 3V_B^2 \Big) r_{AB}^{-1} + k \sum_{\substack{B \\ B\neq A}} m_B \Big(\overset{B}{\overline{\Phi}} - \tfrac{3}{2} V_B^2 \Big) r_B^{-1} + \tfrac{1}{4} k^2 \sum_{\substack{B \\ B\neq A}} m_B^2 r_B^{-2}$$
$$+ \tfrac{1}{8} k^2 \sum_{\substack{B \\ B\neq A}} m_A m_B \Big[r_A^{-1} r_B r_{AB}^{-2} + 7 r_A^{-1} r_B^{-1} - 8 r_A^{-1} r_{AB}^{-1} - 8 r_B^{-1} r_{AB}^{-1} \Big]$$

$$+\tfrac{1}{2}k^2\sum_{\substack{B\\B\neq A}}\sum_{\substack{C\\C\neq A\\C\neq B}} m_C m_B\Big(r_B^{-1}r_C^{-1}-2r_B^{-1}r_{BC}^{-1}\Big)\Big\}-16\pi\sum_A m_A\partial_k\hat{\delta}(\bar{r}_A)\dot{\chi}_k$$

$$+8k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\partial_k\partial_s\Phi\partial_k^A\partial_s^B\ln S_{AB}\,. \qquad (4.26)$$

In order to solve Eq. (4.26), we have to make regularization of the following integrals, which do not vanish only for $\bar{x}=\bar{\xi}_A$

$$\begin{aligned}
&\int\hat{\delta}(\bar{r}'_A)r_A^{\prime -1}N_A^{\prime k}N_A^{\prime s}|\bar{x}-\bar{x}'|^{-1}(d\bar{x}')\,,\\
&\int\hat{\delta}(\bar{r}'_A)r_A^{\prime -1}|\bar{x}-\bar{x}'|^{-1}(d\bar{x}')\,,\\
&\int\hat{\delta}(\bar{r}'_A)r_A^{\prime -1}r_B^{\prime -1}|\bar{x}-\bar{x}'|^{-1}(d\bar{x}')\,,\\
&\int\hat{\delta}(\bar{r}'_A)r_A^{\prime -1}r'_B|\bar{x}-\bar{x}'|^{-1}(d\bar{x}')\,,\\
&\int\hat{\delta}(\bar{r}'_A)r_A^{\prime -2}|\bar{x}-\bar{x}'|^{-1}(d\bar{x}')\,. \qquad (4.27)
\end{aligned}$$

From (3.9) and the following expansion of r'_B and $r_B^{\prime -1}$ into powers series with respect to r'_A:

$$\begin{aligned}
r_B^{\prime -1} &= r_{AB}^{-1}-r_{AB}^{-2}N_{AB}^k N_A^{\prime k}r'_A+O(r_A^{\prime 2})\,,\\
r'_B &= r_{AB}+N_{AB}^k N_A^{\prime k}r'_A+O(r_A^{\prime 2})\,,
\end{aligned}$$

one can easily see that all integrals in (4.27) vanish due to the property (D) of $\hat{\delta}$. Then, the solution of Eq. (4.26) takes the form:

$$\begin{aligned}
\underset{(6)}{h_{00}} &= 2\Phi\ddot{\chi}-4\partial_k\Phi\dot{\chi}_k-8\Phi_k^2-\tfrac{4}{3}\Phi^3-\tfrac{1}{12}k\sum_A m_A(r_A^3)^{\cdot\cdot\cdot\cdot}-\tfrac{2}{3}k^3\sum_A m_A^3 r_A^{-3}\\
&-\frac{7}{4}k\sum_A m_A V_A^4 r_A^{-1}-2k\sum_A m_A\Big(2\overset{A}{\Phi}-3V_A^2\Big)\Phi r_A^{-1}+4k^2\sum_A m_A^2\partial_k\ln r_A W_A^k\\
&+4k\sum_A m_A\overset{A}{\dot{\chi}_k}\partial_k r_A^{-1}+k^2\sum_A m_A^2 r_A^{-1}\partial_k\partial_s r_A V_A^k V_A^s+8k\sum_A m_A\overset{A}{\Phi_k}V_A^k r_A^{-1}\\
&-k\sum_A m_A\overset{A}{\Phi}\Big(\overset{A}{\Phi}+9V_A^2\Big)r_A^{-1}+\tfrac{1}{2}\Big[k\sum_A m_A(2\overset{A}{\Phi}-3V_A^2)r_A\Big]^{\cdot\cdot}+k\sum_A m_A\overset{A}{\ddot{\chi}}r_A^{-1}\\
&-2k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B\Big(\overset{B}{\Phi}-\tfrac{3}{2}V_B^2\Big)r_{AB}^{-1}r_A^{-1}+4k^3\sum_A\sum_{\substack{B\\B\neq A}} m_A^2 m_B r_{AB}^{-2}r_A^{-1}
\end{aligned}$$

$$+k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B[(16V_A^s V_B^k - 12V_A^k V_B^s)\partial_k^A\partial_s^B \ln S_{AB}$$
$$+(16V_B^s V_A^k - 8V_B^s V_B^k - 8V_A^s V_A^k)\partial_k^B\partial_s^B \ln S_{AB} - 8\partial_k^B \ln S_{AB} W_B^k]$$
$$+\tfrac{1}{2}k^3\sum_A\sum_{\substack{B\\B\neq A}} m_A^2 m_B\Big[\partial_b^A\partial_b^B(r_B^{-1}\ln r_A - r_B^{-1}\ln r_{AB}) - \tfrac{1}{2}r_B^{-1}r_A^{-2} - \tfrac{3}{2}r_{AB}^{-2}r_B^{-1}$$
$$+\tfrac{1}{2}r_B r_A^{-2} r_{AB}^{-2}\Big] + 2k^3\sum_A\sum_{\substack{B\\B\neq A}}\sum_{\substack{C\\C\neq A\\C\neq B}} m_A m_B m_C\Big(2r_{AB}^{-1}r_{BC}^{-1} - r_{AB}^{-1}r_{AC}^{-1}\Big)r_A^{-1} + H, \tag{4.28}$$

where H is the solution of the equation:

$$\Delta H = 8k^2\sum_A\sum_{\substack{B\\B\neq A}} m_A m_B \partial_k\partial_s\Phi\partial_k^A\partial_s^B \ln S_{AB}\,. \tag{4.29}$$

The equation above one can solve for the two bodies. In this case, the function H has the form [9]:

$$H = k^2 m_1^2 m_2\{4\Delta_1\partial_b^1\partial_b^2[(r_1 + r_{12})\ln S_{12}] + 8\partial_k^1 \ln S_{12}\partial_k^1 r_{12}^{-1} - 4r_2 r_1^{-2} r_{12}^{-2}$$
$$+4r_{12}^{-2}r_1^{-1} - 2r_{12}^{-1}r_1^{-2} + 4r_2^2 r_1^{-3} r_{12}^{-2} + 6r_2^2 r_1^{-2} r_{12}^{-3} - 4r_1^{-3} - 6r_{12}^{-3}\} + 1 \leftrightarrow 2\,. \tag{4.30}$$

We obtain this expression taking, into account the property $\hat{\delta}(\bar{r}_2)r_1 = \hat{\delta}(\bar{r}_2)r_{12}$, using (4.29), (4.2)–(4.3), (A.14) and the following formulae:

$$\begin{aligned}
\Delta_1 \ln S_{12} &= r_1^{-1}r_{12}^{-1},\\
S_{12}^{-1}(1 + N_1^k N_{12}^k) &= \tfrac{1}{2}(r_2^{-1} + r_1^{-1} - r_{12}r_1^{-1}r_2^{-1}),\\
\partial_k^1\partial_k^2 \ln S_{12} &= \tfrac{1}{2}(r_1^{-1}r_2^{-1} - r_1^{-1}r_{12}^{-1} - r_2^{-1}r_{12}^{-1})\,.
\end{aligned}$$

Finally, our solutions for two bodies have the form:

$$\begin{aligned}
\underset{(2)}{h_{00}} &= -2km_1 r_1^{-1} - 2km_2 r_2^{-1}\,,\\
\underset{(2)}{h_{mn}} &= -2km_1 r_1^{-1}\delta_{mn} - 2km_2 r_2^{-1}\delta_{mn}\,,\\
\underset{(3)}{h_{0n}} &= 4km_1 r_1^{-1}V_1^n + 4km_2 r_2^{-1}V_2^n\,,
\end{aligned} \tag{4.31}$$

$$\underset{(4)}{h_{00}} = km_1 r_1^{-1}[(N_1V_1)^2 - 4V_1^2] + 2k^2 m_1^2 r_1^{-2} + k^2 m_1 m_2[2r_1^{-1}r_2^{-1} + \tfrac{1}{2}r_1 r_{12}^{-3}$$
$$+\tfrac{5}{2}r_2^{-1}r_{12}^{-1} - \tfrac{1}{2}r_1^2 r_2^{-1} r_{12}^{-3}] + 1 \leftrightarrow 2\,, \tag{4.32}$$

$$\underset{(4)}{h_{mn}} = km_1 r_1^{-1}[(N_1V_1)^2\delta_{mn} - 2(N_1^mV_1^n + N_1^nV_1^m)(N_1V_1)]$$
$$-k^2m_1^2r_1^{-2}\left[\delta_{mn} + N_1^mN_1^n\right] + k^2m_1m_2[(\tfrac{1}{2}r_1r_{12}^{-3} + \tfrac{5}{2}r_1^{-1}r_{12}^{-1} - 2r_1^{-1}r_2^{-1}$$
$$-\tfrac{1}{2}r_1^2r_2^{-1}r_{12}^{-3} - 4r_{12}^{-1}S_{12}^{-1})\delta_{mn} + 2r_{12}^{-2}(N_1^mN_{12}^n + N_1^nN_{12}^m)$$
$$+4N_{12}^mN_{12}^n(S_{12}^{-2} + r_{12}^{-1}S_{12}^{-1}) - 4S_{12}^{-2}(N_1^mN_2^n + N_1^nN_2^m + 2N_1^mN_{12}^n$$
$$+2N_1^nN_{12}^m)] + 1 \leftrightarrow 2\,, \tag{4.33}$$

$$\underset{(5)}{h_{0n}} = 2km_1r_1^{-1}V_1^nV_1^2 + k^2m_1^2r_1^{-2}[(N_1V_1)N_1^n - V_1^n]$$
$$+k^2m_1m_2\{N_1^nS_{12}^{-2}[16(N_{12}V_1) + 16(N_2V_1) - 12(N_{12}V_2) - 12(N_2V_2)]$$
$$+N_{12}^n[16S_{12}^{-2}(N_1V_2) - 12S_{12}^{-2}(N_1V_1) - 4S_{12}^{-2}(N_{12}V_1) - 4S_{12}^{-1}r_{12}^{-1}(N_{12}V_1)]$$
$$+V_1^n[4r_1^{-1}r_{12}^{-1} - 8r_2^{-1}r_{12}^{-1} + 4r_{12}^{-1}S_{12}^{-1}]\} + 1 \leftrightarrow 2\,, \tag{4.34}$$

$$\underset{(6)}{h_{00}} = km_1r^{-1}[3(N_1V_1)^2V_1^2 - 4V_1^4 - \tfrac{3}{4}(N_1V_1)^4] + k^2m_1^2r_1^{-2}[V_1^2 - 7(N_1V_1)^2]$$
$$+k^2m_1m_2\{V_1^2[\tfrac{3}{8}r_1^2r_2r_{12}^{-5} - \tfrac{3}{8}r_1^3r_{12}^{-5} + \tfrac{3}{8}r_1r_2^2r_{12}^{-5} - \tfrac{3}{8}r_2^3r_{12}^{-5} + \tfrac{37}{8}r_1r_{12}^{-3} - r_1^2r_2^{-1}r_{12}^{-3}$$
$$-\tfrac{3}{8}r_2r_{12}^{-3} - 2r_2^2r_1^{-1}r_{12}^{-3} - 6r_1^{-1}r_{12}^{-1} + 5r_2^{-1}r_{12}^{-1} + 8r_{12}r_1^{-1}r_2^{-1}S_{12}^{-1} - 16r_{12}^{-1}S_{12}^{-1}]$$
$$+(V_1V_2)[\tfrac{3}{4}r_1^3r_{12}^{-5} - 8r_1^{-1}r_2^{-1} - \tfrac{3}{4}r_1^2r_2r_{12}^{-5} - \tfrac{13}{4}r_1r_{12}^{-3} + 2r_1^2r_2^{-1}r_{12}^{-3} + 6r_1^{-1}r_{12}^{-1}$$
$$+16r_1^{-1}S_{12}^{-1} + 12r_{12}^{-1}S_{12}^{-1}] + (N_{12}V_1)^2[\tfrac{15}{8}r_1^3r_{12}^{-5} - \tfrac{15}{8}r_1^2r_2r_{12}^{-5} - \tfrac{15}{8}r_1r_2^2r_{12}^{-5}$$
$$+\tfrac{15}{8}r_2^3r_{12}^{-5} - \tfrac{57}{8}r_1r_{12}^{-3} + \tfrac{3}{4}r_1^2r_2^{-1}r_{12}^{-3} + \tfrac{33}{8}r_2r_{12}^{-3} - \tfrac{7}{4}r_2^{-1}r_{12}^{-1} + 16S_{12}^{-2} + 16r_{12}^{-1}S_{12}^{-1}]$$
$$+(N_{12}V_1)(N_{12}V_2)[\tfrac{15}{4}r_1^2r_2r_{12}^{-5} - \tfrac{15}{4}r_1^3r_{12}^{-5} + \tfrac{9}{4}r_1r_{12}^{-3} - 12S_{12}^{-2} - 12r_{12}S_{12}^{-1}]$$
$$+(N_1V_1)^2[\tfrac{3}{4}r_2^2r_1^{-1}r_{12}^{-3} - 2r_2^{-1}r_1^{-1} + \tfrac{1}{4}r_1{r_{12}}^{-3} - \tfrac{7}{4}r_1^{-1}r_{12}^{-1} + 8S_{12}^{-2} + 8r_1^{-1}S_{12}^{-1}]$$
$$-4(N_1V_2)(N_2V_2)r_1^{-2} - (N_1V_1)(N_1V_2)[r_1r_{12}^{-3} + 16S_{12}^{-2} + 16r_1^{-1}S_{12}^{-1}]$$
$$+(N_1V_2)^2[8S_{12}^{-2} + 8r_1^{-1}S_{12}^{-1}] + (N_{12}V_1)(N_1V_1)[3r_1^2r_{12}^{-4} - \tfrac{3}{2}r_2^2r_{12}^{-4} - \tfrac{3}{2}r_{12}^{-2} - 16S_{12}^{-2}]$$
$$+(N_{12}V_2)(N_1V_1)[-3r_1^2r_{12}^{-4} + \tfrac{3}{2}r_2^2r_{12}^{-4} - \tfrac{13}{2}r_{12}^{-2} + 40S_{12}^{-2}] + (N_{12}V_1)(N_1V_2)[-\tfrac{3}{2}r_1^2r_{12}^{-4}$$
$$-4r_{12}^{-2} - 16S_{12}^{-2}] + (N_{12}V_2)(N_1V_2)[\tfrac{3}{2}r_1^2r_{12}^{-4} + 3r_{12}^{-2} + 4r_1^{-2} - 16S_{12}^{-2}]$$
$$-16S_{12}^{-2}(N_1V_2)(N_2V_1) + 12S_{12}^{-2}(N_1V_1)(N_2V_2) - 2k^3m_1^3r_1^{-3}$$
$$+k^3m_1^2m_2[\tfrac{1}{4}r_1^3r_{12}^{-6} - 2r_1^{-3} - \tfrac{1}{2}r_2^{-3} - \tfrac{9}{2}r_1^{-2}r_2^{-1} - \tfrac{3}{16}r_1^4r_2^{-1}r_{12}^{-6} + \tfrac{1}{8}r_1^2r_2r_{12}^{-6} - \tfrac{1}{4}r_2^2r_1r_{12}^{-6}$$
$$+\tfrac{1}{16}r_2^3r_{12}^{-6} - \tfrac{5}{4}r_1r_{12}^{-4} + \tfrac{23}{8}r_1^2r_2^{-1}r_{12}^{-4} - \tfrac{43}{8}r_2r_{12}^{-4} + \tfrac{5}{2}r_2^2r_1^{-1}r_{12}^{-4} + 5r_{12}^{-3} - 3r_1r_2^{-1}r_{12}^{-3}$$
$$-3r_2r_1^{-1}r_{12}^{-3} + 3r_2^2r_1^{-2}r_{12}^{-3} - 2r_2^3r_1^{-3}r_{12}^{-3} + \tfrac{1}{2}r_1^{-1}r_{12}^{-2} + \tfrac{1}{4}r_1^2r_2^{-3}r_{12}^{-2} - \tfrac{3}{16}r_2^{-1}r_{12}^{-2}$$
$$-\tfrac{15}{4}r_2r_1^{-2}r_{12}^{-2} + 2r_2^2r_1^{-3}r_{12}^{-2} - 3r_1^{-2}r_{12}^{-1} - 5r_1^{-1}r_2^{-1}r_{12}^{-1} + 2r_2r_1^{-3}r_{12}^{-1} + \tfrac{1}{4}r_1^{-2}r_2^{-3}r_{12}^2]\}$$
$$+1 \leftrightarrow 2. \tag{4.35}$$

In the these formulae the following notations were used: $(N_AV_A) \equiv N_A^kV_A^k$, $(N_AV_B) \equiv N_A^kV_B^k$, $(N_{AB}V_B) \equiv N_{AB}^kV_B^k$, $(V_AV_B) \equiv V_A^kV_B^k$. The symbol $1 \leftrightarrow 2$ means the same term but with the indices 1 and 2 exchanged.

5. The 2PN equations of motion

The equations of motion in the 2PN approximation have the form

$$\frac{d^2\xi_1^n}{dt^2} = \underset{(0)}{F_1^n} + c^{-2}\underset{(2)}{F_1^n} + c^{-4}\underset{(4)}{F_1^n} \,, \tag{5.1}$$

where

$$\underset{(4)}{F_1^n} = \underset{(4)}{F_1^{\prime n}} + \underset{(4)}{F_1^{\prime\prime n}} + \underset{(4)}{F_1^{\prime\prime\prime n}} \,,$$

where $\underset{(4)}{F_1^{\prime\prime n}}$ and $\underset{(4)}{F_1^{\prime\prime\prime n}}$ contain the expressions up to c^{-4}-order arising in $\overset{1}{\overline{\partial_n \underset{(4)}{h_{00}}}}$ (the term $\overset{1}{\overline{\partial_n \ddot{\chi}}}$) and $\underset{(2)}{F_1^n}$ (the term $\overset{1}{\overline{\underset{(3)}{\dot{h}_{0n}}}}$), respectively. Then, we get

$$\underset{(4)}{F_1^{\prime n}} = -\tfrac{1}{2}km_2 \overset{1}{\overline{\partial_n\partial_k r_2 \underset{(2)}{W_2^k}}} \,, \tag{5.2}$$

$$\underset{(4)}{F_1^{\prime\prime n}} = 4km_2 \overset{1}{\overline{r_2^{-1} \underset{(2)}{W_2^n}}} \,. \tag{5.3}$$

Taking into account Eqs. (2.7)–(2.9) and (2.11), we have the following form for $\underset{(4)}{F_1^{\prime\prime\prime n}}$

$$\begin{aligned}
\underset{(4)}{F_1^{\prime\prime\prime n}} = & -\tfrac{1}{2}\overset{1}{\overline{\partial_n \underset{(6)}{h_{00}}}} - \tfrac{1}{2}\overset{1}{\overline{\underset{(2)}{h_{nk}}\,\partial_k \underset{(4)}{h_{00}}}} - \tfrac{1}{2}\overset{1}{\overline{\underset{(4)}{h_{nk}}\,\partial_k \underset{(2)}{h_{00}}}} - \tfrac{1}{2}\overset{1}{\overline{\underset{(2)}{h_{ns}}\,\underset{(2)}{h_{sk}}\,\partial_k \underset{(2)}{h_{00}}}} \\
& + \overset{1}{\overline{\underset{(2)}{h_{nk}}\,\underset{(3)}{\dot{h}_{0k}}}} - \tfrac{1}{2}\overset{1}{\overline{\underset{(3)}{h_{0n}}\,\underset{(2)}{\dot{h}_{00}}}} + \overset{1}{\overline{\underset{(5)}{\dot{h}_{0n}}}} + \tfrac{1}{2}\overset{1}{\overline{\underset{(4)}{\dot{h}_{00}}}}\,V_1^n - \tfrac{1}{2}\overset{1}{\overline{\underset{(2)}{h_{00}}\,\underset{(2)}{\dot{h}_{00}}}}\,V_1^n \\
& - \tfrac{1}{2}\overset{1}{\overline{\underset{(3)}{h_{0k}}\,\partial_k \underset{(2)}{h_{00}}}}\,V_1^n - \overset{1}{\overline{\underset{(3)}{h_{0n}}\,\partial_k \underset{(2)}{h_{00}}}}\,V_1^k + \overset{1}{\overline{\underset{(2)}{h_{nk}}\,\partial_s \underset{(3)}{h_{0k}}}}\,V_1^s - \overset{1}{\overline{\underset{(2)}{h_{nk}}\,\partial_k \underset{(3)}{h_{0s}}}}\,V_1^s \\
& + \overset{1}{\overline{\underset{(4)}{\dot{h}_{nk}}}}\,V_1^k - \overset{1}{\overline{\partial_n \underset{(5)}{h_{0k}}}}\,V_1^k + \overset{1}{\overline{\partial_k \underset{(5)}{h_{0n}}}}\,V_1^k + \overset{1}{\overline{\partial_k \underset{(4)}{h_{00}}}}\,V_1^k V_1^n - \overset{1}{\overline{\underset{(2)}{h_{00}}\,\partial_k \underset{(2)}{h_{00}}}}\,V_1^k V_1^n \\
& - \tfrac{1}{2}\overset{1}{\overline{\partial_n \underset{(4)}{h_{ks}}}}\,V_1^k V_1^s + \overset{1}{\overline{\partial_s \underset{(4)}{h_{nk}}}}\,V_1^k V_1^s - \tfrac{1}{2}\overset{1}{\overline{\underset{(2)}{h_{ns}}\,\partial_s \underset{(2)}{h_{km}}}}\,V_1^k V_1^m \\
& + \overset{1}{\overline{\underset{(2)}{h_{ns}}\,\partial_m \underset{(2)}{h_{ks}}}}\,V_1^k V_1^m + \overset{1}{\overline{\partial_s \underset{(3)}{h_{0k}}}}\,V_1^n V_1^k V_1^s - \tfrac{1}{2}\overset{1}{\overline{\underset{(2)}{\dot{h}_{ks}}}}\,V_1^n V_1^k V_1^s .
\end{aligned} \tag{5.4}$$

Taking into account Eqs. (3.19), (5.2), (5.3) and applying regularization to the divergent integrals (B.6), we get

$$\underset{(4)}{F_1^{\prime n}} = -\tfrac{1}{2}k^2 m_1 m_2 r_{12}^{-3}[3N_{12}^n(N_{12}V_1)^2 + 4N_{12}^n(N_{12}V_2)^2 - 7N_{12}^n(N_{12}V_1)(N_{12}V_2)$$
$$+4V_1^n(N_{12}V_2) - 4V_2^n(N_{12}V_2) - 3V_1^n(N_{12}V_1) + 3V_2^n(N_{12}V_1)]\,, \quad (5.5)$$

$$\underset{(4)}{F_1^{\prime\prime n}} = -16k^3 m_1^2 m_2 r_{12}^{-4} N_{12}^n - 20k^3 m_2^2 m_1 r_{12}^{-4} N_{12}^n - k^2 m_1 m_2 r_{12}^{-3}[-8N_{12}^n V_1^2$$
$$-4N_{12}^n V_2^2 + 16N_{12}^n(V_1V_2) + 6N_{12}^n(N_{12}V_1)^2 + 16V_2^n(N_{12}V_2)$$
$$-16V_1^n(N_{12}V_2) - 12V_2^n(N_{12}V_1) + 12V_1^n(N_{12}V_1)]\,. \quad (5.6)$$

In order to obtain $\underset{(4)}{F_1^{\prime\prime\prime n}}$ we use Eqs. (4.31)-(4.35) and (5.4). After the regularization (B.6), we have

$$\overline{\partial_n \overset{1}{\underset{(6)}{h_{00}}}} = k^3 m_2 r_{12}^{-4} N_{12}^n(6m_2^2 - \tfrac{7}{2}m_1^2 + 11m_2m_1)$$
$$+k^2 m_2^2 r_{12}^{-3}\Big[-2N_{12}^n V_2^2 + 28N_{12}^n(N_{12}V_2)^2 - 14V_2^n(N_{12}V_2)\Big]$$
$$+k^2 m_1 m_2 r_{12}^{-3}\Big[\tfrac{3}{2}N_{12}^n V_1^2 - 5N_{12}^n(V_1V_2) - 41N_{12}^n(N_{12}V_2)^2 - 14N_{12}^n(N_{12}V_1)^2$$
$$+\tfrac{35}{2}N_{12}^n V_2^2 \tfrac{39}{2}V_2^n(N_{12}V_2)^2 - \tfrac{9}{2}V_2^n(N_{12}V_1) - \tfrac{47}{2}V_1^n(N_{12}V_2)$$
$$+57N_{12}^n(N_{12}V_2)(N_{12}V_1) + \tfrac{29}{2}V_1^n(N_{12}V_1)\Big] + k m_2 r_{12}^{-2}\Big[-9N_{12}^n(N_{12}V_2)^2V_2^2$$
$$+6V_2^n(N_{12}V_2)V_2^2 + 4N_{12}^n V_2^4 + \tfrac{15}{4}N_{12}^n(N_{12}V_2)^4 - 3V_2^n(N_{12}V_2)^3\Big]\,, \quad (5.7)$$

$$\overline{\underset{(2)}{h_{nk}}\;\partial_k \overset{1}{\underset{(4)}{h_{00}}}} = 4k^3 m_2^2 r_{12}^{-4} N_{12}^n(2m_2 + m_1) + k^2 m_2^2 r_{12}^{-3}\Big[6N_{12}^n(N_{12}V_2)^2$$
$$-4V_2^n(N_{12}V_2) - 8N_{12}^n V_2^2\Big]\,, \quad (5.8)$$

$$\overline{\underset{(4)}{h_{nk}}\;\partial_k \overset{1}{\underset{(2)}{h_{00}}}} = 2k^3 m_2^2 r_{12}^{-4} N_{12}^n(3m_1 - 2m_2) - 2k^2 m_2^2 r_{12}^{-3}\Big[N_{12}^n(N_{12}V_2)^2$$
$$+2V_2^n(N_{12}V_2)\Big]\,, \quad (5.9)$$

$$\overline{\underset{(2)}{h}_{nm}\quad\underset{(2)}{h}_{mk}\;\partial_k\;\underset{(2)}{h}_{00}}^{\,1} = 8k^3m_2^3r_{12}^{-4}N_{12}^n\,, \tag{5.10}$$

$$\overline{\underset{(2)}{h}_{nk}\quad\underset{(3)}{\dot{h}}_{0k}}^{\,1} = -8k^3m_2^2m_1r_{12}^{-4}N_{12}^n + 8k^2m_2^2r_{12}^{-3}V_2^n(N_{12}V_2)\,, \tag{5.11}$$

$$\overline{\underset{(3)}{h}_{0n}\quad\underset{(2)}{\dot{h}}_{00}}^{\,1} = -8k^2m_2^2r_{12}^{-3}V_2^n(N_{12}V_2)\,, \tag{5.12}$$

$$\begin{aligned}\overline{\underset{(5)}{\dot{h}}_{0n}}^{\,1} &= 2k^2m_2r_{12}^{-4}N_{12}^n(m_1^2+5m_1m_2) + k^2m_2^2r_{12}^{-3}[-N_{12}^nV_2^2 + 4N_{12}^n(N_{12}V_2)^2 \\ &-3V_2^n(N_{12}V_2)] + k^2m_1m_2r_{12}^{-3}[5N_{12}^n(V_1V_2) + 12N_{12}^n(N_{12}V_1)^2 - 16N_{12}^n(N_{12}V_2)^2 \\ &-6N_{12}^nV_1^2 + 6N_{12}^nV_2^2 - 4N_{12}^n(N_{12}V_2)(N_{12}V_1) + 20V_2^n(N_{12}V_2) - 18V_1^n(N_{12}V_2) \\ &+5V_2^n(N_{12}V_1) + 6V_1^n(N_{12}V_1)] + 2km_2r_{12}^{-2}V_2^n(N_{12}V_2)V_A^2\,,\end{aligned} \tag{5.13}$$

$$\begin{aligned}\overline{\underset{(4)}{\dot{h}}_{00}}^{\,1}\; V_1^n = \quad & -4k^2m_1m_2r_{12}^{-3}V_1^n(N_{12}V_1) + 4k^2m_2^2r_{12}^{-3}V_1^n(N_{12}V_2) \\ & +km_2r_{12}^{-2}[-6V_1^nV_2^2(N_{12}V_2) + 3V_1^n(N_{12}V_2)^3]\,,\end{aligned} \tag{5.14}$$

$$\overline{\underset{(2)}{h}_{00}\quad\underset{(2)}{\dot{h}}_{00}}^{\,1}\; V_1^n = 4k^2m_2^2r_{12}^{-3}V_1^n(N_{12}V_2), \tag{5.15}$$

$$\overline{\underset{(3)}{h}_{0k}\;\;\partial_k\;\;\underset{(2)}{h}_{00}}^{\,1}\; V_1^n = 8k^2m_2^2r_{12}^{-3}V_1^n(N_{12}V_2), \tag{5.16}$$

$$\overline{\underset{(3)}{h}_{0n}\;\;\partial_k\;\;\underset{(2)}{h}_{00}}^{\,1}\; V_1^k = 8k^2m_2^2r_{12}^{-3}V_2^n(N_{12}V_1), \tag{5.17}$$

$$\overline{\underset{(2)}{h}_{nk}\;\;\partial_s\;\;\underset{(3)}{h}_{0k}}^{\,1}\; V_1^s = 8k^2m_2^2r_{12}^{-3}V_2^n(N_{12}V_1), \tag{5.18}$$

$$\overline{\underset{(2)}{h_{nk}}\;\partial_k\;\underset{(3)}{h_{0s}}}^{\,1}\; V_1^s = 8k^2 m_2^2 r_{12}^{-3} N_{12}^n (V_1V_2), \tag{5.19}$$

$$\overline{\underset{(2)}{h_{nk}}\;\;\underset{(2)}{\dot{h}_{kb}}}^{\,1}\; V_1^b = 4k^2 m_2^2 r_{12}^{-3} V_1^n (N_{12}V_2), \tag{5.20}$$

$$\begin{aligned}
\overline{\underset{(4)}{\dot{h}_{nk}}}^{\,1}\; V_1^k &= k^2 m_2^2 r_{12}^{-3}[V_2^n(N_{12}V_1) + N_{12}^n(V_1V_2) - 4N_{12}^n(N_{12}V_2)(N_{12}V_1) \\
&-2V_1^n(N_{12}V_2)] + k^2 m_1 m_2 r_{12}^{-3}[3N_{12}^n V_1^2 - 6N_{12}^n(V_1V_2) - 12N_{12}^n(N_{12}V_1)^2 \\
&+12N_{12}^n(N_{12}V_2)(N_{12}V_1) + 5V_1^n(N_{12}V_1) - 6V_2^n(N_{12}V_1)] \\
&+km_2 r_{12}^{-2}[3V_1^n(N_{12}V_2)^3 - 2V_1^n(N_{12}V_2)V_2^2 + 4V_2^n(N_{12}V_2)(V_1V_2) \\
&+2N_{12}^n V_2^2(V_1V_2) + 2V_2^n(N_{12}V_1)V_2^2 - 6N_{12}^n(N_{12}V_2)^2(V_1V_2) \\
&-6V_2^n(N_{12}V_1)(N_{12}V_2)^2]\,,
\end{aligned} \tag{5.21}$$

$$\begin{aligned}
\overline{\partial_n\;\underset{(5)}{h_{0k}}}^{\,1}\; V_1^k - \overline{\partial_k\;\underset{(5)}{h_{0n}}}^{\,1}\; V_1^k &= k^2 m_2^2 r_{12}^{-3}[-V_2^n(N_{12}V_1) + N_{12}^n(V_1V_2)] \\
&+k^2 m_1 m_2 r_{12}^{-3}[5N_{12}^n V_1^2 - 8N_{12}^n(V_2V_1) - 5V_1^n(N_{12}V_1) + 8V_2^n(N_{12}V_1)] \\
&+km_2 r_{12}^{-2}[2V_2^n(N_{12}V_1)V_2^2 - 2N_{12}^n V_2^2(V_1V_2)]\,,
\end{aligned} \tag{5.22}$$

$$\begin{aligned}
\overline{\partial_k\;\underset{(4)}{h_{00}}}^{\,1}\; V_1^k V_1^n &= km_2 r_{12}^{-2}[4V_1^n(N_{12}V_1)V_2^2 + 2V_1^n(N_{12}V_2)(V_1V_2) \\
&-3V_1^n(N_{12}V_1)(N_{12}V_2)^2] - 4k^2 m_2^2 r_{12}^{-3} V_1^n(N_{12}V_1) - 2k^2 m_2 m_1 r_{12}^{-3} V_1^n(N_{12}V_1)\,,
\end{aligned} \tag{5.23}$$

$$\overline{\underset{(2)}{h_{00}}\;\partial_k\;\underset{(2)}{h_{00}}}^{\,1}\; V_1^k V_1^n = -4k^2 m_2^2 r_{12}^{-3} V_1^n(N_{12}V_1)\,, \tag{5.24}$$

$$\begin{aligned}
\overline{\partial_n\;\underset{(4)}{h_{ks}}}^{\,1}\; V_1^k V_1^s &= k^2 m_2^2 r_{12}^{-3}[-2V_1^n(N_{12}V_1) + 2N_{12}^n V_1^2 + 4N_{12}^n(N_{12}V_1)^2] \\
&+k^2 m_1 m_2 r_{12}^{-3}[-4N_{12}^n(N_{12}V_1)^2 + 2V_1^n(N_{12}V_1)] + km_2 r_{12}^{-2}[2V_2^n(N_{12}V_2)V_1^2 \\
&-4V_2^n(N_{12}V_1)(V_1V_2) - 4V_1^n(N_{12}V_2)(V_1V_2) - 3N_{12}^n(N_{12}V_2)^2 V_1^2 \\
&+12N_{12}^n(N_{12}V_2)(N_{12}V_1)(V_1V_2)]\,,
\end{aligned} \tag{5.25}$$

$$\overline{\partial_s \underset{(4)}{h_{nk}}}^{\,1}\; V_1^k V_1^s = k^2 m_2^2 r_{12}^{-3}[V_1^n(N_{12}V_1) - N_{12}^n V_1^2 + 4N_{12}^n(N_{12}V_1)^2]$$
$$+k^2 m_1 m_2 r_{12}^{-3}[-4N_{12}^n(N_{12}V_1)^2 + V_1^n(N_{12}V_1) - N_{12}^n V_1^2]$$
$$+k m_2 r_{12}^{-2}[-3V_1^n(N_{12}V_2)^2(N_{12}V_1) - 2V_2^n(N_{12}V_1)(V_1V_2) - 2V_2^n(N_{12}V_2)V_1^2$$
$$-2N_{12}^n(V_1V_2)^2 + 6N_{12}^n(N_{12}V_2)(N_{12}V_1)(V_1V_2)]\,, \tag{5.26}$$

$$\overline{\underset{(2)}{h_{ns}}\;\partial_s\;\underset{(2)}{h_{km}}}^{\,1}\; V_1^k V_1^m = -4k^2 m_2^2 r_{12}^{-3} N_{12}^n V_1^2\,, \tag{5.27}$$

$$\overline{\underset{(2)}{h_{ns}}\;\partial_m\;\underset{(2)}{h_{ks}}}^{\,1}\; V_1^k V_1^m = -4k^2 m_2^2 r_{12}^{-3} V_1^n(N_{12}V_1)\,, \tag{5.28}$$

$$\overline{\partial_s\;\underset{(3)}{h_{0k}}}^{\,1}\; V_1^n V_1^k V_1^s = -4k m_2 r_{12}^{-2} V_1^n(V_1V_2)(N_{12}V_1)\,, \tag{5.29}$$

$$\overline{\underset{(2)}{\dot{h}_{ks}}}^{\,1}\; V_1^n V_1^k V_1^s = -2k m_2 r_{12}^{-2} V_1^n V_1^2(N_{12}V_2)\,. \tag{5.30}$$

Now we obtain the 2PN equations of motion for two point particles from Eqs. (3.19) and (5.1)–(5.30):

$$\frac{d^2\xi_1^n}{dt^2} = -k m_2 r_{12}^{-2} N_{12}^n + c^{-2}\{k m_2 r_{12}^{-2}[4V_1^n(N_{12}V_1) - 4V_2^n(N_{12}V_1)$$
$$-3V_1^n(N_{12}V_2) + 3V_2^n(N_{12}V_2) - N_{12}^n V_1^2 - 2N_{12}^n V_2^2 + 4N_{12}^n(V_1V_2)$$
$$+\tfrac{3}{2}N_{12}^n(N_{12}V_2)^2] + k^2 m_2(5m_1 + 4m_2) r_{12}^{-3} N_{12}^n\} + c^{-4}\{k m_2 r_{12}^{-2}[4N_{12}^n V_2^2(V_1^k V_2^k)$$
$$-2N_{12}^n V_2^4 - 2N_{12}^n(V_1V_2)^2 + \tfrac{3}{2}N_{12}^n V_1^2(N_{12}V_2)^2 + \tfrac{9}{2}N_{12}^n V_2^2(N_{12}V_2)^2$$
$$-6N_{12}^n(V_1V_2)(N_{12}V_2)^2 - \tfrac{15}{8}N_{12}^n(N_{12}V_2)^4 - 3V_2^n V_1^2(N_{12}V_2) + V_1^n V_1^2(N_{12}V_2)$$
$$+4V_1^n V_2^2(N_{12}V_1) - V_2^n V_2^2(N_{12}V_2) - 5V_1^n V_2^2(N_{12}V_2) + \tfrac{9}{2}V_1^n(N_{12}V_2)^3$$
$$-4V_1^n(V_1V_2)(N_{12}V_1) + 6V_2^n(N_{12}V_1)^2(N_{12}V_2) + 4V_1^n(V_1V_2)(N_{12}V_2)$$
$$+\tfrac{3}{2}V_2^n(N_{12}V_2)^3 + 4V_2^n(V_1V_2)(N_{12}V_2) - 6V_1^n(N_{12}V_1)(N_{12}V_2)^2$$
$$-6V_2^n(N_{12}V_1)(N_{12}V_2)^2] + k^2 m_1 m_2 r_{12}^{-3}[\tfrac{1}{4}N_{12}^n V_1^2 + \tfrac{5}{4}N_{12}^n V_2^2 - \tfrac{13}{2}N_{12}^n(V_1V_2)$$
$$-\tfrac{5}{2}N_{12}^n(N_{12}V_1)^2 + \tfrac{5}{2}N_{12}^n(N_{12}V_2)^2 - 17N_{12}^n(N_{12}V_2)(N_{12}V_1) - \tfrac{15}{4}V_2^n(N_{12}V_2)$$
$$+\tfrac{31}{4}V_1^n(N_{12}V_2) + \tfrac{15}{4}V_2^n(N_{12}V_1) - \tfrac{23}{4}V_1^n(N_{12}V_1)] + k^2 m_2^2 r_{12}^{-3}[4N_{12}^n V_2^2$$
$$-12N_{12}^n(N_{12}V_2)^2 + 2N_{12}^n(N_{12}V_1)^2 - 8N_{12}^n(V_1V_2) - 4N_{12}^n(N_{12}V_2)(N_{12}V_1)$$
$$+4V_2^n(N_{12}V_2) - 2V_1^n(N_{12}V_2) + 2V_2^n(N_{12}V_1) - 2V_1^n(N_{12}V_1)] - 9k^3 m_2^3 r_{12}^{-4} N_n^{12}$$
$$-\tfrac{49}{4}k^3 m_1^2 m_2 r_{12}^{-4} N_{12}^n - \tfrac{57}{2}k^3 m_2^2 m_1 r_{12}^{-4} N_{12}^n\}\,. \tag{5.31}$$

Substituting the index 1 by 2 in (5.31), we obtain the equation of motion for the second body.

6. The 2PN equations of motion in the EIH-coordinates

The solutions $\underset{(2)}{h_{00}}$, $\underset{(2)}{h_{mn}}$, $\underset{(3)}{h_{0n}}$, $\underset{(4)}{h_{00}}$ *i.e.* (4.31) and (4.32) of the field equations (2.1) in the 2PN approximation in EIH coordinates (2.15) have the same form as in the Infeld-coordinates (2.14). However, the solutions $\underset{(4)}{h_{mn}}$, $\underset{(5)}{h_{0n}}$, $\underset{(6)}{h_{00}}$ in EIH-coordinates differ from (4.33)–(4.35) and takes the following form

$$\begin{aligned}\underset{(4)}{h_{mn}}{}^{(\mathrm{EIH})} &= \underset{(4)}{h_{mn}}{}^{(I)} - 4k^2m_1^2r_1^{-2}(\delta_{mn} - 2N_1^mN_1^n)\\ &\quad -8k^2m_1m_2[S_{12}^{-1}(r_1^{-1}\delta_{mn} - r_1^{-1}N_1^mN_1^n)\\ &\quad -S_{12}^{-2}(N_1^mN_1^n + N_1^mN_2^n)] + 1 \leftrightarrow 2\,, \qquad (6.1)\end{aligned}$$

$$\begin{aligned}\underset{(5)}{h_{0n}}{}^{(\mathrm{EIH})} &= \underset{(5)}{h_{0n}}{}^{(I)} + 4k^2m_1^2[r_1^{-2}V_1^n - 2r_1^{-2}N_1^n(N_1V_1)]\\ &\quad +k^2m_1m_2\{4S_{12}^{-1}r_1^{-1}[2V_2^n - 2N_1^n(N_1V_2)] + 4S_{12}^{-2}[N_2^n(N_1V_1)\\ &\quad -2N_1^n(N_{12}V_1) - 2N_2^n(N_{12}V_1) - N_1^n(N_2V_2) - 2N_1^n(N_1V_2)\\ &\quad -2N_2^n(N_1V_2)]\} + 1 \leftrightarrow 2\,, \qquad (6.2)\end{aligned}$$

$$\begin{aligned}\underset{(6)}{h_{00}}{}^{(\mathrm{EIH})} &= \underset{(6)}{h_{00}}{}^{(I)} + 2k^3m_1^2m_2\{2r_2^{-3} + r_1^{-3} + r_{12}^{-3} + r_{12}^{-2}r_1^{-1} - r_1^2r_{12}^{-2}r_2^{-3}\\ &\quad -r_2^2r_1^{-2}r_{12}^{-3} + r_2^{-1}r_{12}^{-2} - r_1^2r_2^{-3}r_{12}^{-2} + r_1^{-2}r_{12}^{-1} + r_1^{-2}r_2^{-1}\\ &\quad -r_{12}^2r_1^{-2}r_2^{-3} + S_{12}^{-1}[4r_1^{-2} + 2r_1^{-3}r_2 + 2r_2^{-1}r_1^{-1} - 2r_{12}^2r_1^{-3}r_2^{-1}\\ &\quad +4r_{12}^{-3}r_2^2r_1^{-1} - 4r_1r_{12}^{-3} - 4r_{12}^{-1}r_1^{-1} + 2r_1^2r_{12}^{-3}r_2^{-1} - 2r_2r_{12}^{-3}\\ &\quad -2r_{12}^{-1}r_2^{-1} - 12r_{12}^{-2}]\} + 4k^3m_1^3r_1^{-3} + 4k^2m_1^2\{2r_1^{-2}(N_1V_1)^2\\ &\quad -r_1^{-2}V_1^2\} + 4k^2m_1m_2\{r_{12}^{-1}S_{12}^{-1}[6V_1^2 - 6(V_1V_2) - 6(N_{12}V_1)^2\\ &\quad +6(N_{12}V_1)(N_{12}V_2)] + r_1^{-1}S_{12}^{-1}[-4V_1^2 - 4(V_1V_2) - 2(N_1V_1)^2\\ &\quad +4(N_1V_2)(N_1V_1)] + S_{12}^{-2}[-2(N_1V_1)^2 + 8(N_{12}V_1)(N_1V_1)\\ &\quad +6(N_{12}V_1)(N_{12}V_2) - 6(N_{12}V_1)^2 + 8(N_{12}V_1)(N_2V_2)\\ &\quad -2(N_1V_1)(N_2V_2) + 4(N_1V_2)(N_{12}V_2) + 4(N_2V_2)(N_1V_2)\\ &\quad -4(N_{12}V_1)(N_1V_2) + 4(N_1V_1)(N_1V_2)]\} + 1 \leftrightarrow 2\,. \qquad (6.3)\end{aligned}$$

The way of deriving Eqs. (6.1)–(6.3) is the same as that one for (4.31)–(4.35).

The post-Newtonian equations of motion in EIH-coordinates have the same form as the equations in the Infeld-coordinates; in the 2PN approximation, however, these equations have different forms; the differences are shown in the following expression

$$\frac{d^2\xi_1^n}{dt^2}^{(\mathrm{EIH})} = \frac{d^2\xi_1^n}{dt^2}^{(I)} + c^{-4}\{4k^2 m_1 m_2 r_{12}^{-3}[8N_{12}^n(N_{12}V_1)(N_{12}V_2) - 4N_{12}^n(N_{12}V_1)^2$$
$$-4N_{12}^n(N_{12}V_2)^2 + N_{12}^n V_1^2 + N_{12}^n V_2^2 - 2N_{12}^n(V_1V_2) + 2V_1^n(N_{12}V_1)$$
$$-2V_1^n(N_{12}V_2) - 2V_2^n(N_{12}V_1) + 2V_2^n(N_{12}V_2)]$$
$$+4k^2m_2^2[8N_{12}^n(N_{12}V_1)(N_{12}V_2) - 4N_{12}^n(N_{12}V_1)^2 - 4N_{12}^n(N_{12}V_2)^2$$
$$+N_{12}^n V_1^2 + N_{12}^n V_2^2 - 2N_{12}^n(V_1V_2) + 2V_1^n(N_{12}V_1) - 2V_1^n(N_{12}V_2)$$
$$-2V_2^n(N_{12}V_1) + 2V_2^n(N_{12}V_2)]$$
$$+2k^3 m_1^2 m_2 r_{12}^{-4} N_{12}^n + 4k^3 m_1 m_2^2 r_{12}^{-4} N_{12}^n + 2k^3 m_2^3 r_{12}^{-4} N_{12}^n\}\,. \quad (6.4)$$

The above expression is obtained by direct calculation of the integrals in (B.7).

7. Coordinate transformations

The form of Eqs. (5.31) differs from that ones obtained in [6,9,11], where the harmonic coordinates were used. Using transformations

$$x^n \longrightarrow x^n - 2c^{-4}\dot{\chi}_n \longrightarrow x^n + c^{-4}[2km_1N_1^kV_1^kV_1^n + 2km_2N_2^kV_2^kV_2^n$$
$$+2k^2m_1m_2(r_1 - r_2)r_{12}^{-2}N_{12}^n)]\,,$$
$$\xi_1^n \longrightarrow \xi_1^n - 2c^{-4}\,\overset{1}{\dot{\chi}_n} \longrightarrow \xi_1^n + c^{-4}[2km_2(N_{12}V_2)V_2^n - 2k^2m_1m_2r_{12}^{-1}N_{12}^n]\,,$$
$$\xi_2^n \longrightarrow \xi_2^n - 2c^{-4}\,\overset{2}{\dot{\chi}_n} \longrightarrow \xi_2^n + c^{-4}[-2km_1(N_{12}V_1)V_1^n + 2k^2m_1m_2r_{12}^{-1}N_{12}^n]\,. \quad (7.1)$$

where

$$\xi_A^n = \int \hat{\delta}(\bar{r}_A)x^n(d\bar{x})\,,$$

we obtain

$$\frac{d^2\xi_1^n}{dt^2} \longrightarrow \frac{d^2\xi_1^n}{dt^2} + c^{-4}\{km_2r_{12}[6V_2^n(N_{12}V_2)^3 - 6V_2^nV_2^2(N_{12}V_2)$$
$$-12V_2^n(N_{12}V_1)(N_{12}V_2)^2 + 4V_2^n(N_{12}V_1)V_2^2 + 8V_2^n(N_{12}V_2)(V_1V_2)$$
$$+6V_2^n(N_{12}V_2)(N_{12}V_1)^2 - 4V_2^n(N_{12}V_1)(V_1V_2) - 2V_2^nV_1^2(N_{12}V_2)]$$
$$+k^2m_1m_2r_{12}^{-3}[10V_2^n(N_{12}V_2) - 12V_2^n(N_{12}V_1) + 8V_1^n(N_{12}V_1) - 6V_1^n(N_{12}V_2)$$
$$-6N_{12}^n(N_{12}V_2)^2 - 16N_{12}^n(N_{12}V_1)^2 + 22N_{12}^n(N_{12}V_2)(N_{12}V_1) + 4N_{12}^nV_1^2$$
$$-4N_{12}^n(V_1V_2)] + 2k^3m_1^2m_2r_{12}^{-4}N_{12}^n - 2k^3m_2^2m_1r_{12}^{-4}N_{12}^n\}\,, \quad (7.2)$$

$$km_2\partial^1_n r^{-1}_{12} \longrightarrow km_2\partial^1_n r^{-1}_{12} + c^{-4}\{k^2m_2^2r_{12}^{-3}[6N_{12}^n(N_{12}V_2)^2 - 2V_2^n(N_{12}V_2)] + k^2m_1m_2[6N_{12}^n(N_{12}V_1)^2 - 2V_1^n(N_{12}V_1)] - 8k^3m_2^2m_1r_{12}^{-4}N_{12}^n\}\,. \tag{7.3}$$

Substitution of (7.2) and (7.3) into (5.31) leads to the 2PN equations of motion in the harmonic coordinates. Similarly, using the transformation

$$\begin{aligned}
&x^n \longrightarrow x^n + c^{-4}[-2\dot{\chi}_n - 4k^2m_1m_2\partial_n \ln S_{12} - 2k^2m_1^2\partial_n \ln r_1 - 2k^2m_2^2\partial_n \ln r_2]\\
&\longrightarrow x^n + c^{-4}[2km_1N_1^kV_1^kV_1^n + 2km_2N_2^kV_2^kV_2^n + 2k^2m_1m_2(r_1-r_2)r_{12}^{-2}N_{12}^n\\
&-4k^2m_1m_2S_{12}^{-1}(N_1^n + N_2^n) - 2km_1^2r_1^{-1}N_1^n - 2km_2^2r_2^{-1}N_2^n]\,,\\
&\xi_1^n \longrightarrow \xi_1^n + c^{-4}\Big[-2\,\overset{1}{\overline{\dot{\chi}_n}} - 4k^2m_1m_2\,\overset{1}{\overline{\partial_n \ln S_{12}}} - 2km_2^2\,\overset{1}{\overline{\partial_n \ln r_2}}\,\Big]\\
&\longrightarrow \xi_1^n + c^{-4}[2km_2(N_{12}V_2)V_2^n - 4k^2m_1m_2r_{12}^{-1}N_{12}^n - 2k^2m_2^2r_{12}^{-1}N_{12}^n]\,,\\
&\xi_2^n \longrightarrow \xi_2^n + c^{-4}\Big[-2\,\overset{2}{\overline{\dot{\chi}_n}} - 4k^2m_1m_2\,\overset{2}{\overline{\partial_n \ln S_{12}}} - 2km_1^2\,\overset{2}{\overline{\partial_n \ln r_1}}\,\Big]\\
&\longrightarrow \xi_2^n + c^{-4}\Big[-2km_1(N_{12}V_1)V_1^n + 4k^2m_1m_2r_{12}^{-1}N_{12}^n + 2k^2m_1^2r_{12}^{-1}N_{12}^n\Big]\,.
\end{aligned} \tag{7.4}$$

in Eqs. (6.4), we get the 2PN equations of motion also in the harmonic coordinates [3–7,9,11]. We obtain the formulae (7.1) and (7.4) by using (37a), (37b) and (38) from [28] and assuming that the transformed metric tensor satisfies the equation

$$\sqrt{-g}g^{\mu\nu} = 0.$$

The last one defines the harmonic coordinates.

Appendix A

The derivatives of the product of singular functions

The identities (2.12), (2.13) we use for the singular function $r_A^{-L}, r_B^{-M}, L = 1,2,3,\ldots, M = 1,2,3,\ldots$.

Putting $f = r_A^{-1}$ and successively $g = r_A^{-1}, r_A^{-2},\ldots$ into (2.12) we obtain

$$\partial_a r_A^{-L} \equiv Lr_A^{-L+1}\partial_a r_A^{-1}. \tag{A.1}$$

Substituting r_A^{-L}, r_B^{-M} into (2.12) and using (A.1) we get

$$\partial_a(r_A^{-L}r_B^{-M}) \equiv Lr_B^{-M}r_A^{-L+1}\partial_a r_A^{-1} + Mr_A^{-L}r_B^{-M+1}\partial_a r_B^{-1}. \tag{A.2}$$

Similarly, substituting $f = r_A^{-1}, g = r_A^{-1}, r_A^{-2},\ldots$ into (2.13) we have

$$\partial_a\partial_b r_A^{-L} \equiv L(L-1)r_A^{-L+2}\partial_a r_A^{-1}\partial_b r_A^{-1} + Lr_A^{-L+1}\partial_a\partial_b r_A^{-1}. \tag{A.3}$$

For r_A^{-L}, r_B^{-M} from (2.13) and (A.3) we obtain

$$\partial_a\partial_b(r_A^{-L}r_B^{-M}) \equiv Lr_B^{-M}r_A^{-L+1}\partial_a\partial_b r_A^{-1} + Mr_A^{-L}r_B^{-M+1}\partial_a\partial_b r_B^{-1}$$
$$+L(L-1)r_B^{-M}r_A^{-L+2}\partial_a r_A^{-1}\partial_b r_A^{-1} + M(M-1)r_A^{-L}r_B^{-M+2}\partial_a r_B^{-1}\partial_b r_B^{-1}$$
$$+LMr_A^{-L+1}r_B^{-M+1}[\partial_a r_A^{-1}\partial_b r_B^{-1} + \partial_a r_B^{-1}\partial_b r_A^{-1}]\,. \quad \text{(A.4)}$$

In particular, from (A.3) and (A.4) we get

$$\Delta r_A^{-L} \equiv L(L-1)r_A^{-L+2}\partial_a r_A^{-1}\partial_a r_A^{-1} + Lr_A^{-L+1}\Delta r_A^{-1}\,, \quad \text{(A.5)}$$

$$\Delta(r_A^{-L}r_B^{-M}) \equiv Lr_B^{-M}r_A^{-L+1}\Delta r_A^{-1} + Mr_A^{-L}r_B^{-M+1}\Delta r_B^{-1}$$
$$+L(L-1)r_B^{-M}r_A^{-L+2}\partial_a r_A^{-1}\partial_a r_A^{-1}M(M-1)r_A^{-L}r_B^{-M+2}\partial_a r_B^{-1}\partial_a r_B^{-1}$$
$$+2LMr_A^{-L+1}r_B^{-M+1}\partial_a r_A^{-1}\partial_a r_B^{-1}\,. \quad \text{(A.6)}$$

Substituting into (A.1)–(A.6) the derivatives (which are taken, similarly as in [11] and [29]) from the distribution theory [30]):

$$\partial_a r_A^{-1} = -r_A^{-2}N_A^a\,, \quad \text{(A.7)}$$

$$\partial_a\partial_b r_A^{-1} = r_A^{-3}[3N_A^aN_A^b - \delta_{ab}] - \tfrac{4}{3}\pi\delta(\bar{r}_A)\delta_{ab} \quad \text{(A.8)}$$

we can determine the derivatives of the product of the functions r_A^{-L}, r_B^{-M} by their derivatives

$$\partial_a r_A^{-L} = -Lr_A^{-L-1}N_A^a\,, \quad \text{(A.9)}$$

$$\partial_a(r_A^{-L}r_B^{-M}) = -Lr_B^{-M}r_A^{-L-1}N_A^a - Mr_A^{-L}r_B^{-M-1}N_B^a, \quad \text{(A.10)}$$

$$\partial_a\partial_b r_A^{-L} = Lr_A^{-L-2}[(L+2)N_A^aN_A^b - \delta_{ab}] - \tfrac{4}{3}\pi Lr_A^{-L+1}\delta(\bar{r}_A)\delta_{ab}\,, \quad \text{(A.11)}$$

$$\partial_a\partial_b(r_A^{-L}r_B^{-M}) = LMr_A^{-L-1}r_B^{-M-1}(N_A^aN_B^b + N_A^bN_B^a) + Mr_A^{-L}r_B^{-M-2}$$
$$\times[(M+2)N_B^aN_B^b - \delta_{ab}] + Lr_B^{-M}r_A^{-L-2}[(L+2)N_A^aN_A^b - \delta_{ab}]$$
$$-\tfrac{4}{3}\pi Mr_A^{-L}r_B^{-M+1}\delta(\bar{r}_B)\delta_{ab} - \tfrac{4}{3}\pi Lr_B^{-M}r_A^{-L+1}\delta(\bar{r}_A)\delta_{ab}\,, \quad \text{(A.12)}$$

$$\Delta r_A^{-L} = L(L-1)r_A^{-L-2} - 4\pi Lr_A^{-L+1}\delta(\bar{r}_A)\,, \quad \text{(A.13)}$$

$$\Delta(r_A^{-L}r_B^{-M}) = L(L-1)r_B^{-M}r_A^{-L-2} + M(M-1)r_A^{-L}r_B^{-M-2}$$
$$-4\pi Lr_B^{-M}r_A^{-L+1}\delta(\bar{r}_A) - 4\pi Mr_A^{-L}r_B^{-M+1}\delta(\bar{r}_B)\,. \quad \text{(A.14)}$$

In order to obtain derivatives of r_A^L, $L = 1, 2, 3, \ldots$ we are using the ordinary differential calculus.

Appendix B

The regularization procedure for the divergent integrals

In (5.2), (5.3), (5.7)–(5.30) we get the following divergent integrals:

$$\begin{aligned}
&\int \hat{\delta}(\bar{r}_A) r_A^K r_B^M d(\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_A^n d(\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_B^n d(\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_A^n N_A^k d(\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_A^n N_B^k d(\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_B^n N_B^k d(\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_A^n N_A^k N_A^s d(\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_A^n N_A^k N_B^s d(\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_A^n N_B^k N_B^s d(\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) r_A^K r_B^M N_B^n N_B^k N_B^s d(\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) \partial_n \partial_b^A \ln S_{AB} (d\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) \partial_n \partial_b^B \ln S_{AB} (d\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) \partial_n \partial_k^A \partial_b^B \ln S_{AB} (d\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) \partial_n^B \partial_k^A \partial_b^A \ln S_{AB} (d\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) \partial_n^B \partial_k^B \partial_b^A \ln S_{AB} (d\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) r_A^{-2} N_A^a \partial_n^B \partial_a^A \ln S_{AB} (d\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) r_B^{-2} N_B^a \partial_n^B \partial_a^A \ln S_{AB} (d\bar{x})\,, \qquad \int \hat{\delta}(\bar{r}_A) r_A^{-2} N_A^a \partial_n^A \partial_a^B \ln S_{AB} (d\bar{x})\,,\\
&\int \hat{\delta}(\bar{r}_A) r_B^{-2} N_A^a \partial_n^A \partial_a^B \ln S_{AB} (d\bar{x})\,. \qquad \text{(B.1)}
\end{aligned}$$

where $K \in \{-4,-3,-2,-1,0,1\}$, $M \in \{-4,-3,-2,-1,0,1\}$ are the power exponents. These integrals are non-vanishing only for $\bar{x} = \bar{\xi}_A$.

In order to calculate the above integrals we need the following expansions of r_B^M, $r_B^K N_B^n$ and S_{AB}^M into a power series with respect to $r_A \approx 0$:

$$\begin{aligned}
r_B^M &= r_{AB}^M + M r_{AB}^{M-1} N_{AB}^k N_A^k r_A + \left[\frac{M(M-2)}{2} r_{AB}^{M-2} N_{AB}^k N_{AB}^s N_A^k N_A^s \right.\\
&\left. + \frac{M}{2} r_{AB}^{M-2}\right] r_A^2 + \left[\frac{M(M-2)(M-4)}{6} r_{AB}^{M-3} N_{AB}^k N_{AB}^s N_{AB}^b N_A^k N_A^s N_A^b \right.\\
&\left. + \frac{M(M-2)}{2} r_{AB}^{M-3} N_{AB}^k N_A^k \right] r_A^3 + O(r_A^4)\,, \qquad \text{(B.2)}
\end{aligned}$$

$$\begin{aligned}
r_B^{(M+1)} N_B^n &= r_{AB}^{(M+1)} N_{AB}^n + \left\{ r_{AB}^M N_A^n + M r_{AB}^M N_{AB}^n N_{AB}^k N_A^k \right\} r_A\\
&+ \left\{ M r_{AB}^{(M-1)} N_{AB}^k N_A^k N_A^n + \frac{M(M-2)}{2} r_{AB}^{(M-1)} N_{AB}^n N_{AB}^k N_{AB}^s N_A^k N_A^s \right.
\end{aligned}$$

$$
+\frac{M}{2}r_{AB}^{(M-1)}N_{AB}^{n}\Big\}r_A^2+\Big\{\frac{M}{2}r_{AB}^{(M-2)}N_A^n+\frac{M(M-2)}{2}r_{AB}^{(M-2)}N_{AB}^nN_{AB}^bN_A^b
+\frac{M(M-2)}{2}r_{AB}^{(M-2)}N_{AB}^bN_{AB}^sN_A^bN_A^sN_A^n
+\frac{M(M-2)(M-4)}{6}r_{AB}^{(M-2)}N_{AB}^nN_{AB}^bN_{AB}^sN_{AB}^kN_A^bN_A^sN_A^k\Big\}r_A^3
+O(r_A^4), \tag{B.3}
$$

$$
S_{AB}^M=(2r_{AB})^M+M(2r_{AB})^{M-1}(1+N_{AB}^KN_A^k)r_A
+\left\{\frac{M(M-1)}{2}(2r_{AB})^{M-2}(1+2N_{AB}^kN_A^k+N_{AB}^kN_{AB}^sN_A^kN_A^s)\right.
\left.+\frac{M}{2}(2r_{AB})^{M-1}(r_{AB}^{-1}-r_{AB}^{-1}N_{AB}^kN_{AB}^sN_A^kN_A^s)\right\}r_A^2
+\left\{\frac{M(M-1)(M-2)}{6}(2r_{AB})^{M-3}(1+N_{AB}^kN_{AB}^sN_{AB}^bN_A^kN_A^sN_A^b\right.
+3N_{AB}^kN_A^k+3N_{AB}^kN_{AB}^sN_A^kN_A^s)+\frac{M(M-1)}{2}(2r_{AB})^{M-2}(r_{AB}^{-1}
+r_{AB}^{-1}N_{AB}^kN_A^k-r_{AB}^{-1}N_{AB}^kN_{AB}^sN_A^kN_A^s-r_{AB}^{-1}N_{AB}^kN_{AB}^sN_{AB}^bN_A^kN_A^sN_A^b)
\left.+\frac{M}{2}(2r_{AB})^{M-1}(r_{AB}^{-2}N_{AB}^kN_{AB}^sN_{AB}^bN_A^kN_A^sN_A^b-r_{AB}^{-2}N_{AB}^kN_A^k)\right\}r_A^3+O(r_A^4)\,. \tag{B.4}
$$

Inserting (B.2), (B.3) and (B.4) into (B.1), we obtain the integrals in the form

$$
\int\hat{\delta}(\bar{r}_A)r_A^{-p}N_A^{k_1}N_A^{k_2}\dots N_A^{k_n}d(\bar{x}),\quad p=0,1,2,\dots L.
$$

Taking into account the property D2) of $\hat{\delta}(\bar{r})$ and using the spherical coordinates, we have

$$
\int\hat{\delta}(\bar{r'}_A)r_A^{'-p}N_A^{'k_1}N_A^{'k_2}\dots N_A^{'k_n}d(\bar{x'})
=\int_0^\infty\hat{\delta}(\bar{r'}_A)r_A^{'-p+2}dr'\oint N_A^{'k_1}N_A^{'k_2}\dots N_A^{'k_n}d\Omega\,,\quad p=0,1,2,\dots L\,,
$$

where the last integral is taken over the solid angle. Returning to $(\bar{x})$ coordinates in the first integral on the right-hand side of the above expression and taking into account that $\oint d\Omega = 4\pi$, we obtain

$$\int \hat{\delta}(\bar{r'}_A) r_A^{\prime -p} N_A^{\prime k_1} N_A^{\prime k_2} \dots N_A^{\prime k_n} d(\bar{x'})$$
$$= \int \hat{\delta}(\bar{r'}_A) r_A^{\prime -p} d(\bar{x'}) \frac{1}{4\pi} \oint N_A^{\prime k_1} N_A^{\prime k_2} \dots N_A^{\prime k_n} d\Omega\,, \quad p = 0, 1, 2, \dots L\,.$$

Taking into account the property (D4) of $\hat{\delta}$ the last integral in the formula above yields

$$\int \hat{\delta}(\bar{r'}_A) r_A^{\prime -p} d(\bar{x'})\, \frac{1}{4\pi} \oint N_A^{\prime k_1} N_A^{\prime k_2} \dots N_A^{\prime k_n} d\Omega = \begin{cases} 0\,, & p = 1, 2, \dots L\,, \\ \frac{1}{4\pi} \int N_A^{\prime k_1} N_A^{\prime k_2} \dots N_A^{\prime k_n} d\Omega, & p = 0\,. \end{cases} \tag{B.5}$$

We calculate the second integral on the right-hand side of (B.5) by integrating over the solid angle or by using the formulae (90.19) and (90.20) from [12].

Finally, from (B.1) by means of (B.5) we obtain the following non-vanishing integrals:

$$\begin{aligned}
\int \hat{\delta}(\bar{r}_A) r_A^{-1} r_B^{-2} N_A^n N_A^b N_A^s (d\bar{x}) &= -\tfrac{2}{15} r_{AB}^{-3} [N_{AB}^n \delta_{bs} + N_{AB}^b \delta_{ns} + N_{AB}^s \delta_{nb}]\,, \\
\int \hat{\delta}(\bar{r}_A) r_A^{-2} N_B^n (d\bar{x}) &= -\tfrac{1}{3} r_{AB}^{-2} N_{AB}^n, \\
\int \hat{\delta}(\bar{r}_A) r_A^{-3} r_B N_A^n (d\bar{x}) &= -\tfrac{1}{15} r_{AB}^{-2} N_{AB}^n, \\
\int \hat{\delta}(\bar{r}_A) r_A^{-3} r_B^{-3} N_A^n (d\bar{x}) &= -r_{AB}^{-6} N_{AB}^n, \\
\int \hat{\delta}(\bar{r}_A) r_A^{-2} r_B^{-4} N_B^n (d\bar{x}) &= \tfrac{5}{3} r_{AB}^{-6} N_{AB}^n\,, \\
\int \hat{\delta}(\bar{r}_A) \partial_n \partial_b^A \ln S_{AB} (d\bar{x}) &= \left[\tfrac{1}{4}\delta_{nb} - \tfrac{1}{4} N_{AB}^n N_{AB}^b\right] r_{AB}^{-2}, \\
\int \hat{\delta}(\bar{r}_A) \partial_n \partial_b^B \ln S_{AB} (d\bar{x}) &= [N_{AB}^n N_{AB}^b - \tfrac{1}{2}\delta_{nb}] r_{AB}^{-2}\,, \\
\int \hat{\delta}(\bar{r}_A) \partial_n \partial_a^A \partial_b^B \ln S_{AB} (d\bar{x}) &= \Big[\tfrac{1}{2} N_{AB}^b \delta_{an} + \tfrac{1}{4} N_{AB}^n \delta_{ab} \\
&\quad + \tfrac{1}{4} N_{AB}^a \delta_{nb} - N_{AB}^n N_{AB}^a N_{AB}^b\Big] r_{AB}^{-3}\,, \\
\int \hat{\delta}(\bar{r}_A) \partial_n^B \partial_a^A \partial_b^A \ln S_{AB} (d\bar{x}) &= \Big[\tfrac{3}{4} N_{AB}^b \delta_{an} + \tfrac{3}{4} N_{AB}^a \delta_{bn}
\end{aligned}$$

$$
\begin{aligned}
& \qquad\qquad +\tfrac{1}{2}N^n_{AB}\delta_{ab} - 3N^n_{AB}N^a_{AB}N^b_{AB}\Big]r^{-3}_{AB}\,,\\
\int \hat{\delta}(\bar{r}_A)\partial^B_n\partial^B_b\partial^A_a \ln S_{AB}(d\bar{x}) &= [-N^b_{AB}\delta_{an} - N^n_{AB}\delta_{ab}\\
&\quad -N^a_{AB}\delta_{nb} + 4N^n_{AB}N^a_{AB}N^b_{AB}]r^{-3}_{AB}\,,\\
\int \hat{\delta}(\bar{r}_A)r^{-2}_B N^a_B\partial^B_n\partial^A_a \ln S_{AB}(d\bar{x}) &= \tfrac{1}{2}r^{-4}_{AB}N^n_{AB}\,,\\
\int \hat{\delta}(\bar{r}_A)r^{-2}_B N^a_B\partial^A_n\partial^B_a \ln S_{AB}(d\bar{x}) &= \tfrac{1}{2}r^{-4}_{AB}N^n_{AB}\,. \qquad \text{(B.6)}
\end{aligned}
$$

These integrals give the contribution in equations of motion (5.31).

The following regularized integrals were used for deriving the 2PN equations in the EIH coordinates:

$$
\begin{aligned}
\int \hat{\delta}(\bar{r}_A)\partial_k\partial_n\partial_m \ln S_{AB}(d\bar{x}) &= r^{-3}_{AB}\Big[\tfrac{5}{2}N^k_{AB}N^n_{AB}N^m_{AB} - \tfrac{1}{2}N^k_{AB}\delta_{nm}\\
&\quad -\tfrac{1}{2}N^n_{AB}\delta_{km} - \tfrac{1}{2}N^m_{AB}\delta_{nk}\Big]\,,\\
\int \hat{\delta}(\bar{r}_A)\partial_k\partial_n\partial^B_m \ln S_{AB}(d\bar{x}) &= r^{-3}_{AB}\Big[-3N^k_{AB}N^n_{AB}N^m_{AB} + \tfrac{3}{4}N^k_{AB}\delta_{nm}\\
&\quad +\tfrac{3}{4}N^n_{AB}\delta_{km} + \tfrac{1}{2}N^m_{AB}\delta_{nk}\Big]\,,\\
\int \hat{\delta}(\bar{r}_A)\partial_k\partial_n\partial^A_m \ln S_{AB}(d\bar{x}) &= r^{-3}_{AB}\Big[\tfrac{1}{2}N^k_{AB}N^n_{AB}N^m_{AB} - \tfrac{1}{4}N^k_{AB}\delta_{nm}\\
&\quad -\tfrac{1}{4}N^n_{AB}\delta_{km}\Big]\,,\\
\int \hat{\delta}(\bar{r}_A)\partial_k\partial^A_n\partial^A_m \ln S_{AB}(d\bar{x}) &= r^{-3}_{AB}\Big[\tfrac{1}{2}N^k_{AB}N^n_{AB}N^m_{AB} - \tfrac{1}{4}N^m_{AB}\delta_{nk}\\
&\quad -\tfrac{1}{4}N^n_{AB}\delta_{km}\Big]\,,\\
\int \hat{\delta}(\bar{r}_A)\partial_k\partial^B_n\partial^B_m \ln S_{AB}(d\bar{x}) &= r^{-3}_{AB}\Big[4N^k_{AB}N^n_{AB}N^m_{AB} - N^k_{AB}\delta_{nm}\\
&\quad -N^n_{AB}\delta_{km} - N^m_{AB}\delta_{kn}\Big]\,,\\
\int \hat{\delta}(\bar{r}_A)r^{-3}_B(3N^k_BN^n_B - \delta_{nk})\ \partial_k \ln S_{AB}(d\bar{x}) &= r^{-4}_{AB}N^n_{AB}\,,\\
\int \hat{\delta}(\bar{r}_A)r^{-2}_B N^k_B\partial_n\partial_k \ln S_{AB}(d\bar{x}) &= -\tfrac{1}{2}r^{-4}_{AB}N^n_{AB}\,. \qquad \text{(B.7)}
\end{aligned}
$$

REFERENCES

[1] J.H. Taylor, J.M. Weisberg, *Astrophys. J.* **345**, 434 (1989).
[2] J.H. Taylor, A. Wolszczan, T. Damour, J.M. Weisberg, *Nature* **355**, 132 (1992).
[3] T. Damour, N. Deruelle, *Phys. Lett.* **87A**, 81 (1981).
[4] T. Damour, N. Deruelle, *C.R. Acad. Sci. Paris* **293** II, 537 (1981); **293** II, 877 (1981).
[5] T. Damour, *C.R. Acad. Sci. Paris* **294** II, 1355 (1982).
[6] T. Damour, in *Gravitational Radiation*, eds. N. Deruelle and T. Piran, North-Holland Publishing Company, Amsterdam 1983, p.59.
[7] L. Bel, T. Damour, N. Deruelle, J. Ibanez, J. Martin, *Gen. Relativ. Gravitation* **13**, 963 (1981).
[8] L. Infeld, *Acta. Phys. Pol.* **13**, 187 (1954).
[9] L. Blanchet, G. Faye, B. Ponsot, *Phys. Rev.* **D58**, 124002 (1998).
[10] L. Blanchet, G. Faye, *J. Math. Phys.* **41**, 7675 (2000).
[11] S.M. Kopejkin, *Astron. Zh.* **62**, 889 (1985).
[12] V.A. Fock, *Theory of Space, Time and Gravitation*, Pergamon Press, London 1959.
[13] Y. Itoh, T. Futamase, H. Asada, *Phys. Rev.* **D62**, 064002 (2000); *Phys. Rev.* **D63**, 064038 (2001).
[14] A. Einstein, L. Infeld, B. Hoffmann, *Ann. Math.* **39**, 65 (1938).
[15] A. Einstein, L. Infeld, *Ann. Math.* **41**, 455 (1940).
[16] A. Einstein, L. Infeld, *Can. J. Math.* **1**, 209 (1949).
[17] M.E. Pati, C.M. Will, *Phys. Rev.* **D62**, 124015 (2000); **D65**, 104008 (2002).
[18] R. Arnowitt, S. Deser, C.M. Misner, *Phys. Rev.* **120**, 313 (1960).
[19] R. Arnowitt, S. Deser, C.M. Misner, in *Gravitation: An Introduction to Current Research*, ed. L. Witten , Wiley, New York 1962.
[20] T. Ohta, H. Okamura, T. Kimura, K. Hiida, *Prog. Theor. Phys.* **51**, 1220 (1974).
[21] T. Ohta, H. Okamura, T. Kimura, K. Hiida, *Prog. Theor. Phys.* **51**, 1598 (1974).
[22] T. Damour, G. Schäfer, *Gen. Relativ. Gravity* **17**, 879 (1985).
[23] S. Bazanski, *The problem of motion* in *Recent Developments in General Relativity*, Pergamon Press, PWN, Warszawa 1962.
[24] L. Infeld, J. Plebanski, *Bull. Acad. Pol. Sci. Cl. III* **4**, 757 (1956).
[25] L. Infeld, J. Plebanski, *Motion and Relativity*, Pergamon Press, New York 1960.
[26] L. Infeld, J. Plebanski, *Bull. Acad. Pol. Sci. Cl. III* **4**, 687 (1956).
[27] J.N. Goldberg, *Phys. Rev.* **99**, 1873 (1959).
[28] V.A. Brumberg, *Relativistic Celestial Mechanics*, Nauka, Moscow 1972, in Russian.
[29] P. Jaranowski, *Mathematics of Gravitation*, Banach Center Publications part II, **41**, 55, ed. A. Królak, Polish Academy of Sciences, Warsaw 1997.
[30] I.M. Gel'fand, G.E. Shilov, *Generalized Functions*, Academic Press, New York 1964.