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STOCHASTIC NETWORK VIEW ON HADRON PRODUCTION

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We demonstrate that hadron production viewed as formation of specific stochastic network can explain in natural way the power-law distributions of transverse mass spectra of pions found recently, which seem to substitute the expected Boltzmann statistical factor.

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Recently it has been pointed out [1] that properly normalized transverse mass spectra of π^0 mesons for $m_{\mathrm{T}} > 1$ GeV$/c^2$ obey specific m_{T} scaling, namely they follow the universal power-law curve,

$$\frac{dN}{m_{\mathrm{T}}^2 dm_{\mathrm{T}}} = c\left(\frac{m_{\mathrm{T}}}{\Lambda}\right)^{-P} \tag{1}$$

(which after integration over transverse momenta results in similar power law in masses and, according to [1], describes well the yields of neutral mesons from η to Υ, *i.e.*, for $m \simeq 0.5$–10 GeV$/c^2$) with $P \sim 8$–10, depending on energy and type of particles. This has been regarded as purely phenomenological observation of the apparent violation of the usual Boltzmann behaviour,

$$\frac{dN}{m_{\mathrm{T}}^2 dm_{\mathrm{T}}} = c_{\mathrm{BG}} \exp\left(-\frac{m_{\mathrm{T}}}{\Lambda}\right), \tag{2}$$

in the region of high m_{T}. In [2] we have noticed that this finding can be regarded also as confirmation that in describing hadronization process one should apply not the usual Boltzmann–Gibbs (BG) statistics but rather its nonextensive generalization, for example in the form of the Tsallis statistics [3], in which Eq. (2) is replaced by

$$\frac{dN}{m_{\mathrm{T}}^2 dm_{\mathrm{T}}} = c_q \left[1-(1-q)\frac{m_{\mathrm{T}}}{\Lambda}\right]^{\frac{1}{1-q}} . \tag{3}$$

Obviously, for large values of m_{T} (where dependence to scale Λ can be neglected) Eq. (3) becomes Eq. (1). However, contrary to Eq. (1) the above distribution offers simple interpretation of the fitted parameters, q and Λ. In particular, as demonstrated in [4] [1], q is entirely given by fluctuations of parameter $1/\Lambda$ in the usual BG approach, *i.e.*, in Eq. (2) [2], with nonextensivity parameter $q = 1 + 1/P$. Notice that for $q \to 1$ Tsallis statistics becomes the usual BG one and Eq. (3) becomes Eq. (2).

This, however, does not solve the puzzle noticed by [1], namely what is dynamical origin leading to the apparent scale-free character of the observed m_{T} spectra seen in Eq. (1) [3]. We would like to propose here one possible scenario resulting in Eq. (1). To this aim let us first mention that Tsallis distribution (3) describes also very well the formation of the so called complex free networks [6] (if one replaces m_{T} by the number of links k)[4]. Inspired by this observation we shall investigate in what follows the possibility that power law seen in Eq. (3) signals that hadronization can be viewed (at least from the limited perspective considered here) as a process of formation of some specific network taking place in the environment of gluons and quark–antiquark pairs ($q\bar{q}$), process in which their original actual energy–momentum distributions would be of second importance in comparison to the fact that, because of their mutual interactions, they connect to each other and that this process of connection has its distinctive dynamical consequences.

The proposed line or reasoning is the following. Suppose that we start with some initial state consisting of number n_0 of already existing ($q\bar{q}$) pairs, which we shall consider as equivalent to vertices in the usual network. We

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[1] We refer interested reader to [3, 4] for details and further references.

[2] Notice that usually $\Lambda = T$, *i.e.*, it is regarded as the "temperature" of the hadronizing system treated as a kind of "heat bath". Fluctuations could arise, for example, from the fact that, in cases considered here, such heat bath is in obvious way finite [5].

[3] Notice that scale parameter Λ in Eq. (1) can be absorbed in the normalization constant, as it was done in [1].

[4] Actually, in [6] the so called escort probability distribution was used, which results in $(\ldots)^{\frac{q}{1-q}}$ instead of $(\ldots)^{\frac{1}{1-q}}$ as here. This has no consequences in what concerns our work as both q can be simply translated to each other.

shall now add to them, in each consecutive time step, another vertex (*i.e.*, a new $(q\bar{q})$ pair), which can have k_0 possible connections to the old state. To be more specific, we regard our quarks as dressed by interaction with surrounding gluons and therefore somehow "excited". Each quark is supposed to interact with k other quarks, what in network terminology would mean that each quark has k links. We shall now assume that the "excitation" of quark mentioned above is proportional to k. We expect also that the number of links k should be proportional to the number of gluons participating in such "excitation", *i.e.*, existing in the vicinity of a given quark. In this case the natural consequence would be that the chances to interact with a given quark grow with the number of links k attached to it and are equal to

$$w(k_i) = \frac{k_i}{\sum k_i}\,, \tag{4}$$

i.e., that new links will be preferentially attached to quarks with already large values of k. This corresponds to building up of the so called preferential network, which evolves due to the occurrence of new $(q\bar{q})$ pairs from decaying gluons. Because of this the number of links is here twice the number of links in the usual network (*cf.* [7]). After time t one has therefore

$$\sum k_i = 2(2k_0 t) = 4k_0 t \tag{5}$$

links. This leads to the following growth equation for the i-th object (the i-th $(q\bar{q})$ pair):

$$\frac{\partial k_i}{k_i} = \frac{1}{\delta}\,\frac{\partial t}{t}\,, \tag{6}$$

where $\delta = 4$. Solving this equation for the initial condition stating that the i-th object appears in time t_i with the number of links equal to k_0 one gets that

$$k_i(t) = k_0\left(\frac{t}{t_i}\right)^{1/\delta}\,. \tag{7}$$

Notice that probability of forming $k_i(t) < k$ links is then given by

$$P(k_i(t) < k) = P\left(t_i > \frac{k_i^{\delta}\,t}{k_0^{\delta}}\right). \tag{8}$$

Assuming now uniform probability distribution of occurrence of objects $(q\bar{q})$, the probability of adding to our system a new such object in the unit of time is given by

$$P(t_i) = \frac{1}{t}\,. \tag{9}$$

It means therefore that, because of Eq. (8),

$$P(k_i(t) < k) = P\left(t_i > \frac{k_i^\delta\, t}{k_0^\delta}\right) = 1 - P\left(t_i \le \frac{k_0^\delta}{k^\delta}\, t\right) = 1 - \left(\frac{k_0}{k}\right)^\delta \qquad (10)$$

and probability of forming an object with k links can be written as

$$P(k) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{k_0^\delta}{1+\delta}\, k^{-(1+\delta)}\,, \qquad (11)$$

i.e., the resulting distribution of the number of links is independent of time and has the characteristic power-like form:

$$P(k) \sim k^{-\gamma} \qquad (12)$$

with $\gamma = 1+\delta$ (for (5) $\gamma = 5$). For the general case of [5]

$$\sum k_i = 2t(2k_0 - 1) \qquad (13)$$

one has $\delta = \frac{2(2k_0-1)}{k_0} = 4 - \frac{2}{k_0}$, *i.e.*, the same Eq. (12) but with $\gamma = 5 - \frac{2}{k_0}$ [6].

The crucial point now is our conjecture that the number of links k determines the transverse mass of emitted particle m_{T}. Assuming that

$$m_{\mathrm{T}} \sim k^\alpha \qquad (14)$$

one gets immediately

$$P(m_{\mathrm{T}}) \sim m_{\mathrm{T}}^{-\frac{\gamma}{\alpha}}\, m_{\mathrm{T}}^{\frac{1-\alpha}{\alpha}} = m_{\mathrm{T}}^{-\beta}\,, \qquad \text{where} \qquad \beta = 1 + \frac{\gamma - 1}{\alpha}\,. \qquad (15)$$

This completes our derivation of (1) with $P = \beta$. It arises because of the conjecture (14) connecting the actual value of m_{T} with the number of links in some directional network growing process defined above. The first parameter here is k_0, which denotes the number of links with which the new vertex (here the $(q\bar{q})$ pair) will join the already existing network of such vertices. However, as one can see, with increasing k_0 one quickly obtains asymptotic value of $\gamma = 5$. The other parameter, α, describes in what way the new links can be chosen in momentum space in respect to allowed directions. For example, for totally random distribution one should choose, analogously

[5] It accounts for the fact that new vertex $(q\bar{q})$ arises from the gluon $\to q\bar{q}$ process, *i.e.*, one gluon disappears and this corresponds to diminishing the number of links by one for each two vertices.

[6] Notice that in the approach to networks using Tsallis statistics [6] one gets $P(k) \sim k^{\frac{q}{1-q}}$, *i.e.*, $\gamma = 5$ corresponds to $q = 1.25$.

to deterministic diffusion process, $\alpha = \frac{1}{2}$ [7]. Other values of parameter α would correspond to other special diffusion cases (see below). In the case of $\alpha = \frac{1}{2}$ one has $\beta = 9$ for Eq. (5) and $\beta = 9 - \frac{4}{k_0}$ for Eq. (13), which numerically gives us values of P equal to 7.66, 8.0 and 8.5 for, respectively, $k_0 = 3, 4$ and 8. As one can see, they agree with those quoted in [1] (and listed before).

It is worth to stress that the proposed approach leads naturally to distinct possible behaviours of transverse mass distributions (all others will be their suitable combination):

(a) The power-like distribution of the type given by Eq. (12) (which was our main motivation). It corresponds to the case of large excitations for which probability of connecting a new quark to the one already existing in the system depends on the number of the actual connections realised so far. Large number of connections results then in large excitation, what means that one has large emission of gluons and what, finally, enhances chances of connection to such a quark.

(b) If one assumes instead that the new quark attaches itself to the already existing one with equal probability (such would be the situation for small excitations, *i.e.*, for small p_{T}) one gets instead exponential distribution of links,

$$P(k) \sim \exp\left(-\frac{k}{\langle k\rangle}\right), \tag{16}$$

which results in different distributions of m_{T} depending on the type of diffusion given by parameter α in Eq. (14) and ranging from

$$P(m_{\mathrm{T}}) \sim \exp\left(-\frac{m_{\mathrm{T}}^2}{\langle m_{\mathrm{T}}^2\rangle}\right) \qquad \text{for} \qquad \alpha = \frac{1}{2} \tag{17}$$

when the full fledged diffusion is allowed, to

$$P(m_{\mathrm{T}}) \sim \exp\left(-\frac{m_{\mathrm{T}}}{\langle m_{\mathrm{T}}\rangle}\right) \qquad \text{for} \qquad \alpha = 1\,, \tag{18}$$

when there is no diffusion. This would be the case of quarks located on the periphery of the region of hadronization in which case they could interact only with interior quarks, and in such case $m_{\mathrm{T}} \sim k$.

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[7] Deterministic diffusion concept is widely discussed in [8]. Notice that lack of such diffusion in momentum space would mean that interactions are highly correlated, its presence indicates chaotic evolution in momentum space.

To summarize — prompted by the observation of the power-like behaviour of the transverse mass spectra reported in [1] we have considered here a possibility that they can be a reflection not so much of any special kind of equilibrium (resulting in some specific statistics) but rather of the formation process, which follows (directs) free networks formation pattern discussed widely in the literature and found in many branches of sciences [6, 7]. Identifing vertices in such network as $(q\bar{q})$ pairs and gluons as links we were able to derive Eq. (1), obtained in [1] by analysis of some experimental data, when we assume that the observed $m_{\rm T}$ reflects somehow the number of links in such network (Eq. (14)). The resulting power-like distribution is rather universal (see, for example [9] where similar power-like distributions observed in other branches of high energy phenomenology have been attributed to the apparent self organizing character of the corresponding processes) and its power index depends mainly on the type of the allowed diffusion process as given by parameter α. The type of this deterministic diffusion process in the momentum space which is allowed in a given experimental scenario leads then to three kinds of distinct characteristic spectra of $m_{\rm T}$ out of which all observed spectra could be composed.

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