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A NEW INTERPRETATION OF ONE CPT VIOLATION TEST FOR K_0–$\bar{K}_0$ SYSTEM

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Using a more accurate approximation than that applied by Lee–Oehme–Yang we show that the interpretation of the tests, measuring the difference between the K_0 mass and the $\bar{K}_0$ mass as the CPT-symmetry test is wrong. We find that in fact such tests should rather be considered as tests for the existence of the hypothetical interaction allowing the first order $|\Delta S| = 2$ transitions $K_0 \rightleftharpoons \bar{K}_0$.

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1. Introduction

CPT symmetry is a fundamental theorem of axiomatic quantum field theory which follows from locality, Lorentz invariance, and unitarity [1]. Many tests of CPT-invariance consist in searching for decay of neutral kaons. The proper interpretation of them is crucial. All known CP- and hypothetically possible CPT-violation effects in neutral kaon complex are described by solving the Schrödinger-like evolution equation [2–9] (we use $\hbar = c = 1$ units)

$$i\frac{\partial}{\partial t}|\psi;t\rangle_{\|} = H_{\|}|\psi;t\rangle_{\|} \tag{1}$$

for $|\psi;t\rangle_{\|}$ belonging to the subspace $\mathcal{H}_{\|} \subset \mathcal{H}$ (where $\mathcal{H}$ is the state space of the physical system under investigation), *e.g.*, spanned by orthonormal neutral kaons states $|K_0\rangle \stackrel{\text{def}}{=} |\mathbf{1}\rangle$, $|\bar{K}_0\rangle \stackrel{\text{def}}{=} |\mathbf{2}\rangle$, and so on, (then states corresponding to the decay products belong to $\mathcal{H}\ominus\mathcal{H}_{\|} \stackrel{\text{def}}{=} \mathcal{H}_{\perp}$). The nonhermitian effective Hamiltonian $H_{\|}$ acts in $\mathcal{H}_{\|}$ and

$$H_{\|} \equiv M - \frac{i}{2}\varGamma\,, \tag{2}$$

where $M = M^{+}$, $\varGamma = \varGamma^{+}$, are (2×2) matrices.

Relations between matrix elements of $H_{\|}$ implied by CPT-transformation properties of the Hamiltonian H of the total system, containing neutral kaon complex as a subsystem, are crucial for designing CPT-invariance and CP-violation tests and for the proper interpretation of their results.

The eigenstates of $H_{\|}$, $|K_{\rm l}\rangle$ and $|K_{\rm s}\rangle$, for the eigenvalues $\mu_{\rm l}$ and $\mu_{\rm s}$, respectively [2–9, 11–21]

$$\mu_{\rm l(s)} = h_0 - (+) h \equiv m_{\rm l(s)} - \frac{i}{2}\gamma_{\rm l(s)}\,, \tag{3}$$

where $m_{\rm l(s)}, \gamma_{\rm l(s)}$ are real, and

$$h_0 \;=\; \frac{1}{2}(h_{11}+h_{22})\,, \tag{4}$$

$$h \;\equiv\; \sqrt{h_z^2 + h_{12}h_{21}}\,, \tag{5}$$

$$h_z \;=\; \frac{1}{2}(h_{11}-h_{22})\,, \tag{6}$$

$$h_{jk} \;=\; \langle \boldsymbol{j}|H_{\|}|\boldsymbol{k}\rangle\,, \quad (j,k=1,2)\,, \tag{7}$$

correspond to the long (the vector $|K_{\rm l}\rangle$) and short (the vector $|K_{\rm s}\rangle$) living superpositions of K_0 and $\bar{K}_0$.

Using the eigenvectors

$$|K_{1(2)}\rangle \stackrel{\rm def}{=} 2^{-1/2}(|\boldsymbol{1}\rangle + (-)|\boldsymbol{2}\rangle) \tag{8}$$

of the CP-transformation for the eigenvalues ± 1 (we define $\mathcal{CP}|\boldsymbol{1}\rangle = -|\boldsymbol{2}\rangle$, $\mathcal{CP}|\boldsymbol{2}\rangle = -|\boldsymbol{1}\rangle$), vectors $|K_{\rm l}\rangle$ and $|K_{\rm s}\rangle$ can be expressed as follows [4,6,10,14]:

$$|K_{\rm l(s)}\rangle \equiv \left(1+|\varepsilon_{\rm l(s)}|^2\right)^{-1/2}\left[|K_{2(1)}\rangle + \varepsilon_{\rm l(s)}|K_{1(2)}\rangle\right]\,, \tag{9}$$

where

$$\varepsilon_{\rm l} \;=\; \frac{h_{12}-h_{11}+\mu_{\rm l}}{h_{12}+h_{11}-\mu_{\rm l}} \equiv -\frac{h_{21}-h_{22}+\mu_{\rm l}}{h_{21}+h_{22}-\mu_{\rm l}}\,, \tag{10}$$

$$\varepsilon_{\rm s} \;=\; \frac{h_{12}+h_{11}-\mu_{\rm s}}{h_{12}-h_{11}+\mu_{\rm l}} \equiv -\frac{h_{21}+h_{22}-\mu_{\rm s}}{h_{21}-h_{22}+\mu_{\rm s}}\,. \tag{11}$$

This form of $|K_{\rm l}\rangle$ and $|K_{\rm s}\rangle$ is used in many papers when possible departures from CP- or CPT-symmetry in the system considered are discussed.

One can easily verify that if $\mu_{\rm l} \neq \mu_{\rm s}$ then:

$$h_{11} = h_{22} \iff \varepsilon_{\rm l} = \varepsilon_{\rm s}\,. \tag{12}$$

The experimentally measured values of the parameters $\varepsilon_{\rm l}, \varepsilon_{\rm s}$ are very small for neutral kaons. Assuming

$$|\varepsilon_{\rm l}| \ll 1\,, \qquad |\varepsilon_{\rm s}| \ll 1\,, \tag{13}$$

one can, *e.g.*, find:

$$h_{11} - h_{22} \simeq (\mu_{\rm s} - \mu_{\rm l})(\varepsilon_{\rm s} - \varepsilon_{\rm l})\,. \tag{14}$$

Keeping in mind that $h_{jk} = M_{jk} - \frac{i}{2}\mathit{\Gamma}_{jk}$, $M_{kj} = M^*_{jk}$, $\mathit{\Gamma}_{kj} = \mathit{\Gamma}^*_{jk}$ (where $M_{jj} \stackrel{\rm def}{=} M_j$, $(j = 1, 2)$) and using (14) one can find, among others that [10,14]

$$\begin{aligned}\Re\,(h_{11} - h_{22}) \equiv M_1 - M_2 \;\simeq\; -(\gamma_{\rm s} - \gamma_{\rm l}) \Bigg[\tan\phi_{SW}\, \Re\, \left(\frac{\varepsilon_{\rm s} - \varepsilon_{\rm l}}{2} \right) \\ -\Im\, \left(\frac{\varepsilon_{\rm s} - \varepsilon_{\rm l}}{2} \right) \Bigg]\,,\end{aligned} \tag{15}$$

where $\Re\,(z)$ denotes the real part, $\Im\,(z)$ — the imaginary part of a complex number (z), and

$$\tan\phi_{SW} \stackrel{\rm def}{=} \frac{2(m_{\rm l} - m_{\rm s})}{\gamma_{\rm s} - \gamma_{\rm l}}\,. \tag{16}$$

Usually it is assumed that M_1, M_2 denote the masses of the particle "1" and its antiparticle "2" [2–15]).

One should remember that relations (14), (15) are valid only if condition (13) holds. On the other hand it is appropriate to emphasize at this point that all relations (9)–(15) do not depend on the specific form of the effective Hamiltonian $H_{||}$ and that they do not depend on the approximation used to calculate $H_{||}$. They are induced by geometric relations between various base vectors in two-dimensional subspace $\mathcal{H}_{||}$ but the interpretation of the relation, *e.g.*, (15), depends on properties of the matrix elements h_{jk} of the effective Hamiltonian $H_{||}$. This means that, if for example, $H_{||} \neq H_{\rm LOY}$, where $H_{\rm LOY}$ is the Lee–Oehme–Yang effective Hamiltonian, then the interpretation of the relation (15), *e.g.*, as the CPT symmetry test, and other similar relations need not be the same for $H_{||}$ and for $H_{\rm LOY}$.

The aim of this note is to analyze the interpretation of the test based on the relation (15) which is commonly considered as the CPT violation test [4–6, 10, 14, 15, 17]. Such an interpretation follows from the properties of the matrix elements of $H_{\rm LOY}$.

In the kaon rest frame, the time evolution is governed by the Schrödinger equation

$$i\frac{\partial}{\partial t}U(t)|\phi\rangle = HU(t)|\phi\rangle\,, \quad U(0) = I\,, \tag{17}$$

where I is the unit operator in $\mathcal{H}$, $|\phi\rangle \equiv |\phi; t_0 = 0\rangle \in \mathcal{H}$ is the initial state of the system:

$$|\phi\rangle \equiv |\psi\rangle_{||} \equiv q_1|\boldsymbol{1}\rangle + q_2|\boldsymbol{2}\rangle \in \mathcal{H}_{||} \subset \mathcal{H} \tag{18}$$

(in our case $|\phi; t\rangle = U(t)|\phi\rangle$), H is the total (selfadjoint) Hamiltonian, acting in $\mathcal{H}$. Thus the total evolution operator $U(t)$ is unitary.

Starting from the Schrödinger equation and using the Weisskopf–Wigner method Lee, Oehme and Yang derived the following formula for the matrix elements $h_{jk}^{\rm LOY}$, $(j,k = 1,2)$ of their effective Hamiltonian $H_{\rm LOY}$ (see [2–4, 16]):

$$h_{jk}^{\rm LOY} = m_0\,\delta_{j,k} - \Sigma_{jk}(m_0) \tag{19}$$

$$= M_{jk}^{\rm LOY} - \frac{i}{2}\varGamma_{jk}^{\rm LOY}\,, \quad (j,k = 1,2)\,, \tag{20}$$

where $\Sigma_{jk}(\epsilon) = \langle \boldsymbol{j}|\Sigma(\epsilon)|\boldsymbol{k}\rangle$, $(j,k = 1,2)$, and

$$\Sigma(\epsilon) = PHQ\frac{1}{QHQ - \epsilon - i0}QHP = \Sigma^R(\epsilon) + i\Sigma^I(\epsilon)\,, \tag{21}$$

and $\Sigma^R(\epsilon = \epsilon^*) = \Sigma^R(\epsilon = \epsilon^*)^+$, $\Sigma^I(\epsilon = \epsilon^*) = \Sigma^I(\epsilon = \epsilon^*)^+$, P is the projector operator onto the subspace $\mathcal{H}_{||}$:

$$P \equiv |\boldsymbol{1}\rangle\langle\boldsymbol{1}| + |\boldsymbol{2}\rangle\langle\boldsymbol{2}|\,, \tag{22}$$

Q is the projection operator onto the subspace of decay products $\mathcal{H}_\perp$:

$$Q \equiv I - P\,, \tag{23}$$

and vectors $|\boldsymbol{1}\rangle$, $|\boldsymbol{2}\rangle$ considered above are the eigenstates of the free Hamiltonian, $H^{(0)}$, (here $H \equiv H^{(0)} + H^I$), for 2-fold degenerate eigenvalue m_0:

$$H^{(0)}|\boldsymbol{j}\rangle = m_0|\boldsymbol{j}\rangle\,, \qquad (j = 1,2)\,, \tag{24}$$

H^I denotes the interaction which is responsible for the decay process. This means that

$$\left[P, H^{(0)}\right] = 0\,. \tag{25}$$

The condition guaranteeing the occurrence of transitions between subspaces $\mathcal{H}_{||}$ and $\mathcal{H}_\perp$, *i.e.*, the decay process of states in $\mathcal{H}_{||}$, can be written as follows

$$\left[P, H^I\right] \neq 0\,, \tag{26}$$

that is

$$[P, H] \neq 0\,. \tag{27}$$

The operator $H_{\rm LOY}$ has the following form

$$H_{\rm LOY} = PHP - \Sigma(m_0) \equiv PHP - \Sigma(m_0)\,, \tag{28}$$

where $PHP \equiv PH^{(0)}P + PH^I P$, and, in the considered case

$$PH^{(0)}P \equiv m_0 P\,, \tag{29}$$

and (see [2–4])

$$PH^I P \equiv 0\,. \tag{30}$$

Following the method used by LOY, assumption (30) may be either kept or dropped. This has no effect on the CP- and CPT-transformation properties of the matrix elements of $H_{\rm LOY}$ and conclusions derived in this paper. The assumption (30) is a reflection of the opinion of physicists following the ideas of [2] and deriving and then applying $H_{\rm LOY}$ that matrix elements of H^I are too small in comparison with m_0 in order to have any measurable effect on time evolution in $\mathcal{H}_{||}$ (see [2–4]). In some papers (see, *e.g.*, [5–7, 17]), instead of (30) the assumption that $PH^I P \neq 0$ is introduced into formulae for matrix elements of $H_{\rm LOY}$. It appears that the properties of such $H_{\rm LOY}$ do not differ from the properties of the LOY effective Hamiltonian derived within the use of the assumption (30). In other words, the approximation applied by LOY in [2] leads to the operator $H_{\rm LOY}$ whose CP- and CPT-transformation properties do not depend on whether $PH^I P \neq 0$, that is $H_{12} \stackrel{\rm def}{=} \langle \boldsymbol{1}|H|\boldsymbol{2}\rangle \neq 0$, or not. (Note that within the LOY assumptions $H_{12} \equiv \langle \boldsymbol{1}|H^I|\boldsymbol{2}\rangle$.) This property of the LOY approximation means that, for example, the verification of the presence (or absence) of the hypothetical superweak interactions causing the direct, first order $K_0 \rightleftharpoons \bar{K}_0$, $|\Delta S| = 2$, transitions [18, 19] in experiments designed within use of the LOY theory is very difficult and that the interpretation of results of such experiments cannot be considered as definitive. The same refers to the problem of how to identify within the LOY theory effects caused by such hypothetical interactions and similar effects predicted by the Standard Model (SM) [20–22]. In this place, in order to avoid possible misunderstandings, one should explain that from the point of view of the problems discussed in this paper, SM effective Hamiltonians, $H_{\rm eff}$, for the problem under consideration, should be identified with $H_{||}$ (or $H_{\rm LOY}$) but they cannot be identified with the operator PHP.

Usually, in LOY and related approaches, it is assumed that

$$\Theta H^{(0)} \Theta^{-1} = H^{(0)\,+} \equiv H^{(0)}\,, \tag{31}$$

where Θ is the anti-unitary operator [23–25]:

$$\Theta \stackrel{\rm def}{=} \mathcal{CPT}\,. \tag{32}$$

The subspace of neutral kaons $\mathcal{H}_{\|}$ is assumed to be invariant under Θ:

$$\Theta P \Theta^{-1} = P^{+} \equiv P\,. \tag{33}$$

Now, if in addition to (31) $\Theta H^{I} \Theta^{-1} = H^{I}$, that is if

$$[\Theta, H] = 0\,, \tag{34}$$

then using, *e.g.*, the following phase convention

$$\Theta|\boldsymbol{1}\rangle \stackrel{\text{def}}{=} \mathrm{e}^{i\alpha_{\Theta}}|\boldsymbol{2}\rangle, \quad \Theta|\boldsymbol{2}\rangle \stackrel{\text{def}}{=} \mathrm{e}^{i\alpha_{\Theta}}|\boldsymbol{1}\rangle\,, \tag{35}$$

and taking into account that $\langle\psi|\varphi\rangle = \langle\Theta\varphi|\Theta\psi\rangle$, one easily finds from (19), (21) that

$$h_{11}^{\mathrm{LOY}} - h_{22}^{\mathrm{LOY}} = 0 \tag{36}$$

in the CPT-invariant system. This is the standard result of the LOY approach and this is the picture which one meets in the literature [2–9, 14, 17]. Property (36) leads to the conclusion that (see (12))

$$\varepsilon_{\mathrm{l}} - \varepsilon_{\mathrm{s}} = 0\,. \tag{37}$$

Therefore the tests based on the relation (15) are considered as the test of CPT-invariance and the results of such tests are interpreted that the masses of the particle "1" (the K_0 meson) and its antiparticle "2" (the $\bar{K}_0$ meson) must be equal if CPT-symmetry holds. Parameters appearing in the right hand side of the relation (15) can be extracted from experiments in such tests and then these parameters can be used to estimate the left side of this relation. The estimation for the mass difference obtained in this way with the use of the recent data [15] reads

$$\frac{|M_1 - M_2|}{m_{K_0}} = \frac{|m_{K_0} - m_{\bar{K}_0}|}{m_{K_0}} \leq 10^{-18} \tag{38}$$

and this estimation is considered as indicating no CPT-violation effect. This interpretation follows from the properties of the H_{LOY}.

Note that in fact the above interpretation of the relation (38) could be considered as the confirmation of the CPT invariance only if the property (36) were the exact relation. The accuracy of the LOY approximation was discussed, *e.g.*, in [9, 26–29]. According to these and other papers one can determine all parameters needed for the time evolution formulae in LOY theory up to the accuracy of $\frac{\Gamma_X}{m_X} \sim 10^{-15}$, where $X = K_{\mathrm{s}}, K_{\mathrm{l}}$, in terms of known quantities. In the light of such considerations an especial meaning has the rigorous result that for the exact effective Hamiltonian h_{11} cannot be equal to h_{22} in CPT invariant and CP noninvariant system (see [30]). All these considerations show that the interpretation of tests for neutral K subsystem within the LOY theory can have more than one meaning and it cannot be considered as crucial. This remark concerns especially tests of type (15) (where the result is of order given in (38)). So one should look for much more accurate approximations describing the time evolution in neutral K complex and should try to interpret results of tests, *e.g.*, of type (15) within the use of this more accurate approximation and of the exact theory.

2. Beyond the LOY approximation

The more exact approximate formulae for $H_{||}$ than those obtained within the LOY approach (*i.e.* than $H_{\rm LOY}$) can be derived using the Krolikowski–Rzewuski (KR) equation for the projection of a state vector [31,32,11–13,33]. KR equation results from the Schrödinger equation (17) for the total system under consideration [31] (see also, *e.g.*, [34]) and it is the exact evolution equation for the subspace $\mathcal{H}_{||} \subset \mathcal{H}$. In the case of initial conditions of the type (18), KR equation takes the following form

$$\left(i\frac{\partial}{\partial t} - PHP\right) U_{\|}(t)|\psi\rangle_{||} = -i\int_0^\infty K(t-\tau)U_{\|}(\tau)|\psi\rangle_{||}d\tau\,, \qquad (39)$$

where $U_{||}(t) \equiv PU(t)P$ is the non-unitary evolution operator for the subspace $\mathcal{H}_{||}$, $U_{||}(t)|\psi\rangle_{||} = |\psi;t\rangle_{||} \in \mathcal{H}_{||}$, $U_{\|}(0) = P$, and

$$\begin{aligned} K(t) &= \Theta(t)PHQ\exp(-itQHQ)QHP\,, \qquad (40)\\ \Theta(t) &= \{1 \text{ for } t \geq 0, \;\; 0 \text{ for } t < 0\}\,. \end{aligned}$$

The integro-differential equation (39) can be replaced by the following differential one (see [31, 32, 11–13, 33])

$$\left(i\frac{\partial}{\partial t} - PHP - V_{||}(t)\right) U_{\|}(t)|\psi\rangle_{||} = 0\,. \qquad (41)$$

This equation has the required form of the Schrödinger-like equation (1) with the effective Hamiltonian $H_{||}$, which in general depends on time t [31, 35],

$$H_{||} \equiv H_{\|}(t) \stackrel{\rm def}{=} PHP + V_{\|}(t)\,. \qquad (42)$$

In the case (27), to the lowest nontrivial order, the following formula for $V_{||}(t)$ has been found in [12, 32]

$$V_{\|}(t) \cong V_{\|}^{(1)}(t) \stackrel{\rm def}{=} -i\int_0^\infty K(t-\tau)\exp\left[i(t-\tau)PHP\right]d\tau\,. \qquad (43)$$

All steps leading to the approximate formula (43) for $V_{||}^{(1)}(t)$ are well defined (see [12, 32]).

We are rather interested in the properties of the system at long time period, at the same for which the LOY approximation was calculated, and therefore we will consider the properties of

$$V_{||} \stackrel{\text{def}}{=} \lim_{t\to\infty} V_{||}^{(1)}(t) \tag{44}$$

instead of the general case $V_{||}(t) \cong V_{||}^{(1)}(t)$.

For simplicity we assume that the CPT-symmetry is conserved in our system, which is that the condition (34) holds. The consequence of this assumption is that

$$H_{11} = H_{22} \stackrel{\text{def}}{=} H_0\,, \tag{45}$$

where

$$H_{jk} = \langle \boldsymbol{j}|H|\boldsymbol{k}\rangle\,, \tag{46}$$

and $(j, k = 1, 2)$. In this case the matrix elements of $\varSigma(\epsilon)$ have the following properties [12, 13, 36]

$$\varSigma_{11}(\epsilon = \epsilon^*) \equiv \varSigma_{22}(\epsilon = \epsilon^*) \stackrel{\text{def}}{=} \varSigma_0(\epsilon = \epsilon^*)\,. \tag{47}$$

So, in the case of the projector P given by the formula (22) for

$$PHP \equiv H_0\,P\,, \tag{48}$$

that is for

$$H_{12} = H_{21} = 0\,, \tag{49}$$

one finds that

$$V_{||} = -\varSigma(H_0)\,. \tag{50}$$

This means that in the case (48)

$$H_{||} = H_0\,P - \varSigma(H_0)\,, \tag{51}$$

(where $H_{||} \stackrel{\text{def}}{=} \lim_{t\to\infty} H_{||}(t) \equiv PHP + \lim_{t\to\infty} V_{||}(t)$), which is exactly as in the LOY approach (see (28)). This also means that in such a case simply $(h_{11} - h_{22}) = 0$ when CPT symmetry is conserved.

On the other hand, in the case

$$H_{12} = H_{21}^* \neq 0\,, \tag{52}$$

and conserved CPT, one obtains [33]

$$\begin{aligned} V_{||} \;=\; & -\frac{1}{2}\Sigma\left(H_0+|H_{12}|\right)\left[\left(1-\frac{H_0}{|H_{12}|}\right)P+\frac{1}{|H_{12}|}PHP\right] \\ & -\frac{1}{2}\Sigma(H_0-|H_{12}|)\left[\left(1+\frac{H_0}{|H_{12}|}\right)P-\frac{1}{|H_{12}|}PHP\right]. \end{aligned} \tag{53}$$

Matrix elements $v_{jk}=\langle \boldsymbol{j}|V_{||}|\boldsymbol{k}\rangle$, $(j,k=1,2)$ of this $V_{||}$ we are interested in take the following form

$$\begin{aligned} v_{j1} = \; - \; & \frac{1}{2}\Big\{\Sigma_{j1}\left(H_0+|H_{12}|\right)+\Sigma_{j1}(H_0-|H_{12}|) \\ + \; & \frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0+|H_{12}|)-\frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0-|H_{12}|)\Big\}, \\ v_{j2} = \; - \; & \frac{1}{2}\Big\{\Sigma_{j2}(H_0+|H_{12}|)+\Sigma_{j2}(H_0-|H_{12}|) \\ + \; & \frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0+|H_{12}|)-\frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0-|H_{12}|)\Big\}. \end{aligned} \tag{54}$$

The form of these matrix elements is rather inconvenient for searching for their properties depending on the matrix elements H_{12} of PHP. It can be done relatively simply assuming [13, 36]

$$|H_{12}| \ll |H_0|\,. \tag{55}$$

Within such an assumption one finds [13, 36]

$$v_{j1} \simeq -\,\Sigma_{j1}(H_0)-H_{21}\frac{\partial\Sigma_{j2}(x)}{\partial x}\bigg|_{x=H_0}\,, \tag{56}$$

$$v_{j2} \simeq -\,\Sigma_{j2}(H_0)-H_{12}\frac{\partial\Sigma_{j1}(x)}{\partial x}\bigg|_{x=H_0}\,, \tag{57}$$

where $j=1,2$. One should stress that due to the presence of resonance terms, derivatives $\frac{\partial}{\partial x}\Sigma_{jk}(x)$ need not be small, and so the products $H_{jk}\frac{\partial}{\partial x}\Sigma_{jk}(x)$ in (56), (57).

From this formulae we conclude that, *e.g.*, the difference between the diagonal matrix elements which plays an important role in designing tests of type (15) for the neutral kaons system, equals to the lowest order of $|H_{12}|$,

$$h_{11}-h_{22} \simeq H_{12}\frac{\partial\Sigma_{21}(x)}{\partial x}\bigg|_{x=H_0} - H_{21}\frac{\partial\Sigma_{12}(x)}{\partial x}\bigg|_{x=H_0} \neq 0\,. \tag{58}$$

So, in a general case, in contradiction to the property (36) obtained within the LOY theory, one finds for diagonal matrix elements of $H_{\|}$ calculated within the above described approximation that in CPT-invariant system the nonzero matrix elements, $H_{12} \neq 0$, of PHP cause that $(h_{11} - h_{22}) \neq 0$.

From the formula (58) it follows that the left side of the relation (15) takes the following form in the case of very weak interactions allowing for the nonzero first order transitions $|\boldsymbol{1}\rangle \rightleftharpoons |\boldsymbol{2}\rangle$

$$M_1 - M_2 = \Re(h_{11} - h_{22}) = 2\,\Im\left(H_{21} \frac{\partial \varSigma_{12}^{I}(x)}{\partial x}\bigg|_{x=H_0} \right) + \ldots \neq 0\,. \tag{59}$$

(Note that as a matter of fact assuming (24) one has $H_{21} \equiv \langle \boldsymbol{2}|H^I|\boldsymbol{1}\rangle$ in (59).) Thus taking into account this result and the implications of the assumptions (48), (49) one can conclude that

$$\Re\,(h_{11} - h_{22}) = 0 \Leftrightarrow |H_{12}| = 0 \tag{60}$$

within the considered approximation. Finally, using result (59) one can replace the relation (15) by the following one:

$$\begin{aligned} 2\,\Im\left(\langle \boldsymbol{2}|H^I|\boldsymbol{1}\rangle \frac{\partial \varSigma_{12}^{I}(x)}{\partial x}\bigg|_{x=H_0} \right) \simeq{}& -(\gamma_{\mathrm{s}} - \gamma_{\mathrm{l}}) \Bigg[\tan\phi_{SW}\, \Re\left(\frac{\varepsilon_{\mathrm{s}} - \varepsilon_{\mathrm{l}}}{2} \right) \\ & - \Im\left(\frac{\varepsilon_{\mathrm{s}} - \varepsilon_{\mathrm{l}}}{2} \right) \Bigg]. \end{aligned} \tag{61}$$

3. Discussion

The results (58) and (59) are in full agreement with the conclusions drawn in [30] on the ground of basic assumptions of quantum theory. Note that similar relation to (58) was obtained for CPT invariant system in [37] by means of another, more accurate than LOY, approximation. In [30] it has been shown that the diagonal matrix elements of the exact effective Hamiltonian governing the time evolution in the subspace of states of an unstable particle and its antiparticle cannot be equal at $t > t_0 = 0$ (t_0 is the instant of creation of the pair) when the total system under consideration is CPT invariant but CP noninvariant. The proof of this property is rigorous. The unusual consequence of this result is that in such a case, contrary to the properties of stable particles, the masses of the unstable particle "1" and its antiparticle "2" need not be equal for $t \gg t_0$. In fact there is nothing strange in this conclusion. From (34) (or from the CPT theorem [1] of axiomatic quantum field theory) it only follows that the masses of particle and antiparticle eigenstates of H (*i.e.*, masses of stationary states for H) should be the same in the CPT invariant system. Such a conclusion cannot be drawn from (34) for particle "1" and its antiparticle "2" if they are unstable, *i.e.*, if states $|\boldsymbol{1}\rangle, |\boldsymbol{2}\rangle$ are not eigenstates of H. There is no axiomatic quantum field theory of unstable particles.

In this place one should explain that the property $H_{12} = H_{21} = 0$, which according to (58) implies that $(h_{11} - h_{22}) = 0$ in the considered approximation, does not mean that the predictions following from the use of the exact effective Hamiltonian (or the more accurate effective Hamiltonian than the LOY theory) should lead to the same masses for K_0 and for $\bar{K}_0$. This does not contradict the above mentioned conclusion about masses of unstable particles drawn in [30] for the exact $H_{||}$: Simply, in the case $H_{12} = H_{21} = 0$ the mass difference is very, very small and should arise at higher orders of the more accurate approximation described in Sec. 2.

Using the above, briefly described formalism, one can find $(h_{11} - h_{22})$ for the generalized Fridrichs–Lee model [9, 12]. Within this toy model one finds [36]

$$\begin{aligned} \Re\,(h_{11} - h_{22}) \stackrel{\text{def}}{=} \Re\,(h_{11}^{\text{FL}} - h_{22}^{\text{FL}}) \;&\simeq\; i\frac{m_{21}\varGamma_{12} - m_{12}\varGamma_{21}}{4(m_0 - \mu)} \\ &\equiv\; \frac{\Im\,(m_{12}\varGamma_{21})}{2(m_0 - \mu)}\,. \end{aligned} \tag{62}$$

This estimation has been obtained in the case of conserved CPT-symmetry for $|m_{12}| \ll (m_0 - \mu)$, which corresponds to (55). In (62) $\varGamma_{12}, \varGamma_{21}$ can be identified with those appearing in the LOY theory, $m_0 \equiv H_{11} = H_{22}$ can be considered as the kaon mass [9], $m_{jk} \equiv H_{jk}\,(j,k = 1,2)$, μ can be treated as the mass of the decay products of the neutral kaon [9].

For neutral K-system, to evaluate $(h_{11}^{\text{FL}} - h_{22}^{\text{FL}})$ one can follow, *e.g.*, [9,14] and one can take $\frac{1}{2}\varGamma_{21} = \frac{1}{2}\varGamma_{12}^{*} \sim \frac{1}{2}\varGamma_{\text{s}} \sim 5 \times 10^{10}\text{sec}^{-1}$ and $(m_0 - \mu) = m_K - 2m_\pi \sim 200$ MeV $\sim 3 \times 10^{23}\text{sec}^{-1}$ [15]. Thus

$$\Re\,(h_{11} - h_{22}) \sim \frac{\varGamma_{\text{s}}}{4(m_K - 2m_\pi)}\,\Im\,(H_{12})\,, \tag{63}$$

that is,

$$\left|\Re\,\left(h_{11}^{\text{FL}} - h_{22}^{\text{FL}}\right)\right| \sim 1,7 \times 10^{-13}\,|\Im\,(m_{12})| \equiv 1,7 \times 10^{-13}\,|\Im\,(H_{12})|\,. \tag{64}$$

Note that the relation (63) is equivalent to the following one

$$\Re\,(h_{11} - h_{22}) \sim -i\frac{\varGamma_{\text{s}}}{4(m_K - 2m_\pi)}\langle \boldsymbol{1}|H_{-}|\boldsymbol{2}\rangle\,, \tag{65}$$

where H_- is the CP odd part of the total Hamiltonian $H \equiv H_+ + H_-$. There are $H_- \stackrel{\text{def}}{=} \frac{1}{2}[H - (\mathcal{CP})H(\mathcal{CP})^+]$ and $H_+ \stackrel{\text{def}}{=} \frac{1}{2}[H + (\mathcal{CP})H(\mathcal{CP})^+]$

(see [17, 21]). H_+ denotes the CP even part of H. We have $\langle \mathit{1}|H_-|\mathit{2}\rangle \equiv i\Im(\langle \mathit{1}|H|\mathit{2}\rangle) = i\,\Im(H_{12})$. According to the literature, in the case of the superweak model for CP violation it should be $\langle \mathit{1}|H_-|\mathit{2}\rangle \equiv i\Im(H_{12}) \neq 0$ to the lowest order and $\langle \mathit{1}|H_-|\mathit{2}\rangle = 0$ in the case of a miliweak model [17, 21].

For the Fridrichs–Lee model it has been found in [12] that $h_{jk}(t) \simeq h_{jk}$ practically for $t \geq T_{as} \simeq \frac{10^2}{\pi(m_0 - |m_{12}| - \mu)}$. This leads to the following estimation of T_{as} for the neutral K-system: $T_{as} \sim 10^{-22}$ sec.

Dividing both sides of (64) by m_0 one arrives at the relation corresponding to (38):

$$\frac{\left|\Re\left(h_{11}^{\mathrm{FL}} - h_{22}^{\mathrm{FL}}\right)\right|}{m_0} \sim 1,7 \times 10^{-13}\frac{|\Im(m_{12})|}{m_0} \equiv 1,7 \times 10^{-13}\frac{|\Im(H_{12})|}{m_0}\,. \quad (66)$$

So, if we suppose for a moment that the result (38) is the only experimental result for neutral K complex then it is sufficient for $\frac{|\Im(H_{12})|}{m_0}$ to be $\frac{|\Im(H_{12})|}{m_0} < 10^{-5}$ in order to fulfill the estimation (38). Of course this could be considered as the upper bound for a possible value of the ratio $\frac{|\Im(H_{12})|}{m_0}$ only if there were no other experiments and no other data for the $K_0, \bar{K}_0$ complex. Note that from such a point o view the suitable order of $\frac{|\Im(H_{12})|}{m_0}$ is easily reached by the hypothetical Wolfenstein superweak interactions [18], which admits first order $|\Delta S| = 2$ transitions $K_0 \rightleftharpoons \bar{K}_0$, that is, which assumes a non-vanishing first order transition matrix $H_{12} = \langle \mathit{1}|H^I|\mathit{2}\rangle \sim gG_F \neq 0$ with $g \ll G_F$. The more realistic estimation for $\frac{|\Im(H_{12})|}{m_0}$ can be found using the property $\frac{|\Im(H_{12})|}{m_0} \equiv \frac{|\langle \mathit{1}|H_-|\mathit{2}\rangle|}{m_0}$. One can assume that $\frac{|\langle \mathit{1}|H_-|\mathit{2}\rangle|}{m_0} \sim \frac{H_-}{H_{\mathrm{strong}}}$. There is $\frac{H_-}{H_{\mathrm{strong}}} \sim 10^{-14}|\varepsilon|$ for the case of the hypothetical superweak interactions (see [17], formula (15.138)) and thus $\frac{|\Im(H_{12})|}{m_0} \sim 10^{-14}|\varepsilon|$. (Using this last estimation one should remember that it follows from the LOY theory of neutral K complex.) This estimation allows one to conclude that

$$\frac{\left|\Re\left(h_{11}^{\mathrm{FL}} - h_{22}^{\mathrm{FL}}\right)\right|}{m_0} \sim 1,7 \times 10^{-27}|\varepsilon|\,. \quad (67)$$

This estimation is the estimation of type (38) and it can be considered as a lower bound for $\frac{|\Re(h_{11} - h_{22})|}{m_0}$ (see also [38]).

Note that contrary to the approximation described in Sec. 2, the LOY approximation, as well as the similar approximation leading to the Bell–Strinberger unitary relations [39] are unable to detect and correctly identify effects caused by the existence (or absence) of the superweak interactions (the interactions for which $H_{12} \neq 0$) in the system.

Let us analyze some important observations following from (59), (61) and from the rigorous result obtained in [30]. The non-vanishing of the right hand side of the relation (15) cannot be considered as the proof that the CPT-symmetry is violated. So, there are two general conclusions following from (59), (60), (61) and [30]. The first one: the tests based on the relation (15) cannot be considered as CPT-symmetry tests and this is the main conclusion of this paper. The second one: such tests should rather be considered as the tests for the existence of new hypothetical (superweak (?)) interactions allowing for the first order $|\Delta S| = 2$ transitions. Simply, the left hand side of the relation (59) can differ from zero only if the matrix element $\langle \boldsymbol{2}|H|\boldsymbol{1}\rangle$ is different from zero and thus the nonzero value of the right hand side of the relation (61) means that it should be $\langle \boldsymbol{2}|H^I|\boldsymbol{1}\rangle \neq 0$.

Note that within the LOY theory one can also obtain nonzero first order $|\Delta S| = 2$ transitions in Standard Model for $K_0 - \bar{K}_0$ complex [22]. The main difference between such an effect and the effect discussed in this paper and connected with the relations (15), (59)–(61) is that within the LOY theory the first order $|\Delta S| = 2$ transitions can appear only for off-diagonal matrix elements, $h_{jk}^{\rm LOY}$, $(j \neq k)$, of the effective Hamiltonian, $H_{\rm LOY}$, whereas within the more accurate approximation, discussed in the previous section, diagonal matrix elements, h_{11}, h_{22}, as well as off-diagonal matrix elements of the effective Hamiltonian $H_{||}$ depend on H_{12}, H_{21}. Within the LOY approach, diagonal matrix elements of $H_{\rm LOY}$ do not depend on H_{12}, H_{21}. Therefore the effect discussed in this paper is absent in the LOY theory.

On the other hand, one should remember that the non-vanishing right hand side of the relations (15), (61) can be considered as the conclusive proof that new interactions allowing for the first order $|\Delta S| = 2$ transitions $K_0 \rightleftharpoons \bar{K}_0$ exist only if an another experiment, based on other principles, definitively confirms that the CPT-symmetry is not violated in $K_0 - \bar{K}_0$ system.

Unfortunately the accuracy of the today's experiments is not sufficient to improve the estimation (38) to the order required by (66). This especially concerns the accuracy required by our "more realistic estimation" for $\frac{|\Im\,(H_{12})|}{m_0}$. Simply it is beyond today's experiments reach. In the light of the above estimations, keeping in mind (59), only much more accurate tests based on the relation (15) can give the answer whether the mentioned new hypothetical interactions exists or not.

Last remark, other results [12, 13, 36] obtained within the approximation described in Sec. 2 suggest also that the form of other parameters usually used to describe properties of $K_0 - \bar{K}_0$ system is different for the case $H_{12} \neq 0$ and for the case $H_{12} = 0$. This can be used as the basis for designing other tests for the hypothetical new interactions.

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