THE DEPENDENCE OF THE NUCLEON–NUCLEON SCATTERING AMPLITUDE ON THE MOMENTUM TRANSFER

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The ratio of the real part to the imaginary part of the nucleon–nucleon elastic scattering amplitude is studied in terms of powers of the momentum transfer squared q^2. This study includes the dependence on the phase variation of the nucleon–nucleon elastic scattering amplitudes. The Gaussian form of the effective nucleon–nucleon interaction is used to calculate the nucleon–nucleon elastic scattering amplitudes. Analytical expressions for the phase variation and for the ratios of the real to the imaginary parts are obtained. The obtained expressions are new formulae and are found to be important for the description of nucleon–nucleon elastic scattering amplitudes. Introducing of the phase variation and the ratio of the real to the imaginary parts of the elastic scattering amplitude proportional to q^2 improves the theoretical calculations.

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1. Introduction

The ratio of the real to the imaginary parts of the nucleon–nucleon (NN) amplitude, ρ, is of major interest both theoretically and experimentally because of its close relationship with the energy integrated inelasticity of the collision via the dispersion relation. In the eikonal models, the dip of the differential cross section is very sensitive to the value of ρ. The ratio of the real to the imaginary parts of the nucleon–nucleon amplitude was parameterized by Alkhazov *et al.* [1] as a linear form of the square of the momentum transfer q^2. The coefficient of q^2 was taken as a free parameter. Glauber's multiple scattering theory [2] of hadron-nucleus based on the eikonal approximation theory has been successfully used to explain the main features of the experimental phenomena [3, 4]. One of the attractive points of this theory

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is that it provides a microscopic description of the nucleus-nucleus scattering data in terms of the nucleon–nucleon elastic scattering amplitude. The phase variation of the NN elastic scattering amplitude plays an important role in the scattering calculations [5]. The phase variation of the NN elastic scattering amplitude with q^2 was used earlier to improve the agreement of the hadron-nucleus [4–6] and nucleus-nucleus [7–9] calculations with the experimental data especially for increasing q^2. Ahmed and Alvi [10] calculated the phase variation parameter η in terms of an effective NN potential, which is consistent with the small angles NN scattering data at 1.75 GeV/c. Block and Pancheri [11] calculated the ratio of the real to the imaginary parts of the forward elastic scattering amplitude of light on light, *i.e.*, the reaction $\gamma+\gamma\rightarrow\gamma+\gamma$ in the forward direction by fitting the total $\gamma\gamma$ cross section data in the high energy region assuming a cross section that rises asymptotically in terms of the energy. They have found a formula for the ratio ρ in terms of both the total cross section and the energy.

The momentum transfer dependence of the NN amplitude phase and the dependence of the ratio of the real to the imaginary parts of the nucleon–nucleon amplitude are presented and discussed. The Gaussian form of the NN effective potential is used to calculate the NN elastic scattering amplitude. In the present work, the coefficient of q^2 is determined by a method similar to that of Ahmed and Alvi [10] for the determination of the phase variation parameter of the scattering amplitude. The formalism of this work is given in Section 2. The calculations and results are presented in Section 3. Finally, the discussion and conclusions are given in Section 4.

2. Theory

The nucleon–nucleon scattering amplitude is given by a Gaussian parameterization in terms of the momentum transfer squared q^2 [2,7,8,10,12–19]

$$f_{NN}(q)=\frac{ik\sigma(1-i\rho)}{4\pi}\exp\left(-\frac{aq^2}{2}\right), \tag{1}$$

where σ is the total nucleon–nucleon cross section, k is the momentum per nucleon of the projectile, q is the momentum transfer, ρ is the ratio of the real to the imaginary parts of the forward NN scattering amplitude, and a is taken to be complex and is given by:

$$a=\beta+i\eta, \tag{2}$$

where the real part is typically the slope parameter, β, of the NN elastic scattering differential cross section given by $d\sigma/dt$, where t is the squared momentum transfer $t=-\hbar^2q^2$. The imaginary part, η, is a free parameter considered as a phase variation of the nucleon–nucleon scattering amplitude. The parameter η leads simply to an overall phase factor $\exp\left(-\frac{i\eta q^2}{2}\right)$, which cannot be obtained, directly from the nucleon–nucleon measurements [7,8]. The phase variation parameter η was treated as a free NN parameter; it was taken to be fixed at any given velocity of the incident nucleus and thus independent of the nuclei involved in the collision, provided that the kinetic energies per nucleon are the same in all cases. Thus, the same value of η would be used in describing all the nucleus–nucleus measurements at a given kinetic energy per nucleon [7,8].

In this work, the parameters η and ρ are not considered as free parameters, but new formulae are given for determining them. The ratio ρ in equation (1) is now considered as a function of the momentum transfer q^2. The nucleon–nucleon scattering amplitude, in equation (1), was parameterized by Alkhazov *et al.* [1] in the form:

$$f_{NN}(q) = \frac{ik\sigma}{4\pi}\left(\tau - i\rho_2 q^2\right)\exp\left(-\frac{aq^2}{2}\right), \tag{3}$$

where

$$\rho = \rho_0 + \rho_2 q^2 \tag{4}$$

and

$$\tau = 1 - i\rho_0 \,. \tag{5}$$

Expanding $\exp\left(-\frac{aq^2}{2}\right)$ in powers of q^2 as:

$$\exp\left(-\frac{aq^2}{2}\right) = 1 - \frac{aq^2}{2} + \frac{a^2q^4}{8} + \ldots \,. \tag{6}$$

Then inserting equation (6) into equation (3) one gets:

$$f_{NN}(q) = \frac{ik\sigma}{4\pi}\left[\tau - \left(\frac{a\tau}{2} + i\rho_2\right)q^2 + \left(\frac{\tau a^2}{8} + i\frac{a}{2}\rho_2\right)q^4 + \ldots\right]. \tag{7}$$

The parameter a, which is in general complex, may be determined by considering the usual two-dimensional integral representation of the nucleon–nucleon amplitude

$$f(q) = \frac{ik}{2\pi}\int d^2b\,\exp(i\boldsymbol{q}\cdot\boldsymbol{b})\Gamma(b)\,, \tag{8}$$

where the profile function $\Gamma(b)$ is given in terms of the phase shift function $\chi(b)$ as:

$$\Gamma(b) = 1 - \exp(i\chi(b))\,. \tag{9}$$

Expanding equation (8) in powers of q^2, one gets:

$$f(q) = ik\left[\mathit{\Gamma}_1 - \mathit{\Gamma}_3\frac{q^2}{4} + \mathit{\Gamma}_5\frac{q^4}{64}\ldots\right], \tag{10}$$

where

$$\mathit{\Gamma}_n = \int\limits_0^\infty \mathit{\Gamma}(b)b^n db\,; \quad n = 1, 3, 5, \ldots\,. \tag{11}$$

Comparing the coefficients of the different powers of q^2 of equation (7) with the eikonal expression (10), one obtains the following terms: The term independent of q gives:

$$\tau = \gamma_1\,. \tag{12}$$

The term proportional to q^2 gives:

$$i\rho_2 = \frac{1}{4}\left(\gamma_3 - 2a\gamma_1\right)\,. \tag{13}$$

The term proportional to q^4 gives:

$$a^2\gamma_1 - a\gamma_3 + \frac{1}{8}\gamma_5 = 0\,, \tag{14}$$

where

$$\gamma_n = \frac{4\pi}{\sigma}\mathit{\Gamma}_n\,. \tag{15}$$

Equation (14) shows that a has two values as follows:

$$a = \frac{\gamma_3 \pm \sqrt{\gamma_3^2 - 0.5\gamma_1\gamma_5}}{2\gamma_1}\,. \tag{16}$$

It can be noticed that there is an ambiguity in determining a due to the presence of the positive and negative signs of the square root. If as an approximation the first term of this equation is only taken, *i.e.*, neglecting the square root, then one gets:

$$a = \frac{\gamma_3}{2\gamma_1}\,. \tag{17}$$

This formula is like the formula that was given by Ahmed and Alvi [10]. If this expression is used to calculate the value of ρ_2 in Eq. (13), one finds it equals zero. This is consistent with the results obtained by Ahmed and Alvi [10] and Block and Kaidalov [20] who did not consider the ρ_2 term, and obtained a simple formula for the parameter a as given by Eq. (17). But in the present work, the value of ρ_2 has been considered and Eq. (14) has been solved by another method. Since a is small, then a^2 is much smaller and can be neglected in Eq. (14), therefore, the solution of this equation yields:

$$a = \frac{1\gamma_5}{8\gamma_3} \,. \tag{18}$$

Since a is complex, as given in Eq. (2), and also γ_n are complex, then the real and imaginary parts of Eq. (18) are given, respectively, as follows:

$$\beta = \frac{1}{8}\frac{\gamma_3^r\gamma_5^r + \gamma_3^i\gamma_5^i}{(\gamma_3^r)^2 + (\gamma_3^i)^2} \,, \tag{19}$$

and

$$\eta = \frac{1}{8}\frac{\gamma_3^r\gamma_5^i - \gamma_3^i\gamma_5^r}{(\gamma_3^r)^2 + (\gamma_3^i)^2} \,, \tag{20}$$

where the superscript of $\gamma(r$ or $i)$ represent the real or imaginary part. The expression for the ρ_2, given in equation (13), is a function of the energy and the total nucleon–nucleon cross section as already seen [11,20]. Separating ρ_2 into the real and imaginary parts, and using Eq. (13), one gets:

$$\mathrm{Re}\,\rho_2 = \frac{1}{4}\left[\gamma_3^i - 2\beta\gamma_1^i - 2\eta\gamma_1^r\right] \,, \tag{21}$$

and

$$\mathrm{Im}\,\rho_2 = -\frac{1}{4}\left[\gamma_3^r - 2\beta\gamma_1^r + 2\eta\gamma_1^i\right] \,. \tag{22}$$

These equations show that when a and γ_n are complex then also ρ_2 is complex. The phase shift $\chi(b)$ can be written in the form [2,10,21]

$$\chi(b) = -\frac{1}{\hbar\nu}\int\limits_{-\infty}^{\infty} V\left(\sqrt{b^2+z^2}\right) dz \,, \tag{23}$$

where ν is the nucleon velocity, b is the impact parameter and V is the effective nucleon–nucleon potential. The effective potential is local and energy dependent and is chosen such that it reproduces the small angle NN scattering data [10]. The effective nucleon–nucleon potential is considered to be in the Gaussian form as:

$$V(r) = -(V_Y + iW_Y)\exp(-\alpha^2 r^2) \,, \tag{24}$$

where V_Y, W_Y, and α^2 are the potential parameters. This gives the following expression for the phase shift function:

$$\chi(b) = \chi_0 \exp(-\alpha^2 b^2) \,. \tag{25}$$

Here χ_0 is a constant, which depends on the potential parameters and the nucleon velocity ν; and is given by:

$$\chi_0 = \sqrt{\frac{\pi}{\alpha^2}} \frac{V_Y + iW_Y}{\hbar\nu} \,. \tag{26}$$

3. Calculations and results

The ratio of the real to the imaginary parts of the forward elastic scattering amplitude, ρ, has been studied in terms of the momentum transfer parameter q. This ratio has been found to be dependent on the phase variation η as shown in Eqs. (21) and (22).

The calculation of the ratio, ρ, requires the knowledge of the NN cross section, σ, which has been calculated in the present work as follows: From equations (8), (9) and (25), one gets:

$$f(q) = \frac{ik}{2\pi} \int d^2 b \exp(i\boldsymbol{q} \cdot \boldsymbol{b}) \left[1 - \exp\left(i\chi_0 \exp\left(-\alpha^2 b^2\right)\right)\right] \,. \tag{27}$$

This equation can be rewritten as:

$$f(q) = -\frac{ik}{2\alpha^2} \sum_{n=1}^{\infty} \frac{i^n \chi_0^n}{n\Gamma(n+1)} \exp\left(-\frac{q^2}{4n\alpha^2}\right) , \tag{28}$$

where χ_0 is complex and may be written as:

$$\chi_0 = \operatorname{Re}\chi_0 + i \operatorname{Im}\chi_0 \,, \tag{29}$$

and the total NN cross section is obtained from the elastic scattering amplitude using the optical theorem as follows:

$$\sigma = \frac{4\pi}{k} \operatorname{Im} f(0) \,. \tag{30}$$

Now, as an approximation considering only the first two terms of the expression (28), one gets:

$$\sigma = \frac{2\pi}{\alpha^2} \left[\operatorname{Im}\chi_0 + \frac{1}{4}\left((\operatorname{Re}\chi_0)^2 - (\operatorname{Im}\chi_0)^2\right)\right] \,. \tag{31}$$

The effective NN potential parameters V_Y, W_Y and α^2 are considered at the energies 1000 MeV and 1630 MeV from Refs. [10,22]. The corresponding values of the real and imaginary parts of χ_0 are calculated according to Eq. (26). The results of these calculations and the calculations of the total nucleon–nucleon cross section, σ, and the ratios of the real to imaginary

TABLE I

The obtained results of the NN total cross section, σ, the slope parameter, β, the phase variation, η, and the ρ's ratios at 1 GeV for $V_Y = 95$ MeV and $W_Y = 131$ MeV [10].

α^2 $(\mathrm{GeV}/c)^2$	$\mathrm{Re}\,\chi_0$	$\mathrm{Im}\,\chi_0$	β (fm^2)	η $(\mathrm{GeV}/c)^{-2}$	σ (mb)	ρ_0	$\mathrm{Re}\,\rho_2$ (fm^2)	$\mathrm{Im}\,\rho_2$ (fm^2)
0.0357[a]	0.61903	0.8536	0.2946	0.2392	66.17	-0.4355	-0.0096	-0.1739
0.041[a]	0.5776	0.7965	0.2557	0.196	53.44	-0.45	-0.0038	-0.151
0.14[b]	0.3126	0.4311	0.0733	0.0328	8.134	-0.5578	0.0092	-0.0423

[a] Ref. [22]
[b] Ref. [10]

TABLE II

The obtained results of the NN total cross section, σ the slope parameter, β, the phase variation, η, and the ρ's ratios at 1.63 GeV for $V_Y = 95$ MeV and $W_Y = 131$ MeV [10].

α^2 $(\mathrm{GeV}/c)^2$	$\mathrm{Re}\,\chi_0$	$\mathrm{Im}\,\chi_0$	β (fm^2)	η $(\mathrm{GeV}/c)^{-2}$	σ (mb)	ρ_0	$\mathrm{Re}\,\rho_2$ (fm^2)	$\mathrm{Im}\,\rho_2$ (fm^2)
0.0357	0.4849	0.6686	0.2915	0.1925	50.8	-0.485	0.0084	-0.1719
0.041	0.4524	0.6239	0.2532	0.1574	41.07	-0.4975	0.0115	-0.149
0.14	0.2448	0.3377	0.0729	0.02602	6.3	-0.59	0.0124	-0.0414

parts of the NN elastic scattering amplitude are given in Tables I and II. Also, the slope parameter, β, and the phase variation; η which represents the real and the imaginary parts of the parameter a are obtained from equations (19) and (20) and the results are shown in Tables I and II for the energies 1 GeV and 1.63 GeV, respectively. Lombard and Maillet [6] have found that the phase factor η has its strongest effects at places where the multiple scattering contributions interfere and the influence is less obvious when only one term dominates. As pointed out by Franco and Yin [7,8], the phase factor is independent of the type of the nucleus. In the present work, as shown in Tables I and II, the phase variation η is inversely proportional to the parameter α^2.

The results of the calculations of the ratio of the real to the imaginary parts of the NN scattering amplitude for $E = 1000$ MeV with $\alpha^2 = 0.0357$ $(\mathrm{GeV}/c)^2$ are $\rho_0 = -.4355$ and $\rho_2 = -0.00961$–$0.17385i$. These values are found to be $\rho_0 = -0.45$ and $\rho_2 = -0.0038$–$0.15104i$ for $\alpha^2 = 0.041$ $(\mathrm{GeV}/c)^2$. The results of these values for $\alpha^2 = 0.14$ $(\mathrm{GeV}/c)^2$ are $\rho_0 = -0.55778$ and $\rho_2 = 0.0092$–$0.04231i$. These results are shown in Table I. At 1 GeV there is an uncertainty in the values of the parameter ρ, see for example Refs.[4,23,24]. The different values of ρ lead to different values of the potential parameters χ_0 and α^2. The calculations have been done also for $E = 1.63$ GeV as shown in Table II. The ratios are found to be $\rho_0 = -0.4847$ and $\rho_2 = 0.0084–0.1719i$ for $\alpha^2 = 0.0357$ (GeV/c)^2; $\rho_0 = -0.4975$ and $\rho_2 = 0.0115–0.149i$ for $\alpha^2 = 0.041$ (GeV/c)^2 and $\rho_0 = -0.58997$ and $\rho_2 = 0.01237–0.04136i$ for $\alpha^2 = 0.14$ (GeV/c)^2. The values of ρ_0 and ρ_2, given by Alkhazov *et al.* [1], are 0.25 and 0.065 fm^2, respectively.

One can notice that the values obtained in the present work are slightly different from the values of Refs. [10,22], which may be due to using different approaches. Calculations of these values permit the description of the region of the minimum in the differential cross sections for the scattering of the proton with different nuclei [5] such as ^{4}He, ^{12}C and ^{16}O. The dependence on the momentum transfer of the ratio $\mathrm{Re}\, f_{NN} \,/\, \mathrm{Im}\, f_{NN}$ appreciably influences the fitting of the diffraction minima. However, in the case of the proton scattering by nuclei having a large deformation ($l \geq 1$), such as ^{9}Be and ^{11}B, the principal mechanism of fitting the minima is due to the non-sphericity of the nucleus and to the presence of several incoherent channels of scattering, among which an important role is played by the channel in which the rearrangement of the nucleus occurs.

4. Discussion and conclusions

The dependence on the momentum transfer of the ratio for the real part to the imaginary part of the forward elastic scattering amplitude has been studied. The phase variation has also been considered. The nucleon–nucleon Gaussian form has been used to calculate the nucleon–nucleon elastic scattering amplitude. New analytical expressions for the ratios ρ in terms of the momentum transfer q^2 and new formulae for the phase variation η have been obtained. The obtained results agree well with the previous results given by other authors such as Alkhazov *et al.* [1].

In the literature the parameter, a, was considered real. Then, introducing an imaginary part of it as fitting parameter improved the fitting with the experimental data. Ahmed and Alvi [10] derived analytical expressions to determine the imaginary part. The dependence of ρ_2 on q^2 was introduced as a free parameter. The obtained formulae in the present work show that ρ_2 is complex.

The formulae obtained in this work can be used to determine the phase variation parameter and the ratio of the real to the imaginary parts of the NN scattering amplitude as long as the values of the effective nucleon–nucleon potential are found.

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