EVIDENCE FOR A LIQUID-GAS PHASE TRANSITION IN THE FRAGMENTATION OF Pb NUCLEI AT 158 *A*GeV

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A liquid-gas phase transition in the fragmentation of Pb nucleus at 158 *A*GeV energy has been investigated. The results of the analysis of moments of the charge distributions of fragments are consistent with the postulated occurrence of a liquid-gas phase transition in the process of fragmentation of heavy nuclei. Values of critical exponents are found and compared to those obtained at lower energies.

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1. Introduction

Since many years investigations of the nucleus–nucleus interactions have been carried out at the highest available energies and masses of interacting nuclei. These investigations were focused on two different processes considerably separated in time. The first occurs in time of the order of the time-of-flight of the projectile nucleons through the target nucleus. During this short time particles are produced in inelastic collisions between projectile and target nucleons and the prompt nucleons are knocked out of the projectile nucleus. After the collision the excited nuclear remnant expands and cools down to the state where fragments are formed from the high temperature and low density vapor of nucleons. This second process, delayed in time in comparison with the first one, is the subject of our present investigations. In particular we focus on one of the most important problem *i.e.* the search for the existence of phase transitions in the process of fragmentation of heavy nuclei. The transition of nuclear matter from liquid to gaseous state should be observed at energies attainable in conventional fixed target experiments. As a matter of fact, until now, there were published several papers *e.g.* Refs. [1–10] devoted to this problem yet, the questions whether the phase transition from liquid to gas state takes place and what role it plays in the process of nuclear fragmentation still remain open. Therefore any additional experimental data concerning these questions seem to be valuable.

The present study is a continuation of our previous analyses [11–14] of Pb interactions with nuclear targets at 158 AGeV. After describing experimental details we present analysis of multiplicity and the charges of fragments emitted from the Pb projectile nucleus. The behavior of these observables is in agreement with that anticipated in the presence of the liquid-gas phase transition. In the second part of the paper we focused on the analysis of moments of the charge distributions of fragments. The results are compared to the data obtained at different energies and masses of the fragmenting nuclei.

2. Experiment

Results presented in this paper are based on the data obtained from 158 AGeV Pb interactions with Pb and plastic ($C_5H_4O_2$) targets. Interactions were registered in several lead–emulsion chambers irradiated at CERN-SPS. Details about the chambers layout, scanning of Pb interactions and measurements of emission angles and charges of particles emitted from the interaction vertex can be found elsewhere [11, 15, 16].

In our previous paper [12] we have shown that the fragmentation mode of the Pb projectile depends mainly on the amount of energy deposited into the fragmenting Pb nucleus, but not on the mass of the target. This observation enables one to combine interactions with Pb and plastic targets. We have also shown [12, 14] that at 158 AGeV Pb–Pb collisions the electromagnetic dissociation processes become competitive with the nuclear ones. Although a majority of these electromagnetic interactions appears as events with a heavy residue accompanied by a few nucleons, for which the scanning efficiency is very poor, our experimental material still is contaminated by electromagnetic interactions.

The analyzed sample of events consists of 904 Pb collisions with lead and plastic targets. This sample does not contain fission and fission-like events, which constitute about 9% of the total number of found events. The rejected fission events are characterized by the emission of two heavy fragments each with the charge close to half the charge of the primary nucleus. The fission-like events were defined as those with two heavy fragments accompanied by less than about 20 spectator particles. The majority of these spectator particles are protons, some are alpha particles and in few cases light fragments.

Spectator particles are defined as projectile fragments with charge $Z > 2$ (N_f), alpha particles (N_α) and projectile spectator protons (N_p). Their total multiplicity is $N = N_f + N_\alpha + N_p$. In each interaction the number N_p of spectator protons was estimated as a fraction of singly charged relativistic particles emitted at angles smaller than a given $\theta_{\rm cut}$ value. The emission cone, defined by $\theta_{\rm cut}$, should be chosen to contain almost all spectator protons and the contamination by produced particles and prompt protons should be as small as possible. The value of $\theta_{\rm cut} = (10^3\langle\theta_0\rangle + 1.1) = 1.9$ mrad, where $\langle\theta_0\rangle = 0.12/p_0$ [17] is the mean emission angle of spectator protons and p_0 is the momentum per nucleon of the projectile in GeV/c. An addition of 1.1 mrad to the average emission angle, corresponds to adding one sigma of the θ distribution of spectator protons, assuming that the distribution is isotropic in the rest frame of the fragmenting nucleus, (it can be, then, well approximated by a Gaussian distribution in pseudorapidity, $\eta_{\rm lab} = -\ln\tan(\theta/2)$, with a width of about 0.9). In the following we use the normalized multiplicities $m = N/Z_{\rm sp}$ as a control parameter to study the nuclear fragmentation phenomena. Here $Z_{\rm sp}$ denotes the total charge of spectator particles, including spectator protons. It was shown [18] that the total multiplicity of fragments is proportional to the excitation energy of the system created after the collision. For peripheral collisions (large impact parameter, low excitation energy) the value of m is small, $m \approx 0$, then it increases with increasing centrality of the collision and finally becomes close to one, $m \approx 1$, for very central collisions (small impact parameter,

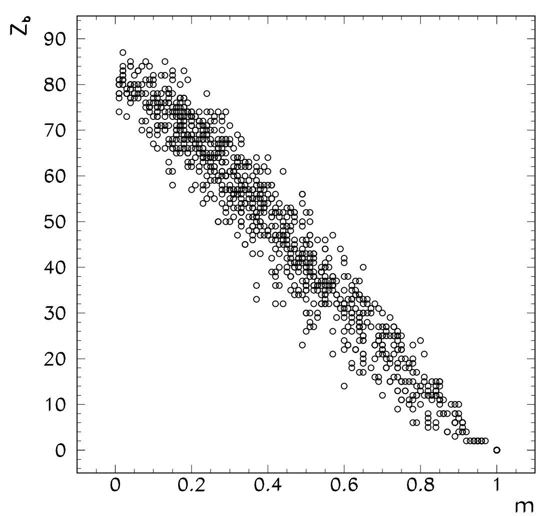

Fig. 1. Correlation between the total number, Z_b, of multiply charged fragments and normalized multiplicity, m, of charged spectator particles.

high excitation energy). Additionally it was shown that the total charge Z_b bounded in multicharged fragments is also correlated with the degree of the nucleus excitation [19]. Thus, quantities m and Z_b should be strongly correlated. Indeed, such strong correlation is observed as illustrated in Fig. 1. Both quantities have been frequently used in the analysis of fragmentation processes.

3. Number of fragments and their charges as a function of the multiplicity

In Fig. 2 the dependence of several observables on the control parameter m is shown. In Fig. 2(a) we present the mean number $\langle N_f \rangle$ of fragments with $Z \geq 3$ and the mean number $\langle N_{\rm IMF} \rangle$ of the intermediate mass fragments. The latter are usually defined as fragments with $3 \leq Z \leq 30$. This definition is somewhat arbitrary, nevertheless one can see in Fig. 2(a) that for $m > 0.5$ there are only the intermediate mass fragments while for $m < 0.5$ fragments with $Z > 30$ are also present. This might be an indication that the vicinity of $m \approx 0.5$ is a borderline between the two phases of nuclear matter *i.e.* liquid and gaseous regions. If so, it follows that in the gas phase there are practically only intermediate mass fragments and this seems to be a reasonable conjuncture.

Investigation of the charge distribution of fragments and its dependence on the parameter m is a widely used method in the analysis of nuclear fragmentation. Following the first observation of a power law of the charge distribution of fragments it became a common analysis tool to fit the charge distributions to a power law, $Z^{-\tau}$, and to investigate the dependence of the exponent τ as a function of m. Furthermore, it was anticipated that at the critical point, defined by a certain value of m where the phase transition might occur, the value of the exponent τ of the power law should attain a lower value than those obtained from fits performed away from the critical point [20–22]. In Fig. 2(b) we present the dependence of the exponent τ of the power law fits to the charge distribution of fragments, performed at different ranges of m. In this analysis the fits are restricted to fragment charges smaller than $Z = 16$. This was done because of low statistics of events characterized by the emission of heavier fragments. At small values of m a system has few light fragments and the power law is steep; at large values of m there are many light fragments and little else leading again to a steep power law. At the moderate excitation energies where heavier fragments appear and where we expect the phase transition, the exponent τ has its lowest value. As can be seen from Fig. 2(b) it happens for m values between 0.35 and 0.55.

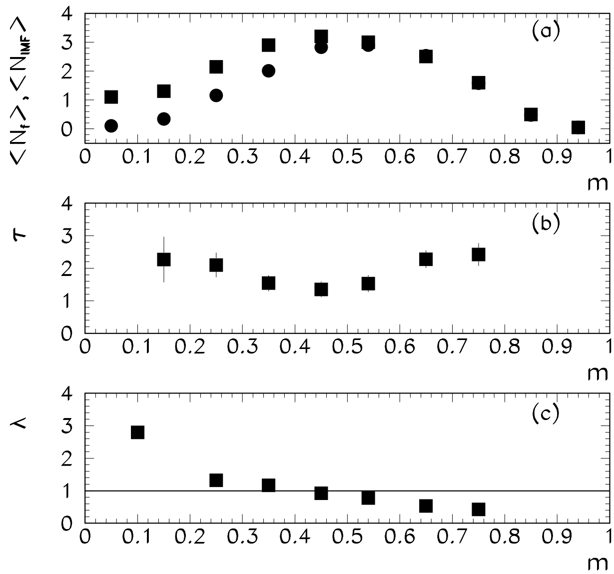

Fig. 2. (a) — Mean number, $\langle N_f \rangle$, of fragments (squares) and mean number, $\langle N_{\mathrm{IMF}} \rangle$, of intermediate mass fragments (circles) as a function of the normalized multiplicity m. Error bars are smaller than the size of the squares and circles. (b) — Power law exponent, τ, of the charge distribution of fragments in different intervals of m. (c) — Power law exponent, λ, in the Zipf's law (see text) in different intervals of m. Error bars are smaller then data points.

Additional information indicating a possible occurrence of a liquid-gas phase transition in the process of nuclear fragmentation comes from the application of the Zipf's law. The Zipf's law [23] has been known as a statistical phenomenon in the field of linguistics and also in other fields of sciences such as *e.g.* molecular biology [24]. It was also applied to analyze the cluster-size distribution in percolation processes [25] and to the study of nuclear phenomena [12, 26, 27]. The Zipf's law for the multifragmentation processes states that the relation between the sizes of nuclear fragments and their rank n should be described by $\langle Z_n \rangle \sim 1/n$ if a nuclear liquid-gas phase transition occurs. $\langle Z_n \rangle$ is the average charge of fragments of rank n in a charge list arranged according to the decreasing fragment size. The relation between Z_n and n has been examined theoretically for the fragmentation of Xe nuclei at different temperatures [26]. The higher the temperature the smaller the slope parameter λ in the relation $\langle Z_n \rangle \sim n^{-\lambda}$. It was shown that the Zipf's law $Z_n \sim n^{-\lambda}(\lambda = 1)$ is satisfied at the temperature that coincides with the critical temperature of the liquid-gas phase transition. This theoretical conjuncture proves that the Zipf's law can be a valid signature of a liquid-gas phase transition in nuclear matter.

Our studies [12] of multifragmentation processes of lead nuclei in collisions with heavy and light targets at 158 AGeV have shown that the analyzed multifragmentation data are consistent ($\lambda \approx 1$) with the Zipf's law. In the present study we examine the dependence of the power law exponent λ on the control parameter m. In Fig. 2(c) the dependence of λ obtained from the fits $\langle Z_n \rangle \sim n^{-\lambda}$, on m is depicted. The exponent λ decreases with increasing m in agreement with the theoretical predictions [26, 27]. In the region of m from about 0.3 to 0.5 the value of λ is close to unity and the Zipf's law is satisfied. This suggests that at this value of m the liquid-gas phase transition occurs. It has been checked that $\lambda = 1$ occurs in the same region of m irrespectively of the mass of the target. This means that the liquid-gas phase transition occurs when a given amount of energy is deposited into the nucleus and does not depend on the mass of the target. Interestingly, the previously shown maxima in the frequency distributions of multiply charged fragments (Fig. 2(a)) as well as a minimum of the power law parameter τ (Fig. 2(b)), all occur at the same values of m, where the Zipf's law is fulfilled.

The analysis presented above indicates the possible occurrence of liquid-gas phase transition in fragmentation of the excited Pb nucleus. Frequency distribution of fragments, the power law distribution of their charges with the minimum value of the exponent τ and the validity of the Zipf's law yield only a necessary condition for the phase transition. Nevertheless, the fact that they all point to the same critical value of the control parameter m seems to be significant.

4. Moments of the charge distributions of fragments

The following analysis is based on the moments of the charge distributions. The analysis of moments of the charge distributions of fragments is motivated by the Fisher model [28] of droplet condensation when applied to nuclear fragmentation. This analysis was suggested by Campi [1] and since then has been widely used. The k-th moment of the charge distribution of N charged fragments in an event is given by $M(k) = 1/Z_{\rm sp} \sum_{n=1}^{N-1} Z_n^k$. The sum runs over all fragments including spectator protons except for the heaviest fragment in the collision. Moments are normalized to the total charge $Z_{\rm sp}$ of spectator particles. In Fig. 3 we plot second moments M_2 as a function of the control variable m for the analyzed sample of events. The second moment M_2 has a broad maximum that extends from m of about 0.3 to 0.5. Such a maximum may be an indication of the existence of phase transition at $m = m_{\rm c}$ from the interval $(0.3 \div 0.5)$ where $m_{\rm c}$ denotes the critical value. It is expected that near $m_{\rm c}$, M_2 should exhibit large fluctuations, as a manifestation of the coexistence of two phases: liquid phase ($m < m_{\rm c}$) and the gas phase ($m > m_{\rm c}$). The broadness of the maximum seen in Fig. 3 is due to the finite size of the system that is represented here by the fragmenting nucleus.

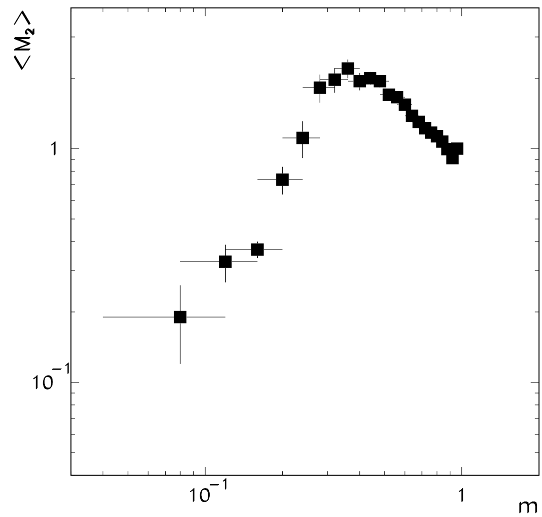

Fig. 3. Mean value, $\langle M_2 \rangle$, of the second moment of charge distribution of fragments as a function of the normalized multiplicity m.

In order to further check that the $m_{\rm c}$ represents a critical value, we examine the behaviour of M_2 and the charge $Z_{\rm max}$, of the heaviest fragment in an event, as a function of m as well as the charge distribution of fragments in the region near the critical point. If we can prove that the above observables can be described by power laws and that the critical exponents of these power laws fulfill the scaling relation (see below), this will be a sufficient signal of the existence of phase transition [9].

Power law dependencies of different observables are defined as follows:

$$M_2(\mathit{\Delta}) \sim \mid \mathit{\Delta} \mid^{-\gamma}, \tag{4.1}$$

$$Z_{\rm max}(\mathit{\Delta}) \sim \mid \mathit{\Delta} \mid^{\beta} \qquad \text{for} \quad \mathit{\Delta} < 0\,, \tag{4.2}$$

$$n(Z) \sim Z^{-\tau} \qquad \text{for} \quad \mathit{\Delta} > 0\,, \tag{4.3}$$

where $\mathit{\Delta} = m - m_{\rm c}$ measures the distance from the critical point. The scaling relation between critical exponents γ , β and τ reads:

$$\tau = 2 + \frac{\beta}{\beta + \gamma}\,. \tag{4.4}$$

This relation has to be satisfied if the phase transition takes place [29]. This type of analysis was applied by the EOS Collaboration [4] for 1 AGeV

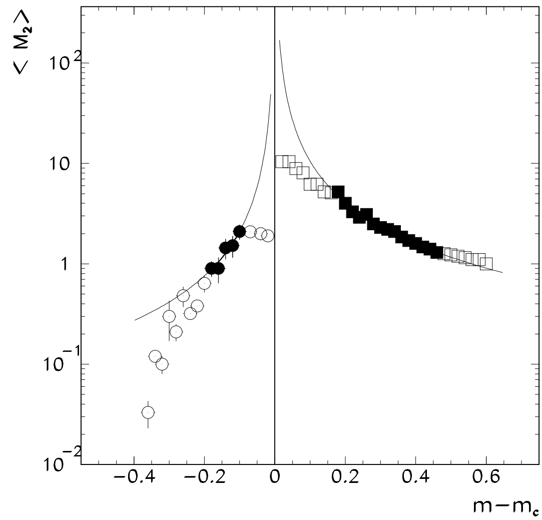

Fig. 4. Dependence of the mean value, $\langle M_2 \rangle$, of the second moment of charge distribution of fragments on the multiplicity difference, $(m - m_{\rm c})$, for $m_{\rm c} = 0.39$. Circles and squares denote liquid and gas phase respectively. Full circles and squares denote values used to fit the critical exponent γ in liquid and gas phase. Curves represent the power law fits.

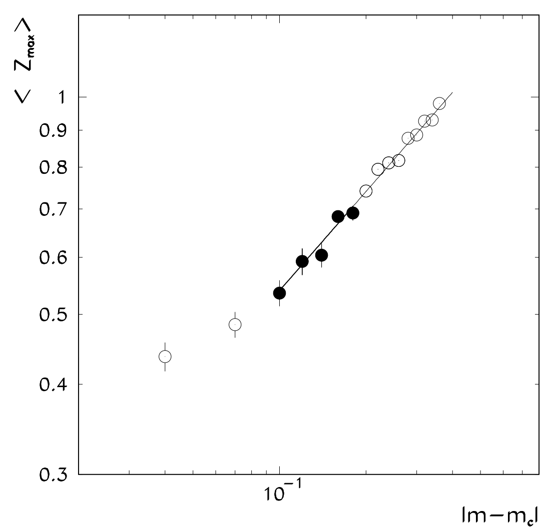

Fig. 5. Mean value, $\langle Z_{\rm max} \rangle$, of the heaviest fragment in the liquid phase $(m < m_{\rm c})$ for $m_{\rm c} = 0.39$. Full circles denote values used to fit the critical exponent β. Straight line is the result of power law fit.

gold nuclei fragmenting in collisions with the carbon target. Since then it was frequently applied to the analysis of a liquid-gas phase transition in multifragmentation of Au nuclei at 4 and 10.6 AGeV and Pb nuclei at 158 AGeV (see *e.g.* [3, 5–8, 10]).

We begin with the determination of the exponent γ. We choose several different values of $m_{\rm c}$ from the interval where M_2 exhibits maximum ($m_{\rm c} = 0.3 \div 0.5$, see Fig. 3) and find γ for $\mid \mathit{\Delta} \mid > 0$. The fit range cannot cover values of m too close to $m_{\rm c}$ and too far from $m_{\rm c}$. In the first case large fluctuations of M_2 are expected and in the second case the signature of critical behaviour vanishes. On the liquid side ($m < m_{\rm c}$) we remove the heaviest fragment ($Z_{\rm max}$) that is considered as the percolating cluster. On the gas side ($m > m_{\rm c}$) there is no liquid drop and the largest fragment is the largest gas particle. Using the above procedure we try to find a value of $m_{\rm c}$ for which $\mid \delta\gamma \mid = \mid \gamma_{\rm gas} - \gamma_{\rm liq} \mid$ is close to zero and fits giving $\gamma_{\rm gas}$ and $\gamma_{\rm liq}$ have sufficiently good χ^2 values. It should be admitted that this procedure is somewhat subjective. The exponents γ, particularly that in the liquid phase, depend on the choice of the value of $m_{\rm c}$ and on the range of $\mathit{\Delta}$ used for fitting the power law dependence. Also, one has to remember about the pre-selection of events done at the very beginning of this analysis. Without rejecting fission and fission-like events (see Section 2), finding critical exponents by means of charge moments would be ineffective. We found that the value of $m_{\rm c}$ selected from the interval $(0.35 \div 0.40)$ satisfies $\mid \delta\gamma \mid \approx 0$. Fig. 4 shows the dependence of $\langle M_2 \rangle$ on $\mathit{\Delta}$ for $m_{\rm c} = 0.39$ in liquid and gas phases. Full squares and dots denote the values used to fit the γ exponents. We get for the liquid phase $\gamma_{\rm liq} = 1.48 \pm 0.70$ ($\chi^2/ndf = 0.21$) and for the gas phase $\gamma_{\rm gas} = 1.37 \pm 0.05$ ($\chi^2/ndf = 0.9$). The mean value is $\langle\gamma\rangle = 1.43 \pm 0.35$. For a different value of $m_{\rm c}$, e.g $m_{\rm c}$=0.35, we get $\gamma_{\rm liq} = 1.52 \pm 0.38$ ($\chi^2/ndf = 1.3$) and $\gamma_{\rm gas} = 1.44 \pm 0.03$ ($\chi^2/ndf = 1.3$) with the mean $\langle\gamma\rangle = 1.48 \pm 0.19$.

Using the critical value $m_{\rm c} = 0.39$ we found the exponent β from the Eq. (4.2) (see Fig. 5). The fit was performed on the liquid side, where $Z_{\rm max}$ is well defined and in the same interval of $\mathit{\Delta}$ as that used in finding $\gamma_{\rm liq}$. We get $\beta = 0.46 \pm 0.10$. Now, we can use the scaling relation Eq. (4.4) to calculate the third exponent τ. We get $\tau = 2.24 \pm 0.06$ using the found values of γ and β exponents. The exponent τ can be found independently from the relation $\ln M_3/\ln M_2 = (\tau - 4)/(\tau - 3)$ studied in the gas phase only [2]. Fig. 6 shows a power law fit to the dependence of $\ln M_3$ on $\ln M_2$. It gives the τ value of 2.20 ± 0.05 ($\chi^2/ndf = 0.23$). The fit was performed in the same interval of $\mathit{\Delta}$ as that used in calculating the exponent $\gamma_{\rm gas}$. We see that indeed the scaling relation (Eq. (4.4)) is valid. The difference between the exponents τ obtained from the analysis of moments of charge distribution of fragments on the one side and from the yield of fragments at different intervals of m (see Section 3) on the other side is not surprising. In

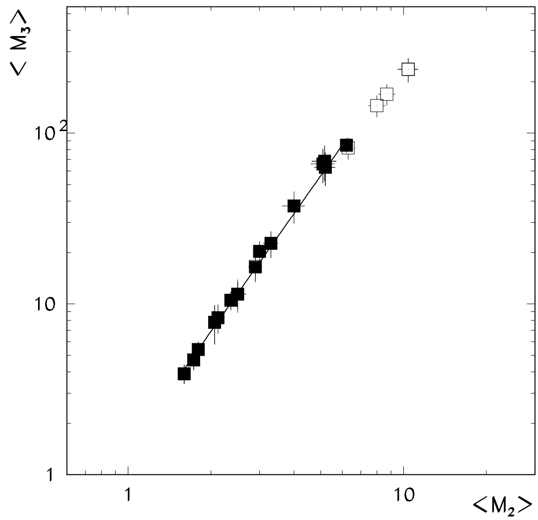

Fig. 6. Relation between the mean value, $\langle M_3 \rangle$, of the third and the mean value, $\langle M_2 \rangle$, of the second moments of charge distributions of fragments in the gas phase. Full squares denote values used to fit the power law dependence depicted by the straight line.

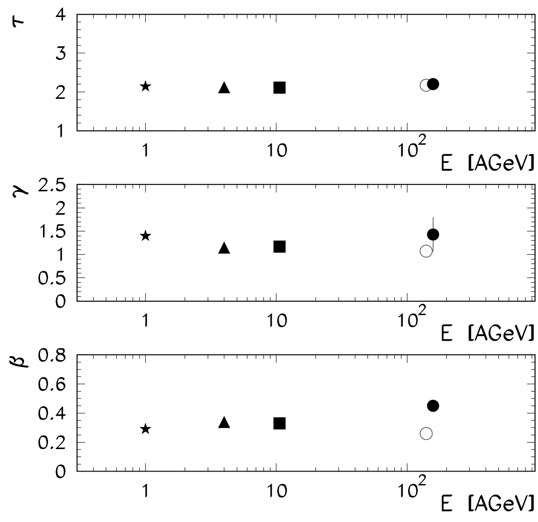

Fig. 7. Critical exponents γ, β and τ as a function of the energy of the projectile nucleus for: 1 AGeV, Au–C [8] (stars), 4 AGeV, Au–Emulsion, [10] (triangles), 10.6 AGeV, Au–Emulsion, [10] (squares), 158 AGeV, Pb–Emulsion, [6] (open circles), 158 AGeV, Pb–(Pb+Plastic), this experiment (full circles).

the latter case fragments with $Z > 16$ were omitted, while in the analysis of moments only events characterized by $m > m_c$ (belonging to the gas phase) were taken into account. Thus, data samples used in these two approaches of calculating the exponent τ are not necessarily the same. In Fig. 7 power law exponents γ, β and τ are plotted and compared to the values obtained in other experiments [6, 8, 10] analysed multifragmentation of Au and Pb nuclei at different energies.

5. Summary

Interactions of Pb nuclei with Pb and plastic ($C_5H_4O_2$) targets at 158 AGeV energy were used to study the multifragmentation of the projectile Pb nucleus.

The maxima of frequency distributions of multiply charged fragments and intermediate mass fragments as well as the minimum value of the exponent τ of the power law distribution of charges of fragments, they all show up at the value of the control parameter m of about 0.4. The Zipf's statistical law is also fulfilled at the same value of m. All these observations suggest that the liquid-gas phase transition occurs at the critical value m_c of about 0.4. This conclusion is supported by the analysis of moments of the charge distributions of fragments. Critical exponents γ, β and τ were found from the analysis of the charge moments of Pb fragments. The values of the critical exponents are consistent with those anticipated for the liquid-gas phase transition. As could be expected critical exponents do not show dependence on the energy of the projectile nucleus. However, one observes a significant spread of the critical exponents obtained in different experiments. In particular this refers to the critical exponent γ and may be due to the different methods of selecting multifragmentation events.

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