FOLDED LOCALIZED EXCITATIONS OF THE MACCARI SYSTEM

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Using the variable separation approach, a quite general variable separation solution of the (2+1)-dimensional Maccari systems can be derived. Special type of soliton solutions, folded solitary waves (FSWs) and foldons, are obtained by selecting some types of multi-valued functions appropriately. The FSWs and foldons may be "folded" in quite complicated waves and possess interesting interaction properties.

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1. Introduction

In the study of nonlinear physical models, to find some accurate localized soliton solutions and study the interaction of the soliton solutions in the cases of (2+1) dimensions is very important. In recent years much effort has been focused on this aspect and many types of localized excitations such as solitoffs, dromions, rings, lumps, breathers, instantons, peakons , campactons and localized chaotic and fractal patterns, *etc.*, are found in many (2+1)-dimensional physical models. Meanwhile many interesting interaction properties for these soliton solutions or localized soliton structures were revealed [1–8]. However, in the real natural world, there exist very complicated folded phenomena such as the folded protein [9], folded brain and skin surface and many other kinds of folded biology system [10]. The bubbles on (or under) a fluid surface may be thought to be the simplest folded waves. Various ocean waves are really folded waves, too. The loop solitons [11,12], which are thought to be class of the simplest folded waves in(1+1)-dimensional case, have been found in many (1+1)-dimensional integrable systems and have been applied in some possible physical fields like

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the string interaction with external field [13], quantum field theory [14] and particle physics [15]. Recently, Tang and Lou [16] first considered these special folded localized excitations and found folded solitary waves in some (2+1)-dimensional nonlinear models, such as the (2+1)-dimensional dispersive long wave equation, the(2+1)-dimensional Broer–Kaup–Kupershmidt equation and the (2+1)-dimensional Burgers equation, *etc.* They defined a new type of soliton, *i.e.* foldon if the interactions among the folded solitary waves are completely elastic. In this paper, we consider the Maccari system, so called "new (2+1)-dimensional nonlinear system" [2,3]

$$iA_t + A_{xx} + LA = 0\,, \tag{1}$$

$$iB_t + B_{xx} + LB = 0\,, \tag{2}$$

$$L_y = (AA^* + BB^*)_x\,. \tag{3}$$

Using the variable separation approach, we convert the systems into simple variable separation solution. Because some types of the usual localized excitations of Eqs. (1)–(3) such as solitoffs, dromions, dromion lattice, breathers, instantons, peakons and compactons are obtained by selecting some types of lower-dimensional appropriate functions, [4,5] here we only try to find some kinds of FSWs and foldon structures. In particular, we are interested in the possible interaction behavior of the foldons.

2. Variable separation solutions for Maccari system

In Ref. [4], using the variable separation approach, we have proved that the Maccari system (1)–(3) possesses a quite general solution

$$A = \frac{2\delta_1\delta_2\sqrt{\lambda a_1 a_2 p_x q_y}\exp(ir+is_1)}{a_1p - a_2q + a_3pq}\,, \tag{4}$$

$$B = \frac{2\delta_3\delta_4\sqrt{(2-\lambda) a_1 a_2 p_x q_y}\exp(ir+is_2)}{a_1p - a_2q + a_3pq}\,, \tag{5}$$

$$L = 2\left(\frac{a_1p_{xx} + a_3p_{xx}q}{a_1p - a_2q + a_3pq} - \frac{(a_1p_x + a_3p_xq)^2}{(a_1p - a_2q + a_3pq)^2}\right) + L_{0x}\,, \tag{6}$$

where $\lambda,\ a_1,\ a_2,\ a_3$ are arbitrary constants and $\delta_1^2 = \delta_1^2 = \delta_3^2 = \delta_4^2 = 1$. In Eqs. (4)–(6), $p \equiv p(x,t)$ is an arbitrary function of (x,t) due to the arbitrariness of the introduced seed function $L_0 = p_0(x,t)$,

$$p_{0x} = (4p_x^2)^{-1}(4r_tp_x^2 + 4p_x^2r_x^2 + p_{xx}^2 - 2p_xp_{xx})\,, \tag{7}$$

$r \equiv r(x,t)$ is related to p with

$$p_t + 2p_xr_x = c_1(-a_2 + a_3p)^2 + c_2(-a_2 + a_3p) - a_1a_2c_3\,, \tag{8}$$

$s_i \equiv s_i(y)(i=1,2)$ are arbitrary functions of (y) satisfies

$$s_{it} = 0\,, \tag{9}$$

and $q \equiv q(y,t)$ satisfies the following Riccati equation

$$q_t = -c_3(a_1 + a_3 q)^2 - c_2(a_1 + a_3 q) + a_1 a_2 c_1\,, \tag{10}$$

with $c_i \equiv c_i(t)(i=1,2,3)$. If we select

$$c_1 = \frac{-1}{a_1 a_2 a_3^2 H_1}\left[(a_1 + a_3 H_2)^2 H_{3t} + \left(a_1 a_3 + a_3^2 H_2\right) H_{1t} - a_3^2 H_1 H_{2t}\right], \tag{11}$$

$$c_2 = \frac{-1}{a_3^2 H_1}\left[2(a_1 + a_3 H_2) H_{3t} + a_3 H_{1t}\right], \tag{12}$$

$$c_3 = \frac{H_{3t}}{a_3^2 H_1}\,, \tag{13}$$

then the Riccati Eq. (10) has a solution

$$q = \frac{H_1}{H_3 + F(y)} + H_2\,, \tag{14}$$

where $H_i \equiv H_i(t)(i=1,2,3)$ are arbitrary functions of (t) and $F \equiv F(y)$ is an arbitrary function of (y), which means that $q = q(y,t)$ can also be viewed as an arbitrary function of (y,t) due to the arbitrariness of $H_i(t)(i=1,2,3)$ and $F \equiv F(y)$. Especially, the module squares of the field A and B read

$$U = |A|^2 = \frac{\lambda a_1 a_2 p_x q_y}{(a_1 p - a_2 q + a_3 pq)^2}\,, \tag{15}$$

$$V = |B|^2 = \frac{(2-\lambda) a_1 a_2 p_x q_y}{(a_1 p - a_2 q + a_3 pq)^2}\,. \tag{16}$$

3. Folded solitary waves and foldons of the Maccari system

Because of the arbitrariness of the functions of p and q, many kinds of stable coherent localized excitations for the field U (or V) can be obtained. Here we construct the FSWs and foldons directly from the expression (15) and (16). For briefness, we only discuss the field U, and the field V can be treated in the same way. It is considered that these special excitations should be described by multi-valued functions, we first concentrate on how to find some types of FSWs and foldons of the field U briefly. A localized functions p_x can be written in the form

$$p_x = \sum_{j=1}^{M} f_j(\xi + v_j t)\,, \quad x = \xi + \sum_{j=1}^{M} g_j(\xi + v_j t)\,, \tag{17}$$

where $v_1 < v_2 < \dots v_M$ are all arbitrary constants and $(f_j, g_j),\ \forall_j$ are all localized functions with the properties $f_j(\pm\infty),\ g_j(\pm\infty) = G_j^{\pm} = constants$. From the second equation of (17), we know that ξ may be a multi-valued functions in some possible regions of x by selecting the functions g_j suitably. Therefore the functions p_x may be a multi-valued function of x in these regions though it is a single valued function of ξ. Besides, p_x is an interaction solution of M localized excitations due to the property $\xi\mid_{x\to\infty}\to\infty$. Actually, most of the known (1+1)-dimensional multi-loop solutions are the special cases of (17). If we take the arbitrary functions of formula (15), we can get various types of FSWs and/or foldons.

In Fig. 1, six typical special FSWs are plotted for the field quantity U shown by (15) with p_x in Eq. (17) and $p = \int^{\xi} p_x x_{\xi} d\xi + K_0$ and the functions q_y, q being given in a similar way

$$q_y = \sum_{j=1}^{M} Q_j(\theta)\,, \quad y = \theta + \sum_{j=1}^{M} R_j(\theta)\,, \quad q = \int^{\theta} q_y y_{\theta} d\theta + H_0\,, \tag{18}$$

where K_0 and H_0 are arbitrary integration constants. In (18), $Q_j(\theta)$ and $R_j(\theta)$, $\forall j$ are localized functions of θ. The more detailed function selections of the figures of the field U (15) are given as

$$p_x = \operatorname{sech}^2(\xi - v_1 t)\,, \tag{19}$$

$$q_y = \operatorname{sech}^2\theta + L_0\operatorname{sech}^6\theta\,, \tag{20}$$

$$x = \xi + k_0\tanh(\xi - v_1 t) + k_1\tanh^2(\xi - v_1 t) + k_2\tanh^3(\xi - v_1 t)\,, \tag{21}$$

$$y = \theta + l_0\tanh\theta + l_1\tanh^2(\theta) + l_2\tanh^3(\theta)\,, \tag{22}$$

$$p = \int^{\xi} p_x x_{\xi} d\xi + K_0\,, \tag{23}$$

$$q = \int^{\theta} q_y y_{\theta} d\theta + H_0\,, \tag{24}$$

with $\lambda = 1,\ a_1 = 2,\ a_2 = 1,\ a_3 = 0$ at $t = 0$. The corresponding selections of the parameters are

$$k_0 = -1.2,\ l_0 = 1, K_0 = 8,\ L_0 = H_0 = k_1 = k_2 = l_1 = l_2 = 0 \tag{25}$$

for Fig. 1(a),

$$k_0 = -1.2,\ l_0 = -1.1, K_0 = 8,\ L_0 = H_0 = k_1 = k_2 = l_1 = l_2 = 0 \tag{26}$$

for Fig. 1(b),

$$k_0 = -1.2,\ k_1 = l_0 = 1,\ K_0 = 8,\ L_0 = H_0 = k_2 = l_1 = l_2 = 0 \tag{27}$$

for Fig. 1(c),

$$L_0 = k_1 = l_1 = 1,\ k_0 = l_0 = 2, k_2 = 0,\ l_2 = -5.5,\ K_0 = H_0 = 50 \tag{28}$$

for Fig. 1(d),

$$L_0 = k_1 = l_1 = 1,\ k_0 = l_0 = 2,\ k_2 = l_2 = -5.5,\ K_0 = H_0 = 50 \tag{29}$$

for Fig. 1(e), and

$$L_0 = k_1 = l_1 = 1,\ k_0 = l_0 = 2,\ k_2 = l_2 = -10,\ K_0 = 50,\ H_0 = 250 \tag{30}$$

for Fig. 1(f).

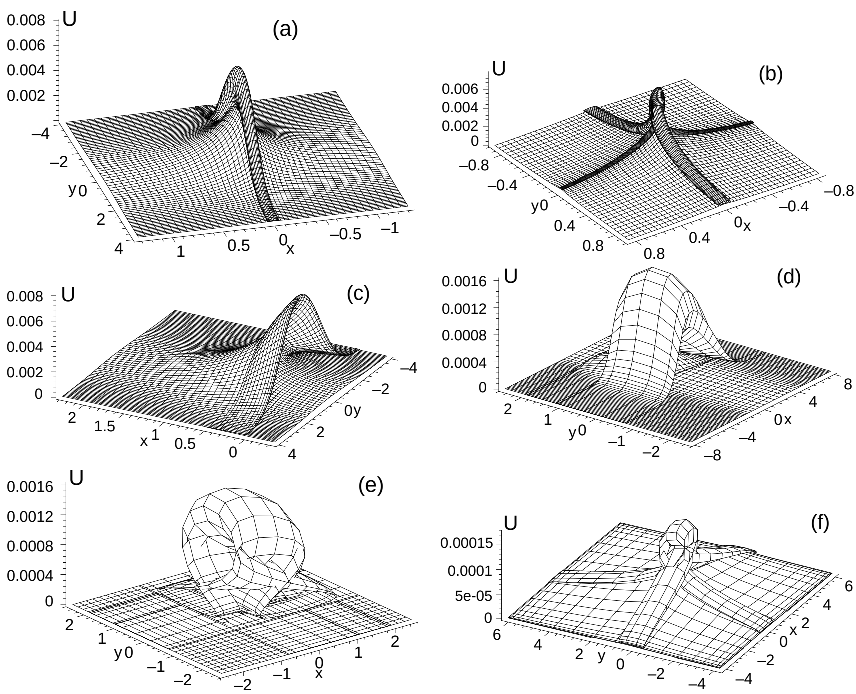

Fig. 1. Six typical FSWs for the field U expressed by (15) with $\lambda = 1$, $a_1 = 2$, $a_2 = 1$, $a_3 = 0$ at t=0 and the selections (19)–(24) and the related concrete parameter selections are given in (25) for (a), (26) for (b), (27) for (c), (28) for (d), (29) for (e) and (30) for (f), respectively.

From Fig. 1 we can see that the FSWs may be "folded" in quite complicated waves and possess rich structures.

Figs. 2(a)–(e) are plotted to show the possible existence of foldons which are given by (15) and the concrete function selections are given

$$p_x = 0.8\mathrm{sech}^2\xi + 0.5\mathrm{sech}^2(\xi - 0.25t)\,, \tag{31}$$

$$q_y = \mathrm{sech}^2\theta\,, \tag{32}$$

$$x = \xi - 1.5\tanh\xi - 1.5\tanh(\xi - 0.25t)\,, \tag{33}$$

$$y = \theta - 2\tanh\theta\,, \tag{34}$$

$$p = \int^{\xi} p_x x_\xi d\xi + 20\,, \tag{35}$$

$$q = \int^{\theta} q_y y_\theta d\theta \tag{36}$$

with $\lambda = 1,\ a_1 = 2,\ a_2 = 1,\ a_3 = 0$ at times (a)$t = -18$,(b)$t = -8$, (c) $t = 0$, (d)$t = 8$, (e) $t = 18$.

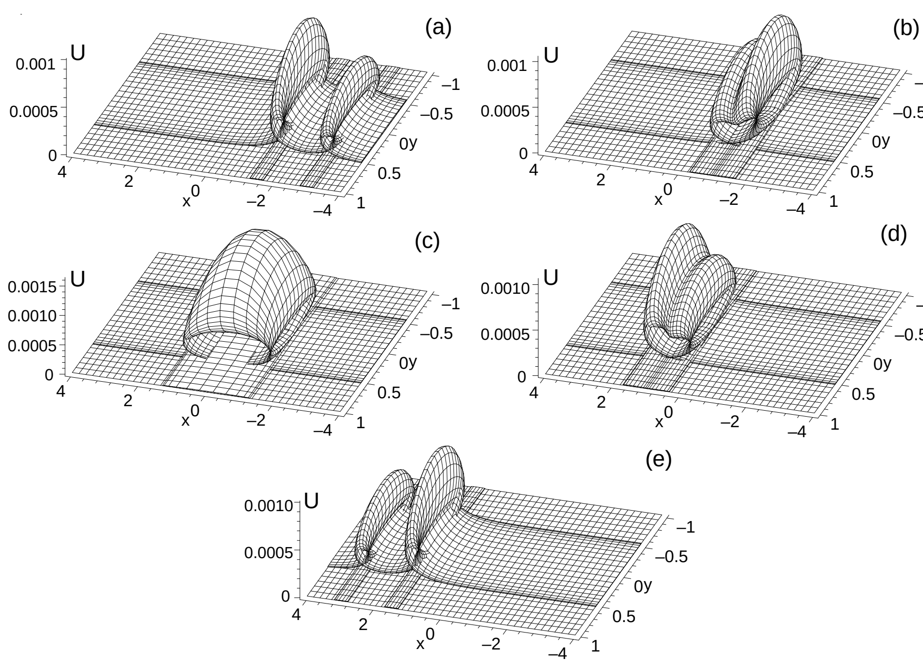

Fig. 2. Evolution plots of two foldon for the field U expressed by (15) with $\lambda = 1,\ a_1 = 2,\ a_2 = 1,\ a_3 = 0$ and the selection (31)–(36) at the times (a) $t = -18$, (b) $t = -8$, (c) $t = 0$, (d) $t = 8$, and (e) $t = 18$, respectively.

From Fig. 2(a) and Fig. 2(e), we can see that the interaction of two foldons is completely elastic. Because one of the velocities of foldons has been selected as zero, which makes it easy to survey their phase shifts. It can also be seen that there are phase shifts for two foldons. Especially, before the interaction, the static foldon (the large one) is located at $x = -1.5$ and after the interaction, the large foldon is shifted to $x = 1.5$.

4. Summary and discussion

In summary, using the variable separation approach, a quite general variable separation solution of the (2+1)-dimensional Maccari systems can be derived. For the (2+1)-dimensional Maccari system, there are folded excitations. Starting from the formula of the module quantity U expressed by (15) of the Maccari system directly, we obtain many types of FSWs and foldons by selecting arbitrary multi-valued functions appropriately. The foldons may be folded quite freely and complicated and possess quite rich structures and interaction behaviors. In reality, there are a large number of complicated folded and/or the multi-valued phenomena, so to find the FSWs and foldons in other high dimensional nonlinear models and put the general (or special) foldons into its real possible applications is worthy of further study.

REFERENCES

[1] S.Y. Lou, H.Y. Ruan, *J. Phys. A: Math. Gen.* **34**, 305 (2001); S.Y. Lou, X.Y. Tang, C.L. Chen, *Chin. Phys. Lett.* **19**, 769 (2002); J. Lin, *Chin. Phys. Lett.* **19**, 765 (2002); S.Y. Lou, *J. Phys. A: Math. Gen.* **35**, 10619 (2002).

[2] A. Maccari, *J. Phys. A: Math. Gen.* **38**, 4151 (1997); D.W.C. Lai, K.W. Chow, *J. Phys. Soc. Jpn.* **70**, 666 (2001).

[3] X.Y. Tang, S.Y. Lou, Y. Zhang, *Phys. Rev.* **E66**, 046601 (2002); S.Y. Lou, C.L. Chen, X.Y. Tang, *J. Math. Phys.* **43**, 4078 (2002).

[4] J.F. Zhang, W.H. Huang, C.L. Zheng, *Acta Phys. Sin.* **51**, 2676 (2002), in Chinese.

[5] J.F. Zhang, J.P. Meng, W.H. Huang, *Commun. Theor. Phys.* **40**, 443 (2003).

[6] V. B. Matvev, M.A. Shalle, *Darboux Transformations and Solitons*, Spinger-Verlag, Berlin, Heidelberg 1991; F. Kako, N. Yajima, *J. Phys. Soc. Jpn.* **49**, 2063 (1980).

[7] H.Y. Ruan, Y.X. Chen, *J. Math. Phys.* **40**, 248 (1999); H.Y. Ruan, Y.X. Chen, *Phys. Rev.* **E62**, 5738 (2000); S.Y. Lou, H.Y. Ruan, *J. Phys. A: Math. Gen.* **24**, 1979 (2001); H.Y. Ruan, *Acta Phys. Sin.* **50**, 586 (2001), in Chinese; H.Y. Ruan, Y.X. Chen, *Z. Naturforsch.* **52a**, 948 (2002); H.Y. Ruan, Y.X. Chen, *Acta Phys. Sin.* **52**, 1313 (2003), in Chinese.

[8] J. Lin, H.M. Li, *Z. Naturforsch. A: Phys. Sci.* **57**, 929 (2002); J. Lin, M.Q. Xian, *Phys. Lett.* **A313**, 93 (2003).

[9] S.C. Trewick, T.F. Henshaw, R.P. Hansinger, T. Lindahl, B. Sedgwick, *Nature* **419**, 174 (6903); S.W. Lockless, R. Ranganathan, *Science* **268**, 295 (1999); P.A. Lindagard, H. Bohr, *Phys. Rev. Lett.* **77**, 779 (1996).

[10] M.B. Goodman, G.G. Ernstrom, D.S. Chelur, R.O. Hagan, C.A. Yao, M. Chalfie, *Nature* **415**, 1039 (2002); B.L. Maclnnis, R.B. Campenot, *Science* **295**, 1536 (2002).

[11] K. Konno, Y. Chkawa, M. Wadati, *J. Phys. Soc. Jpn.* **50**, 1025 (1981).

[12] V.O. Vakhnenko, *J. Phys. A: Math. Gen.* **25**, 4181 (1992); V.O. Vakhnenko, E.J. Parks, *Nonlinearity* **11**, 1457 (1998); A.J. Morrison, E.J. Parks, V.O. Vakhnenko, *Nonlinearity* **12**, 1427 (1999).

[13] H. Kakuhata, K. Konno, *J. Phys. Soc. Jpn.* **68**, 757 (1999).

[14] S. Matsutani, *Mod. Phys. Lett.* **A10**, 717 (1985); *J. Geom. Phys.* **43**, 146 (2002).

[15] M. Schleif, R. Wunsch, *Eur. J. Phys.* **A1**, 171 (1998); M. Schleif, R. Wunsch, T. Meissner, *Int. J. Mod. Phys.* **E7**, 121 (1998).

[16] X.Y. Tang, S.Y. Lou, *Commun. Theor. Phys.* **40**, 62 (2003); X.Y. Tang, S.Y. Lou, *J. Math. Phys.* **44**, 4000 (2003).