SPECTROSCOPIC MONTE CARLO CALCULATION WITH THE MONOPOLE PLUS QUADRUPOLE PAIRING PLUS QUADRUPOLE INTERACTION FOR ^{170}Yb

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The Spectroscopic Monte Carlo Method is applied to the nucleus ^{170}Yb using the Monopole plus Quadrupole Pairing plus Quadrupole interaction for yrast states up to $J^\pi = 12^+$. The unusually small statistical errors in the Monte Carlo calculations of the yrast energies ($\approx 40 \div 70$Kev) despite moderately strong sign oscillations, are discussed. Energy levels, expectation values of pairing potential and the pairing strength to $J = 12$ for the neutron intruder orbit and to $J = 10$ for the proton intruder orbit are evaluated. It is shown that the Hartree–Fock–Bogoliubov wave functions obtained with variation after projection to good particle number and to good z-component of the angular momentum and reprojected to good J^2, J_z after variation, give excitation energies in reasonable agreement with the Monte Carlo values. The intrinsic HFB wave functions have a considerable angular momentum dependence. Wave functions which are projected to good angular momentum after variation (PAV), lead to excitation energies larger than the corresponding Monte Carlo values.

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1. Introduction

The study of the yrast levels in strongly deformed medium and heavy nuclei has attracted considerable interest over the years because of the backbending phenomena, the superconducting to normal phase transition and the change in shape at high spin (Ref. [1]) and shape competition. Theoretically, within a purely fermionic approach, they have been described by variants of cranked Hartree–Fock–Bogoliubov (HFB) approximation, which approximately restores the exact angular momentm quantum number, (see

for instance Ref. [1] and references therein, and Ref. [2]), by the VAMPIR approach of Ref. [3], which is the full HFB method with variation after projection onto good quantum numbers, also by the EXCITED VAMPIR of Ref. [4], which extends the above approach with a suitable selection of many quasi-particle vacua with good quantum numbers by minimization of the energy with Graham–Schmidt orthogonalization, and with the projected shell model approach (Ref. [6]) which constructs a Nilsson+BCS shell model basis in which a truncated Hamiltonian matrix is diagonalized. More recently, the Monte Carlo Shell Model via the Quantum Monte Carlo diagonalization method of Ref. [5], which has similarities with the EXCITED VAMPIR approach, has been introduced. It selects the shell model basis by applying the Hubbard–Stratonovich trasformation (Ref. [7]) to the HF or HFB wave functions.

Full shell model calculations for medium heavy and heavy nuclei are outside of the reach of present day computers, because of the enormous size of the Hamiltonian matrix evaluated in the full Hilbert space based on the selected single particle basis. In order to overcome this limitation, the methods of Refs. [4] and [5] operate an optimal choice of the shell model basis in which the shell model Hamiltonian is diagonalized.

Recently (Ref. [8]) we have introduced a method that allows the evaluation of yrast energies (as well as other observables) entirely within Monte Carlo methods by evaluating $\langle \psi, N, Z | \hat{P}_{J,J_z=J} \hat{H} \exp^{-\beta \hat{H}} | \psi, N, Z \rangle / \langle \psi, N, Z | \hat{P}_{J,J_z=J} \exp^{-\beta \hat{H}} | \psi, N, Z \rangle$, where $|\psi NZ\rangle$ is a HFB wave function, $\hat{P}_{J,J_z=J}$ is the projector to good angular momentum J and good $J_z = J$ and $\hat{H}$ is the Hamiltonian, by sampling the full Hilbert space with the Monte Carlo Metropolis method (Ref. [9]), without actually performing a shell model diagonalization in a truncated basis. Up to now the calculation with this Spectroscopic Monte Carlo method have been carried out using the monopole pairing plus quadrupole model, but it is widely accepted that the Hamiltonian should also contain a quadrupole pairing interaction.

The purpose of this paper is to perform a Spectroscopic Monte Carlo calculation including also the quadrupole pairing interaction for the nucleus ^{170}Yb. At present, perhaps the strongest limitation of this kind of calculations for deformed nuclei is the limitation to one major shell (the neutron 82–126 and the proton 50–82 major shells), because of the computational requirements. This also forces a redetermination of the parameters of the Hamiltonian and a limitation in the number of the yrast levels to be studied. It would be most interesting to study the backbending phenomena in an exact Monte Carlo fashion as well as the high spin behavior beyond the HFB approximation for well deformed nuclei as well as transitional nuclei. Because of the computation cost, however, we limited the calculations up to the

12^+ state. However, several things can be learned. First of all the feasibility of the Monte Carlo calculation itself for several yrast levels with this type of Hamiltonian. This Hamiltonian displays moderately strong sign oscillations when the functional integral for $\langle\psi, N, Z|\hat{P}_{J,J_z=J}\hat{H}\exp^{-\beta\hat{H}}|\psi, N, Z\rangle$ is evaluated with the Monte Carlo Metropolis method. Despite these sign oscillations, as shown below, the use of HFB wave functions leads to small statistical uncertainties, which are necessary for the determination of the rather small excitation energies.

Moreover, the degree of accuracy of the HFB approximation [1] with particle number projection and a partially (in the sense explained below) before variation angular momentum projection can be tested. Also, from a physical point of view, we can learn about the changes in the theoretical spectra brought by the rearrangements of the intrinsic wave function for different J values. In some approximations to the ground state band (Ref. [6]), the same intrinsic wave function is used and then is angular momentum projected to several J values to obtain the members of the ground-state band.

In Section 2 we review the Spectroscopic Monte Carlo method and discuss its application to the nucleus ^{170}Yb, and in Section 3 we present the conclusions.

2. The Spectroscopic Monte Carlo method

The method used in this work has been introduced in Ref. [8]. The Hamiltonian we consider is the monopole+quadrupole pairing+quadrupole model given by

$$\hat{H} = \sum_{\tau=n,p}\left(\hat{H}_{\tau 0} - G_\tau \hat{P}^\dagger_\tau \hat{P}_\tau - G_{2\tau}\sum_{\mu=-2}^{2}\hat{P}^\dagger_{\tau\mu}\hat{P}_{\tau\mu}\right) - \frac{k}{2}\hat{Q}\hat{Q}\,, \qquad (1)$$

where τ labels neutron and protons, $\hat{H}_0$ is the spherical single-particle Hamiltonian, with the single-particle energies taken from Ref. [10], $\hat{P}$ is the monopole pair destruction operator and $\hat{P}_\mu = 1/2\sum_{i,j}\langle i|r^2Y_{2\mu}|j\rangle a_i a_{\bar{j}}$ (where $\bar{j}$ is the time reversal of the single particle state j) is the quadrupole pairing operator. In Eq. (1), $\hat{Q}$ is the standard isoscalar quadrupole operator of Ref. [10]. The single-particle space is restricted to the 82–126 major shell for the neutrons and to the 50–82 major shell for the protons. The calculation starts with the determination of particle-number projected (before variation) HFB wave functions. These wave functions are assumed to be products of neutron and proton wave functions with good parity, but

[1] Accuracy in the sense of a many-body method for approximating solutions of the Schroedinger equation.

otherwise no other symmetries (such as axial and time reversal symmetry) are enforced and all terms in the energy functional are kept (quite often exchange terms and pairing terms associated with the quadrupole force are neglected). Subsequently, these wave functions are varied anew after projection to good z-component of the angular momentum $J_z = J$. To avoid the awkward projection to $J_x = J$, in this method, it is natural to take as rotation axis not the x-axis (as in the cranking method) but the z-axis. The intrinsic symmetry axis (if there is any symmetry as a result of the unrestricted HFB variation) will point in some other direction; for example in the $x - y$ plane. To avoid any possible source of confusion, which may be caused by this convention, the fully angular momentum projected wave-function to good J and $J_z = J$ values will have $K = 0$ in the case of axial symmetry where K is the projection of the angular momentum along the intrinsic symmetry axis , and a small value of K in the case of no axial symmetry. The wave functions thus obtained are used as input wave functions for the Spectroscopic Monte Carlo method (Ref. [8]). The reason why no symmetries (apart from parity) are imposed to the HFB wave functions is to minimize as much as possible the energy of the HFB wave functions, although in the case of ^{170}Yb studied in this work axial symmetry is nearly exact with a very small tendency toward triaxiality with increasing values of J_z. The HFB wave function is

$$|\psi NZ\rangle = \hat{P}_{J_z=J}\hat{P}_N\hat{P}_Z|\phi\rangle\,, \tag{2}$$

where $|\phi\rangle$ is the intrinsic wave function parametrized as an exponential of a pair operator acting on the particle vacuum, *i.e.* $\exp\sum_{ij} 1/2 a_i^\dagger X_{ij} a_J^\dagger|0\rangle$ and $\hat{P}_{J_z=J}, \hat{P}_N, \hat{P}_Z$ are the projectors to good $J_z = J$ and to good neutron and proton particle numbers respectively. The coupling constants were determined by fitting the spectrum of ^{170}Yb up to $J = 12^+$ with a final full angular momentum projection on the wave functions of Eq. (2). The resulting deformation variables of the particle-number projected intrinsic wave functions, which are J-dependent, are the following. The γ variable (in degrees) increases from 0.1 to 0.8, while the β variable increases with the angular momentum from 0.24 to 0.25. The long elongation axis of intrinsic states (which can be found by determination of the principal axis of the tensor of inertia) turn out to have a small tilting angle (the deviation from orthogonality between the long intrinsic axis and the rotation axis) which decreases from about 7.7 to 6.7 degrees. This later quantity is associated with the small axial asymmetry. The strength of the monopole pairing force was determined with the requirement that $\Delta_\tau = G_\tau\sqrt{\langle\hat{P}^\dagger\hat{P}\rangle}$ for $\tau = n, p$ approximately reproduces the known gaps for neutrons and protons. We did not change the strength of the quadrupole force compared to the choice

of Ref. [10]. The values of the monopole pairing strength is 1.1 larger than the values of Ref. [10] and the strength of the quadrupole pairing force was taken to be $0.5\,G_\tau$ in order to reproduce the moment of inertia as inferred from the spectrum.

The energies and the excitation energies obtained after a final full angular momentum projection are listed in Table I, under the column HFB. For the purposes of monitoring the change in the intrinsic structure caused by the J_z projection, under the column labeled PAV, also the excitation energies obtained without the J_z projection before variation, are shown (that is, the particle-number projected HFB wave functions are directly fully angular momentum projected)[2].

TABLE I

Experimental and HFB energies for selected yrast states of ^{170}Yb. Together with the excitation energies $E^\star$, also the absolute values are listed. Δ_n and Δ_p are the gaps. S_n and S_p are the pair strength for the intruder orbits defined in the text. The parameters of the Hamiltonian have been obtained by adjusting $E^\star_{\rm HFB}$ to the experimental values.

State	$E^*_{\rm exp}$	$E_{\rm PAV}$	$E^\star_{\rm PAV}$	$E_{\rm HFB}$	$E^*_{\rm HFB}$	Δ_n	Δ_p	S_n	S_p
0^+	0	−126.532	0	−126.602	0	0.90	0.90	1.9	4.1
2^+	0.0843	−126.438	0.094	−126.515	0.087	0.89	0.89	1.9	4.1
4^+	0.2775	−126.218	0.314	−126.315	0.287	0.87	0.88	1.9	4.2
6^+	0.5736	−125.876	0.656	−126.016	0.586	0.82	0.85	2.0	4.2
8^+	0.9636	−125.414	1.118	−125.635	0.967	0.76	0.82	2.0	4.2
10^+	1.4379	−124.839	1.693	−125.192	1.410	0.69	0.80	2.0	4.3
12^+	1.9837	−124.161	2.371	−124.717	1.885	0.63	0.79	2.1	4.3

The wave functions obtained in this way are used to evaluate the following quantity

$$E(\beta,N,Z,J)=\frac{\langle\psi NZ|\hat{P}_{JJ}\hat{H}\exp^{-\beta(\hat{H}-\omega\hat{J}_z-\mu_n\hat{N}_n-\mu_p\hat{N}_p)}|\psi NZ\rangle}{\langle\psi NZ|\hat{P}_{JJ}\exp^{-\beta(\hat{H}-\omega\hat{J}_z-\mu_n\hat{N}_n-\mu_p\hat{N}_p)}|\psi NZ\rangle}\,, \qquad (3)$$

where ω is a cranking frequency and μ_n and μ_p are the neutrons and protons chemical potentials and $\hat{P}_{JJ}$ is the three dimensional projector to good angular momentum J with z-component $J_z = J$. The exponential of the Hamiltonian is then expressed in terms of the functional integral of Ref. [8],

[2] The experimental spectrum can be reproduced with PAV wave functions by changing the coupling constants at the price of having disagreement between the Monte Carlo excitation energies and the experimental data.

which schematically we write as

$$\exp^{-\beta(\hat{H}-\omega\hat{J}_z-\mu_n\hat{N}_n-\mu_p\hat{N}_p)} = \int dx G(x)\hat{W}(x)\,, \tag{4}$$

where x is a multidimensional integration point, $G(x)$ is a gaussian weight and $\hat{W}(x)$ is the propagator. Eq.(3) is evaluated as the ratio of the following two quantities $\mathcal{E}$ and $\mathcal{O}$ given by

$$\mathcal{E} = \frac{\mathrm{Re}\int dx G(x)\langle\psi NZ|\hat{W}|\psi NZ\rangle \frac{\langle\psi NZ|\hat{P}_{JJ}\hat{H}\hat{W}(x)|\psi NZ\rangle}{\langle\psi NZ|\hat{W}|\psi NZ\rangle}}{\int dx|\mathrm{Re}\left[G(x)\langle\psi NZ|\hat{W}|\psi NZ\rangle\right]|}\,, \tag{5}$$

$$\mathcal{O} = \frac{\mathrm{Re}\int dx G(x)\langle\psi NZ|\hat{W}|\psi NZ\rangle \frac{\langle\psi NZ|\hat{P}_{JJ}\hat{W}(x)|\psi NZ\rangle}{\langle\psi NZ|\hat{W}|\psi NZ\rangle}}{\int dx|\mathrm{Re}\left[G(x)\langle\psi NZ|\hat{W}|\psi NZ\rangle\right]|}\,, \tag{6}$$

where Re means real part. Both $\mathcal{E}$ and $\mathcal{O}$ are evaluated with the Metropolis Monte Carlo method using as a probability distribution the absolute value of the real part of $G(x)\langle\psi NZ|\hat{W}|\psi NZ\rangle$. The yrast energies of Eq. (3) are given by the ratio $\mathcal{E}/\mathcal{O}$. The statistical error for the energies are obtained from the standard expression for the error of the ratio of two quantities (*cf.* Eq. (12) below). As a short hand notation for the quantities appearing inside the integrals in Eqs. (5) and (6), let us set

$$\mathrm{e}^{A(x)+iB(x)} = G(x)\langle\psi NZ|\hat{W}|\psi NZ\rangle\,, \tag{7}$$

$$F_E = \frac{\langle\psi NZ|\hat{P}_{JJ}\hat{H}\hat{W}(x)|\psi NZ\rangle}{\langle\psi NZ|\hat{W}|\psi NZ\rangle}\,, \tag{8}$$

$$F_O = \frac{\langle\psi NZ|\hat{P}_{JJ}\hat{W}(x)|\psi NZ\rangle}{\langle\psi NZ|\hat{W}|\psi NZ\rangle} \tag{9}$$

then, if s_B is the sign of $\cos B$, the Monte Carlo probability distribution is proportional to $p(x) = \exp[A(x) + \ln|\cos(B(x))|$ and

$$\mathcal{E},\mathcal{O} = \frac{\int dx p(x) f_{E,O}}{\int dx p(x)} \equiv \langle f_{E,O}\rangle_p \tag{10}$$

with

$$f_{E,O} = s_B[\mathrm{Re}\,F_{E,O} - \tan B\mathrm{Im}\,F_{E,O}]\,. \tag{11}$$

As discussed in Ref. [8] the role of the cranking frequency which is contained in the propagator $\hat{W}$ is to enhance the contribution of the desired J value and the role of the chemical potentials is to enhance the contribution of the desired neutron and proton particle numbers. The results of

TABLE II

Monte Carlo energies in MeV for the selected yrast states of ^{170}Yb. All energies have been evaluated with $\beta = 1\,\mathrm{MeV}^{-1}$, $\langle s_B \rangle$ is the average sign and $\mathcal{O}$ is defined by Eq. (6). For the states evaluated with J_z projection only, the Monte Carlo $\langle J^2 \rangle$ is listed.

State	E	$E^\star$	$\langle s_B \rangle$	$\mathcal{O}$	$\langle J^2 \rangle$
0^+	-127.011 ± 0.048	0.000	0.73 ± 0.08	0.054 ± 0.009	
2^+	-126.934 ± 0.056	0.077 ± 0.074	0.50 ± 0.10	0.048 ± 0.014	
4^+	-126.864 ± 0.054	0.147 ± 0.072	0.71 ± 0.07	0.068 ± 0.011	
6^+	-126.630 ± 0.062	0.381 ± 0.078	0.68 ± 0.07	0.081 ± 0.013	
8^+	-126.058 ± 0.042	0.953 ± 0.064	0.59 ± 0.06	0.085 ± 0.011	77 ± 1
10^+	-125.731 ± 0.056	1.280 ± 0.074	0.68 ± 0.06	0.095 ± 0.018	114 ± 3
12^+	-125.387 ± 0.070	1.624 ± 0.085	0.65 ± 0.06	0.066 ± 0.008	158 ± 2

the calculation for $E(\beta NZJ) = \mathcal{E}/\mathcal{O}$ are shown in Table II. As previously mentioned the functional integrals show moderately strong sign fluctuations (the average sign is denoted as $\langle s_B \rangle$ in Table II), however they do not seem to affect appreciably the statistical uncertainty. First of all the average value of s_B decreases for increasing β. Since the HFB wave functions are already a good approximation to the exact wave functions, only low values of β are necessary. Within the statistical error $E(\beta = 1\,\mathrm{MeV}^{-1}, N, Z, J = 0) \approx E(\beta = 0.5\,\mathrm{MeV}^{-1}, N, Z, J = 0$ indicating that, at least for the ground state, $\beta = 0.5\,\mathrm{MeV}^{-1}$ is sufficently large. All Monte Carlo excitation energies are evaluated at $\beta = 1\,\mathrm{MeV}^{-1}$. The average sign of s_B ranges from 0.7 to 0.5 depending on the angular momentum. To understand the nature of the statistical error of the Monte Carlo estimate of $E(\beta, J)$ let us consider the expression for the statistical error of $E(\beta, J)$. If $X(f_{E,O})$ denotes the average of N_s statistically independent samples of $f_{E,O}$ distributed according to $p(x)$, the statistical error of the Monte Carlo estimate of $E(\beta, J) = X(f_E)/X(f_O)$ is the variance of the ratio $X(f_E)/X(f_O)$, which according with the central limit theorem (see for example Ref. [11] for a discussion of the central limit theorem in Monte Carlo calculations) is given by

$$\sigma(E(\beta, J)) = |\frac{\mathcal{E}}{\mathcal{O}}| \sqrt{\left[\frac{\sigma^2_{f_E}}{\mathcal{E}^2} + \frac{\sigma^2_{f_O}}{\mathcal{O}^2} - 2 \frac{\langle f_E f_O \rangle_p - \langle f_E \rangle_p \langle f_O \rangle_p}{\langle f_E \rangle_p \langle f_O \rangle_p} \right] \frac{1}{N_s}}, \qquad (12)$$

where the variances $\sigma_{f_E}, \sigma_{f_O}$ and the averages $\langle ... \rangle_p$ are evaluated with the distribution p defined above. In practice these variances and the expectation values are replaced with the corresponding Monte Carlo quantities.

Since the angular momentum projected HFB wave functions are good approximations to the exact eigenstates of the Hamiltonian we expect f_E/f_O and therefore $X(f_E)/X(f_O)$ to be almost constant (this last ratio is a random variable which depends on the chosen set of decorrelated samples) , and equal to the yrast energy. This argument does not seem to depend on the sign fluctuations. In order to understand the origin of the statistical error quantitatively, let us write $X(f_E) = E_0(X(f_O) + \eta)$, where η is a random variable and E_0 is a constant. The quantities $X(f_E), X(f_O)$ are random variables whose joint probability distribution is given by the central limit theorem with averages $\mathcal{E}, \mathcal{O}$ respectively. Selecting $E_0 = \mathcal{E}/\mathcal{O}$ one has $\langle\eta\rangle = 0$ (with respect to the joint probability distribution of $X(f_E)$ and $X(f_O)$). If the random variable η has a very narrow distribution as we change the sample, the statistical error would be small. It is easy to prove that $\sigma(X(f_E)/E_0X(f_O)) = \sigma(\eta)/\mathcal{O}$ (variances and expectation values refer to the joint probability distribution). Typically the variance of η is about 2×10^{-5}. Provided $X(f_O)$ is not too small, one obtains small statistical errors for the energies. For example, if we require relative accuracy for the energies of about 5×10^{-4} then the method can withstand values of $\mathcal{O} \sim 0.04$ or larger. Small values of $X(f_O)$ imply a strong suppression of the ratio of Eq. (6) caused either by the angular momentum projection (mostly) or by sign fluctuations (only moderately strong). Typically $X(f_O)$ ranges from 0.048 to 0.095 and it is usually determined with a relative accuracy ranging from 12 to 29% at the worst. Therefore we conclude that the smallness of the statistical error for the yrast energies is due to the smallness of the variance of the variable η, which has zero expectation values. Differently stated, the variables $X(f_E)$ and $X(f_O)$ which are Monte Carlo estimate (and the ratio of which is the yrast energy) are almost proportional to each other, *i.e.* their ratio is sample independent. From Eq. (10) and (11), and from the definitions (8) and (9) it follows that F_E and F_O are proportional to each other regardless of the integration variable x, which is due to the excellent approximation made by the HFB method. Presumably much stronger sign fluctuations would have to occur to decrease the value of $\mathcal{O}$ and increase the statistical error bejond acceptable values. The values of $X(f_O)$, are shown in Table II under the heading $\mathcal{O}$. It should be mentioned that some of the fluctuations seen in this calculation may be due to the finite accuracy of the matrix inversion and diagonalization routines.

Comparing the experimental excitation energies with the corresponding HFB and Monte Carlo value, of Table I and Table II, we see that the Monte Carlo values are lower then the corresponding HFB values giving a discrepancy of about 100 KeV with the experimental data.

As in Ref. [8] at high angular momentum, the full three dimensional angular momentum projector in Eq. (3) has been replaced by the projector to good J_z value. As a test of whether higher angular momentum states contribute to the energies, for these cases in Table II we also show the expectation values of $\hat{J}^2$. The number of samples, for fully angular momentum projected quantities, is about 100. Only for J_z projected quantities the number of samples is considerably larger.

TABLE III

Monte Carlo energy gaps (in MeV) and the pair strength for the neutron and proton intruder orbits for the angular momentum pair 12 (neutrons) and 10 (protons) evaluated at $\beta = 1\,\mathrm{MeV}^{-1}$.

State	Δ_n	Δ_p	S_n	S_p
0^+	0.95 ± 0.04	0.93 ± 0.05	2.1 ± 0.2	3.3 ± 0.5
2^+	0.84 ± 0.05	0.81 ± 0.04	1.7 ± 0.2	4.1 ± 0.3
4^+	0.89 ± 0.03	0.88 ± 0.03	1.7 ± 0.1	4.6 ± 0.3
6^+	0.95 ± 0.05	0.87 ± 0.06	2.1 ± 0.2	4.0 ± 0.3
8^+	0.80 ± 0.04	0.87 ± 0.03	1.9 ± 0.2	4.3 ± 0.1
10^+	0.69 ± 0.06	0.76 ± 0.04	2.2 ± 0.3	4.5 ± 0.2
12^+	0.48 ± 0.17	0.78 ± 0.03	2.1 ± 0.1	4.1 ± 0.2

In Table III we list the Monte Carlo results for the neutron and proton gaps defined as $\Delta_\tau = G_\tau \sqrt{\langle JJ|\hat{P}^\dagger \hat{P}|JJ\rangle}$, where $|JJ\rangle = \exp^{-\beta \hat{H}/2}\, \hat{P}_{JJ}|\psi NZ\rangle$ which can be compared with the corresponding HFB results of Table I. Also shown are the values of pair strengths for the intruder orbits, defined as $S_{n,p} = \langle JJ|[a^\dagger \times a^\dagger]^{J_p} \cdot [a \times a]^{J_p}|JJ\rangle$, with $J_p = 12, 10$ for the neutrons and protons, respectively. The agrement between the HFB and the corresponding Monte Carlo values is good.

3. Conclusions

In this work we have shown that yrast energies can be obtained using three dimensional angular momentum projected Monte Carlo methods. So far these methods have been used only with the pairing + quadrupole model (*i.e.* without quadrupole pairing). Here we have included such quadrupole pairing terms and shown that despite moderately strong sign oscillations in the Monte Carlo calculations, because of the use of a good approximation the yrast eigenfunctions, the statistical errors in the Monte Carlo calculation of the spectrum remains small. According to the argument given in the text either the HFB would have to be a poor approximation to the yrast wave

functions or much stronger sign fluctuations would have to occur in order to have large statistical errors in the evaluation of the yrast energies. These methods differ considerably from the method used in Ref. [12] which cannot give excitation energies and require very small values of the temperature in order to converge to the ground state.

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