

CAN ONE OF THREE RIGHTHANDED NEUTRINOS  
BE LIGHT ENOUGH TO PRODUCE  
A SMALL LSND EFFECT?\*

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(Received May 24, 2004)

It is shown on the ground of a simple  $6 \times 6$  neutrino mixing model that one of three conventional sterile (righthanded) neutrinos, if light enough, may be consistently used for explaining a *small* LSND effect. Then, it is still considerably heavier than the three active (lefthanded) neutrinos, so that a kind of *soft* seesaw mechanism can work. The usual condition that the Majorana lefthanded component of the overall  $6 \times 6$  neutrino mass matrix ought to vanish, *implies* the smallness of active-neutrino masses *versus* sterile-neutrino masses, when three mixing angles between both sorts of neutrinos are small. In the presented model, the mass spectrum of active neutrinos comes out roughly degenerate, lying in the range  $(5-7.5) \times 10^{-2}$  eV, *if* there is a *small* LSND effect with the amplitude of the order  $10^{-3}$  and with the mass-squared splitting  $\sim 1$  eV<sup>2</sup>.

PACS numbers: 12.15.Ff, 14.60.Pq, 12.15.Hh

It is well known that the neutrino experiments with solar  $\nu_e$ 's [1], atmospheric  $\nu_\mu$ 's [2], long-baseline accelerator  $\nu_\mu$ 's [3] and long-baseline reactor  $\bar{\nu}_e$ 's [4] are very well described by oscillations of three active neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ , where the mass-squared splittings of the related neutrino mass states  $\nu_1, \nu_2, \nu_3$  are estimated to be  $\Delta m_{\text{sol}}^2 \equiv \Delta m_{21}^2 \sim 7 \times 10^{-5}$  eV<sup>2</sup> and  $\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 \sim 2.5 \times 10^{-3}$  eV<sup>2</sup> [5]. The neutrino mixing matrix  $U^{(3)} = (U_{\alpha i}^{(3)})$  ( $\alpha = e, \mu, \tau$  and  $i = 1, 2, 3$ ), responsible for the unitary transformation

$$\nu_\alpha = \sum_i U_{\alpha i}^{(3)} \nu_i, \quad (1)$$

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\* Work supported in part by the Polish State Committee for Scientific Research (KBN), grant 2 P03B 129 24 (2003-2004).

is experimentally consistent with the global bilarge form

$$U^{(3)} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (2)$$

where  $c_{12} = \cos \theta_{12}$  and  $s_{12} = \sin \theta_{12}$  with  $\theta_{12} \sim 33^\circ$ , while  $U_{e3}^{(3)} = s_{13} \exp(-i\delta)$  is neglected according to the negative results of neutrino disappearance experiments with short-baseline reactor  $\bar{\nu}_e$ 's, in particular the Chooz experiment [6] that estimates the experimental upper bound for  $s_{13}^2$  as  $s_{13}^2 < 0.03$ .

However, the signal of  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  appearance reported by the LSND experiment with short-baseline accelerator  $\bar{\nu}_\mu$ 's [7] requires for its interpretation in terms of neutrino oscillations a third neutrino mass-squared splitting, say,  $\Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2$ . This cannot be justified by the use of only three neutrinos (unless the CPT invariance of neutrino oscillations is seriously violated, leading to considerable mass splittings of neutrinos and antineutrinos [8]; in the present note the CPT invariance is assumed to hold). The LSND result will be tested soon in the ongoing MiniBooNE experiment [9]. If this test confirms the LSND result, we will need the light sterile neutrinos in addition to three active neutrinos to introduce extra mass splittings.

While the 3+1 neutrino models with one light sterile neutrino are considered to be disfavored by present data [10], the 3+2 or 3+3 neutrino schemes with two or three light sterile neutrinos may *a priori* provide a better description of current neutrino oscillations including the LSND effect (for a statistical discussion showing the better compatibility of all short-baseline neutrino experiments within 3+2 models than within 3+1 models *cf.* Ref. [11]; in Ref. [12] we argue, however, that the simple 3+2 models are not more effective in this description than the simple 3+1 models: both kinds of them may be consistent with a *small* LSND effect having the amplitude of, say, the order  $10^{-3}$ ).

In the present note, we discuss the question to what extent three conventional sterile (righthanded) neutrinos may help to reconcile the possible LSND effect with the well established results of solar and atmospheric oscillation experiments (such a possibility was discussed before, *cf.* Ref. [13], where one of the righthanded neutrinos becomes nearly massless due to some imposed lepton symmetries).

To this end, consider the usual neutrino theory, where the Majorana lefthanded component  $M^{(L)}$  of the overall  $6 \times 6$  neutrino mass matrix  $M^{(6)} = \left( M_{\alpha\beta}^{(6)} \right)$  ( $\alpha, \beta = e, \mu, \tau, e_s, \mu_s, \tau_s$ ) is zero:

$$M^{(6)} = \begin{pmatrix} 0 & M^{(D)} \\ M^{(D)T} & M^{(R)} \end{pmatrix}. \tag{3}$$

Here, three active neutrinos  $\nu_{e,\mu,\tau} \equiv \nu_{e,\mu,\tau L}$  and three conventional sterile antineutrinos  $\nu_{e_s,\mu_s,\tau_s} \equiv (\nu_{e,\mu,\tau R})^c$  form the basis of a 3+3 neutrino model. Then, the overall  $6 \times 6$  neutrino mixing matrix  $U^{(6)} = (U_{\alpha i}^{(6)})$  transforms unitarily flavor neutrinos  $\nu_\alpha$  ( $\alpha = e, \mu, \tau, e_s, \mu_s, \tau_s$ ) into mass neutrinos  $\nu_i$  ( $i = 1, 2, 3, 4, 5, 6$ ) :

$$\nu_\alpha = \sum_i U_{\alpha i}^{(6)} \nu_i. \tag{4}$$

In the flavor representation, where the charged-lepton mass matrix is diagonal, the  $6 \times 6$  mixing matrix  $U^{(6)}$  is at the same time the  $6 \times 6$  diagonalizing matrix for the  $6 \times 6$  mass matrix  $M^{(6)}$  :

$$U^{(6)\dagger} M^{(6)} U^{(6)} = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6) \tag{5}$$

and so, inversely

$$M_{\alpha\beta}^{(6)} = \sum_i U_{\alpha i}^{(6)} m_i U_{\beta i}^{(6)*}. \tag{6}$$

To proceed further we will assume the simple  $6 \times 6$  neutrino mixing model, where

$$U^{(6)} = \begin{pmatrix} U^{(3)} & 0^{(3)} \\ 0^{(3)} & 1^{(3)} \end{pmatrix} \begin{pmatrix} C^{(3)} & S^{(3)} \\ -S^{(3)} & C^{(3)} \end{pmatrix} = \begin{pmatrix} U^{(3)}C^{(3)} & U^{(3)}S^{(3)} \\ -S^{(3)} & C^{(3)} \end{pmatrix} \tag{7}$$

with  $U^{(3)}$  given in Eq. (2) and

$$C^{(3)} = \begin{pmatrix} c_{14} & 0 & 0 \\ 0 & c_{25} & 0 \\ 0 & 0 & c_{36} \end{pmatrix}, \quad S^{(3)} = \begin{pmatrix} s_{14} & 0 & 0 \\ 0 & s_{25} & 0 \\ 0 & 0 & s_{36} \end{pmatrix}, \tag{8}$$

where  $c_{14} = \cos \theta_{14}$ ,  $s_{14} = \sin \theta_{14}$  and so on. Thus, in Eq. (7)

$$U^{(3)}C^{(3)} = \begin{pmatrix} c_{12}c_{14} & s_{12}c_{25} & 0 \\ -\frac{1}{\sqrt{2}}s_{12}c_{14} & \frac{1}{\sqrt{2}}c_{12}c_{25} & \frac{1}{\sqrt{2}}c_{36} \\ \frac{1}{\sqrt{2}}s_{12}c_{14} & -\frac{1}{\sqrt{2}}c_{12}c_{25} & \frac{1}{\sqrt{2}}c_{36} \end{pmatrix},$$

$$U^{(3)}S^{(3)} = \begin{pmatrix} c_{12}s_{14} & s_{12}s_{25} & 0 \\ -\frac{1}{\sqrt{2}}s_{12}s_{14} & \frac{1}{\sqrt{2}}c_{12}s_{25} & \frac{1}{\sqrt{2}}s_{36} \\ \frac{1}{\sqrt{2}}s_{12}s_{14} & -\frac{1}{\sqrt{2}}c_{12}s_{25} & \frac{1}{\sqrt{2}}s_{36} \end{pmatrix}. \tag{9}$$

Due to Eq. (7) with (9), the unitary mixing transformation  $\nu_i = \sum_{\alpha} U_{\alpha i}^{(6)*} \nu_{\alpha}$ , inverse to (4), reads explicitly

$$\begin{aligned}
 \nu_1 &= c_{14} \left( c_{12} \nu_e - s_{12} \frac{\nu_{\mu} - \nu_{\tau}}{\sqrt{2}} \right) - s_{14} \nu_{e_s}, \\
 \nu_2 &= c_{25} \left( s_{12} \nu_e + c_{12} \frac{\nu_{\mu} - \nu_{\tau}}{\sqrt{2}} \right) - s_{25} \nu_{\mu_s}, \\
 \nu_3 &= c_{36} \frac{\nu_{\mu} + \nu_{\tau}}{\sqrt{2}} - s_{36} \nu_{\tau_s}, \\
 \nu_4 &= s_{14} \left( c_{12} \nu_e - s_{12} \frac{\nu_{\mu} - \nu_{\tau}}{\sqrt{2}} \right) + c_{14} \nu_{e_s}, \\
 \nu_5 &= s_{25} \left( s_{12} \nu_e + c_{12} \frac{\nu_{\mu} - \nu_{\tau}}{\sqrt{2}} \right) + c_{25} \nu_{\mu_s}, \\
 \nu_6 &= s_{36} \frac{\nu_{\mu} + \nu_{\tau}}{\sqrt{2}} + c_{36} \nu_{\tau_s}.
 \end{aligned} \tag{10}$$

Here,  $\nu_{\mu}$  and  $\nu_{\tau}$  mix maximally, since  $(\nu_{\mu} - \nu_{\tau})/\sqrt{2}$  and  $(\nu_{\mu} + \nu_{\tau})/\sqrt{2}$  do not mix at all. More generally,  $\nu_e, (\nu_{\mu} - \nu_{\tau})/\sqrt{2}, \nu_{e_s}$  and  $\nu_{\mu_s}$  do not mix at all with  $(\nu_{\mu} + \nu_{\tau})/\sqrt{2}$  and  $\nu_{\tau_s}$ .

Applying Eqs. (6) and (7) with (9), we obtain

$$M^{(L)} = U^{(3)} \begin{pmatrix} c_{14}^2 m_1 + s_{14}^2 m_4 & 0 & 0 \\ 0 & c_{25}^2 m_2 + s_{25}^2 m_5 & 0 \\ 0 & 0 & c_{36}^2 m_3 + s_{36}^2 m_6 \end{pmatrix} U^{(3)\dagger}, \tag{11}$$

$$M^{(D)} = U^{(3)} \begin{pmatrix} c_{14} s_{14} (m_4 - m_1) & 0 & 0 \\ 0 & c_{25} s_{25} (m_5 - m_2) & 0 \\ 0 & 0 & c_{36} s_{36} (m_6 - m_3) \end{pmatrix} \tag{12}$$

and

$$M^{(R)} = \begin{pmatrix} c_{14}^2 m_4 + s_{14}^2 m_1 & 0 & 0 \\ 0 & c_{25}^2 m_5 + s_{25}^2 m_2 & 0 \\ 0 & 0 & c_{36}^2 m_6 + s_{36}^2 m_3 \end{pmatrix}. \tag{13}$$

Due to Eq. (11), the condition  $M^{(L)} = 0$  tells us that

$$m_1 = -t_{14}^2 m_4, \quad m_2 = -t_{25}^2 m_5, \quad m_3 = -t_{36}^2 m_6, \tag{14}$$

where  $t_{14} = s_{14}/c_{14} = \tan \theta_{14}$  and so on. Then, Eqs. (12) and (13) take the forms

$$M^{(D)} = U^{(3)} \begin{pmatrix} t_{14}m_4 & 0 & 0 \\ 0 & t_{25}m_5 & 0 \\ 0 & 0 & t_{36}m_6 \end{pmatrix} \tag{15}$$

and

$$M^{(R)} = \begin{pmatrix} (1 - t_{14}^2)m_4 & 0 & 0 \\ 0 & (1 - t_{25}^2)m_5 & 0 \\ 0 & 0 & (1 - t_{36}^2)m_6 \end{pmatrix}. \tag{16}$$

Hence, we calculate

$$-M^{(D)} \frac{1}{M^{(R)}} M^{(D)T} = U^{(3)} \begin{pmatrix} \frac{m_1}{1-t_{14}^2} & 0 & 0 \\ 0 & \frac{m_2}{1-t_{25}^2} & 0 \\ 0 & 0 & \frac{m_3}{1-t_{36}^2} \end{pmatrix} U^{(3)\dagger}. \tag{17}$$

If  $t_{14}^2 = |m_1/m_4| \ll 1$ ,  $t_{25}^2 = |m_2/m_5| \ll 1$  and  $t_{36}^2 = |m_3/m_6| \ll 1$ , as it is the case in the seesaw mechanism, two expressions

$$-M^{(D)} \frac{1}{M^{(R)}} M^{(D)T} \simeq U^{(3)} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^{(3)\dagger} \tag{18}$$

and

$$M^{(R)} \simeq \begin{pmatrix} m_4 & 0 & 0 \\ 0 & m_5 & 0 \\ 0 & 0 & m_6 \end{pmatrix} \tag{19}$$

describe approximately the Majorana mass matrices for active (lefthanded) and sterile (righthanded) neutrinos, respectively (the second mass matrix is here diagonal). But, *a priori*, it is not necessary for the small ratios  $t_{14}^2 = |m_1/m_4| \ll 1$ ,  $t_{25}^2 = |m_2/m_5| \ll 1$  and  $t_{36}^2 = |m_3/m_6| \ll 1$  to be so drastically small as in the case of seesaw mechanism. We will see that this alternative scenario may be consistently realized, when one of three sterile (righthanded) neutrinos produces a small LSND effect.

In the case of  $6 \times 6$  mixing matrix  $U^{(6)}$  given in Eqs. (7) and (9), we obtain the following neutrino oscillation probabilities in the vacuum, if  $x_{31} \simeq x_{32}$ ,  $x_{41} \simeq x_{42} \simeq x_{43}$ ,  $x_{51} \simeq x_{52} \simeq x_{53}$ ,  $x_{61} \simeq x_{62} \simeq x_{63}$  and  $c_{14}^2 \gg s_{14}^2$ ,  $c_{25}^2 \gg s_{25}^2$ ,  $c_{36}^2 \gg s_{36}^2$  :

$$P(\nu_e \rightarrow \nu_e) \simeq 1 - 4c_{12}^2 s_{12}^2 \sin^2 x_{21} - 4c_{12}^2 s_{14}^2 \sin^2 x_{41} - 4s_{12}^2 s_{25}^2 \sin^2 x_{51}, \tag{20}$$

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - c_{12}^2 s_{12}^2 \sin^2 x_{21} - \sin^2 x_{31} - 2s_{12}^2 s_{14}^2 \sin^2 x_{41} - 2c_{12}^2 s_{25}^2 \sin^2 x_{51} - 2s_{36}^2 \sin^2 x_{61} \tag{21}$$

and

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \simeq 2c_{12}^2 s_{12}^2 \sin^2 x_{21} + 2c_{12}^2 s_{12}^2 (s_{14}^2 - s_{25}^2)(s_{14}^2 \sin^2 x_{41} - s_{25}^2 \sin^2 x_{51}), \tag{22}$$

where  $x_{ji} \equiv 1.27\Delta m_{ji}^2 L/E$  and  $\Delta m_{ji}^2 \equiv m_j^2 - m_i^2$ . In Eqs. (20) and (21), quadratic terms with respect to the small parameters  $s_{14}^2$ ,  $s_{25}^2$  and  $s_{36}^2$  are neglected.

Hence, for solar  $\nu_e$ 's, Chooz reactor  $\bar{\nu}_e$ 's, atmospheric  $\nu_\mu$ 's and LSND accelerator  $\bar{\nu}_\mu$ 's, where  $(x_{21})_{\text{sol}} \sim O(\pi/2)$ ,  $(x_{31})_{\text{Chooz}} \simeq (x_{31})_{\text{atm}} \sim O(\pi/2)$  and  $(x_{41})_{\text{LSND}} \sim O(\pi/2)$ , respectively, we deduce the following oscillation probabilities :

$$P(\nu_e \rightarrow \nu_e)_{\text{sol}} \simeq 1 - 4c_{12}^2 s_{12}^2 \sin^2(x_{21})_{\text{sol}} - 2(c_{12}^2 s_{14}^2 + s_{12}^2 s_{25}^2), \tag{23}$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\text{Chooz}} \simeq 1 - 2(c_{12}^2 s_{14}^2 + s_{12}^2 s_{25}^2), \tag{24}$$

$$P(\nu_\mu \rightarrow \nu_\mu)_{\text{atm}} \simeq 1 - \sin^2(x_{31})_{\text{atm}} - (s_{12}^2 s_{14}^2 + c_{12}^2 s_{25}^2 + s_{36}^2) \tag{25}$$

and

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)_{\text{LSND}} \simeq 2c_{12}^2 s_{12}^2 (s_{14}^2 - s_{25}^2) \left( s_{14}^2 \sin^2(x_{41})_{\text{LSND}} - \frac{1}{2}s_{25}^2 \right), \tag{26}$$

if  $x_{21} \ll x_{31} \ll x_{41}, x_{51}, x_{61}$  *i.e.*,  $m_1^2, m_2^2, m_3^2 \ll m_4^2, m_5^2, m_6^2$ . In Eq. (26), it is assumed in addition that  $x_{41} \ll x_{51}$  *i.e.*,  $m_4^2 \ll m_5^2$ . Of course, for solar  $\nu_e$ 's the MSW matter mechanism is significant, leading to the experimentally accepted LMA solar solution.

If there is a small LSND effect with the amplitude of the order  $10^{-3}$ , then due to Eq. (26) we can write

$$(s_{14}^2 - s_{25}^2)^{1/2} \left( s_{14}^2 - \frac{s_{25}^2}{2 \sin^2(x_{41})_{\text{LSND}}} \right)^{1/2} \sim \left( \frac{10^{-3}}{2c_{12}^2 s_{12}^2} \right)^{1/2} \sim 0.049, \tag{27}$$

where  $\theta_{12} \sim 33^\circ$  giving  $c_{12}^2 \sim 0.70$  and  $s_{12}^2 \sim 0.30$ . In the case of  $1 \gg s_{14}^2 \gg s_{25}^2 \gg s_{36}^2$  *i.e.*,  $1 \gg t_{14}^2 = |m_1/m_4| \gg t_{25}^2 = |m_2/m_5| \gg t_{36}^2 = |m_3/m_6|$  (even if  $m_1^2 < m_2^2 < m_3^2$ ), Eq. (27) gives

$$s_{14}^2 \sim \left( \frac{10^{-3}}{2c_{12}^2 s_{12}^2} \right)^{1/2} \sim 0.049. \tag{28}$$

Hence,  $|m_1/m_4| = t_{14}^2 \sim 0.052 \ll 1$ , though this ratio is not so dramatically small as in the seesaw mechanism. If  $\Delta m_{41}^2 \sim 1 \text{ eV}^2$ , then  $|m_4| \sim 1 \text{ eV}$  and we predict that  $|m_1| \sim 5.2 \times 10^{-2} \text{ eV}$ . In this case, from the experimental estimates  $\Delta m_{21}^2 \sim 7 \times 10^{-5} \text{ eV}^2$  and  $\Delta m_{32}^2 \sim 2.5 \times 10^{-3} \text{ eV}^2$  we deduce that

$|m_2| \sim 5.3 \times 10^{-2}$  eV and  $|m_3| \sim 7.3 \times 10^{-2}$  eV. Thus, in this model, the mass spectrum of active neutrinos is roughly degenerate, although  $\Delta m_{21}^2 \ll \Delta m_{32}^2 \simeq \Delta m_{31}^2$ .

Making use of the estimate (28), we get from Eqs. (23), (24) and (25) the following estimations compatible with neutrino experimental data:

$$P(\nu_e \rightarrow \nu_e)_{\text{sol}} \sim 1 - 0.83 \sin^2(x_{21})_{\text{sol}} - 0.069, \quad (29)$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\text{Chooz}} \sim 1 - 0.069 \quad (30)$$

and

$$P(\nu_\mu \rightarrow \nu_\mu)_{\text{atm}} \simeq 1 - \sin^2(x_{31})_{\text{atm}} - 0.014, \quad (31)$$

where  $\theta_{12} \sim 33^\circ$ . The neglected quadratic terms in  $s_{14}^2$ ,  $s_{25}^2$  and  $s_{36}^2$  would make the values of the shifts 0.069 and 0.014 as well as the oscillation amplitudes 0.83 and 1 in Eqs. (29), (30) and (31) a little bit smaller.

For larger LSND effect the parameter  $s_{14}^2$  is larger, and thus the small deviations in Eqs. (29), (30) and (31) from pure three-active-neutrino oscillations grow, becoming more significant.

In conclusion, we have shown in this note on the ground of a simple  $6 \times 6$  neutrino mixing model that one of three conventional sterile (righthanded) neutrinos, if light enough, may be consistently used for explaining a *small* LSND effect. Then, it is still considerably heavier than the three active (lefthanded) neutrinos, so that a kind of a *soft* seesaw mechanism can work.

The usual condition that the Majorana lefthanded mass matrix  $M^{(L)}$  ought to vanish, *implies* the smallness of active-neutrino masses *versus* sterile-neutrino masses, when three mixing angles  $\theta_{14}, \theta_{25}, \theta_{36}$  between both sorts of neutrinos are small (more precisely,  $\theta_{14}, \theta_{25}, \theta_{36}$  are mixing angles between active neutrinos and the corresponding conventional sterile antineutrinos). In the present model, the mass spectrum of active neutrinos comes out roughly degenerate, lying in the range  $(5-7.5) \times 10^{-2}$  eV, *if* there is a *small* LSND effect with the amplitude of the order  $10^{-3}$  and with the mass-squared splitting  $\sim 1$  eV<sup>2</sup>.

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