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THE ASSOCIATED Σ PRODUCTION AND THE Σ-NUCLEUS POTENTIAL*

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Kaon spectrum from (π^-, K^+) reaction on ^{28}Si target in the region of Σ production is analyzed in impulse approximation for different strengths of the Σ single particle potential. It is concluded that this potential is repulsive, and its strength is consistent with the Nijmegen model F of the hyperon–nucleon interaction.

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1. Introduction

Our present knowledge of the Σ hyperon interaction with nuclear matter, represented by the single particle (s.p.) potential U_Σ, comes from the following sources:

— Strong interaction shifts ϵ_a and widths Γ_a of the observed Σ^- atomic levels [1, 2].

— Final state interaction of Σ hyperons in the strangeness exchange (K^-, π) reactions, which affects the observed pion spectrum (see, *e.g.*, [3], and [4] — hereafter referred to as I).

— Free space hyperon–nucleon scattering data, to which the hyperon–nucleon interaction potential may be fitted (see, *e.g.*, [5–8]), and with this potential one may calculate U_Σ (see, *e.g.*, [9–11]) [1].

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[1] This indirect way of determining U_Σ is burdened by the scarcity of the hyperon–nucleon two-body data. In fact, the procedure is often used in the reversed direction: knowing U_Σ helps to determine the hyperon–nucleon interaction (see, *e.g.*, [2]).

All these sources imply that the ΣN interaction is well represented by the Nijmegen model F of the baryon–baryon interaction [6], which leads to a repulsive U_Σ with the strength of the repulsion of about 24 MeV at the equilibrium density of nuclear matter [11, 12].

Recently, a new source of information on U_Σ became available: the final state interaction of Σ hyperons in the associated production reaction, (π, K^+). The first measurement of the inclusive K^+ spectrum from the (π^-, K^+) reaction on a silicon target was performed in KEK at pion momentum of 1.2 GeV/c [13].

In the present paper we analize the inclusive K^+ spectrum reported in [13] and try to determine U_Σ consistent with this spectrum. We extend the procedure applied in I in the case of the strangeness exchange reaction to the case of the associated production. We simply calculate the cross section for the inclusive (π^-, K^+) reaction in the impulse approximation with different strengths of U_Σ, and compare it with experimental results.

The paper is organized as follows. In Section 2, we calculate in the impulse approximation the cross section for the (π^-, K^+) reaction, and obtain final expressions for the inclusive spectrum of the produced kaons. In Section 3, we present our results for the (π^-, K^+) reaction on the ^{48}Si target and compare them with the experimental results of [13]. Discussion of our results and conclusions are presented in Section 4. The determination of the transition matrix for the elementary process $\pi^+ + P \to K^+ + \Sigma^-$ is presented in Appendix A.

2. Cross section for the inclusive (π^-, K^+) reaction in the impulse approximation

In the Σ s.p. model, the motion of Σ^- in the hypernuclear state produced in the (π^-, K^+) reaction is described by the wave function $\psi_\Sigma(\boldsymbol{r})$ which is the solution of the s.p. Schrödinger equation with the s.p. potential $U_\Sigma(r) = V_\Sigma(r) + iW_\Sigma(r)$, where W_Σ represents the absorption due to the $\Sigma\Lambda$ conversion process $\Sigma^- P \to \Lambda N$. The target protons involved in the elementary process $\pi^- P \to K^+ \Sigma^-$ are described by the wave functions $\psi_P(\boldsymbol{r})$ which are bound state solutions of the s.p. Schrödinger equation with the shell model potential $V_P(r)$.

We want to calculate the cross section for the (π^-, K^+) reaction in which the pion π^- with momentum $\boldsymbol{k}_\pi$ (in units of $\hbar$) and energy E_π hits a target proton in the state ψ_P (with s.p. energy e_P) and emerges in the final state as kaon K^+ in the direction $\hat{k}_K$ with energy E_K (both E_π and E_K are total energies including rest masses), whereas the hit proton emerges in the final state as a Σ^- hyperon with momentum $\boldsymbol{k}_\Sigma$. We apply the impulse approximation with K^+ and π^- plane waves, assume a zero-range spinindependent interaction for the elementary process $\pi^- P \rightarrow K^+ \Sigma^-$ (with a constant transition matrix t) and obtain (with spins suppressed in the notation):

$$\frac{d^3\sigma}{d\hat{k}_\Sigma d\hat{k}_K dE_K} = \frac{E_K E_\pi M_\Sigma c^2 k_K k_\Sigma}{(2\pi)^5 (\hbar c)^6 k_\pi} \left| t \int d\boldsymbol{r} \exp(-i\boldsymbol{qr}) \psi_{\Sigma,\boldsymbol{k}_\Sigma}(\boldsymbol{r})^{(-)*} \psi_P(\boldsymbol{r}) \right|^2 , \tag{1}$$

where the momentum transfer $\boldsymbol{q} = \boldsymbol{k}_K - \boldsymbol{k}_\pi$, and $\psi_{\Sigma,\boldsymbol{k}_\Sigma}(\boldsymbol{r})^{(-)}$ is the Σ scattering wave function which behaves asymptotically as $\exp(i\boldsymbol{k}_\Sigma \boldsymbol{r})$ + incoming wave.

The energy conservation imposes the following relation between the energies of the particles involved in the (K^-, π^+) reaction:

$$\epsilon_\Sigma = \frac{\hbar^2 k_\Sigma^2}{2M_\Sigma} = M_P c^2 - M_\Sigma c^2 + e_P + E_\pi - E_K \,. \tag{2}$$

Notice that the recoil of the hypernucleus is neglected here.

If only the energy spectrum of kaons at fixed $\hat{k}_K$ is measured then this spectrum, $d^2\sigma/d\hat{k}_K dE_K$, is obtained by integrating cross section (1) over $\hat{k}_\Sigma$:

$$\frac{d^2\sigma(l_P j_P)}{d\hat{k}_K dE_K} = \int d\hat{k}_\Sigma \left\{ \frac{d^3\sigma(l_P j_P)}{d\hat{k}_\Sigma d\hat{k}_K dE_K} \right\} . \tag{3}$$

Here, we have indicated explicitly the quantum numbers $l_P j_P$ of the s.p. state ψ_P of the proton on which the elementary process $\pi^- P \rightarrow K^+ \Sigma^-$ takes place. To get the experimental pion spectrum, we have to sum expression (3) over all states occupied by target protons.

Let us consider the case when the $l_P j_P$ proton shell is closed, *i.e.*, when all the $2j_P + 1$ magnetic substates of the $l_P j_P$ shell are occupied by protons (as is the case with the ^{28}Si target considered in the next section). When we introduce the spin coordinate ξ and the Σ and P magnetic quantum numbers μ_Σ and m_P, we may write the total contribution of the $l_P j_P$ proton shell to the pion spectrum in the form:

$$\frac{d^2\sigma(l_P j_P)}{d\hat{k}_K dE_K} = \frac{E_K E_\pi M_\Sigma c^2 k_K k_\Sigma}{(2\pi)^5 (\hbar c)^6 k_\pi} |t|^2 S(l_P j_P) \,, \tag{4}$$

$$S(l_P j_P) = \sum_{\mu_\Sigma m_P} \int d\hat{k}_\Sigma \left| \int d\tau \exp(-i\boldsymbol{qr}) \psi_{\Sigma,\boldsymbol{k}_\Sigma \mu_\Sigma}(\boldsymbol{r},\xi)^{(-)*} \psi_{P,l_P j_P m_P}(\boldsymbol{r},\xi) \right|^2 , \tag{5}$$

where the Σ scattering wave function $\psi^{(-)}_{\Sigma,\boldsymbol{k}_\Sigma \mu_\Sigma}$ behaves asymptotically as a plane wave with momentum k_Σ and spin projection μ_Σ + incoming wave,

$\psi_{P,l_P j_P m_P}$ is the normalized proton wave function in the s.p. state with quantum numbers $l_P j_P m_P$, and $\int d\tau$ denotes the $\boldsymbol{r}$ integration and the ξ summation.

A straightforward calculation (similar to that in [14]) leads to the following expression for $S(l_P j_P)$ in terms of Wigner 3-j symbols [valid in the case of a pure central potential U_Σ (without spin–orbit coupling) considered in the present paper]:

$$S(l_P j_P) = (4\pi)^2 (2j_P + 1) \sum_{Ll} (2L+1)(2l+1) \times \begin{pmatrix} L & l_P j_P & l \\ 0 & 0 & 0 \end{pmatrix}^2 |\langle l | j_L(qr) | l_P \rangle|^2, \tag{6}$$

with

$$\langle l | j_L(qr) | l_P j_P \rangle = \int dr u_l(k_\Sigma; r)^{(-)*} j_l(qr) R_{l_P j_P}(r), \tag{7}$$

where $R_{l_P j_P}(r)/r$ is the radial part of $\psi_{P,l_P j_P m_P}$ and $u_l(k_\Sigma; r)^{(-)}/r$ is the radial part of the l component of $\psi_{\Sigma,\boldsymbol{k}_\Sigma \mu_\Sigma}$, whose asymptotic behavior is $u_l(k_\Sigma; r)^{(-)}/r - j_l(k_\Sigma r) \sim h_l^{(2)}(k_\Sigma r)$. Notice that the dependence on the kaon scattering angle θ_K enters through the relation: $q^2 = k_\pi^2 + k_K^2 - 2k_\pi k_K \cos\theta_K$.

3. Results for the ^{28}Si target

Similarly as in I we assume for the Σ s.p. potential the form of a square well

$$U_\Sigma(r) = -(V_{\Sigma 0} + iW_{\Sigma 0})\theta(R - r), \tag{8}$$

where the radius R is the same as the radius R in expression (9) for the proton s.p. potential. For the depth of the absorptive potential, we use the value $W_{\Sigma 0} = 2.5$ MeV, obtained in [15] and [16] from the $\Sigma^- P \to \Lambda n$ cross section. For the depth $V_{\Sigma 0}$ we assume values varying from -40 to 20 MeV. Notice that $V_{\Sigma 0}$ is positive for an attractive and negative for a repulsive potential.

For the proton s.p. potential, we use — as in I — the form:

$$U_P(r) = -V_{P0}\theta(R - r) - V_{Pls} \boldsymbol{ls} \delta(R - r), \tag{9}$$

with $V_{P0} = 46$ MeV and $V_{Pls} = 15$ MeV fm. The radius $R = 3.7559$ fm has been adjusted to the proton separation energy $B_P = 11.585$ MeV. This potential leads to the s.p. proton energies in the $d_{5/2}$, $p_{1/2}$, $p_{3/2}$, and $s_{1/2}$ states: $e_P(d_{5/2}) = -B_P$, $e_P(p_{1/2}) = -22.91$ MeV, $e_P(p_{3/2}) = -26.88$ MeV, and $e_P(s_{1/2}) = -35.70$ MeV, which agree qualitatively with the separation energies determined in the $(p, 2p)$ and $(e, e'p)$ reactions (see [17]).

The Coulomb interaction of Σ^- and the target proton is not taken into account explicitly. Its average value inside the nuclear core is ± 6 MeV, and we assume that it is included into $V_{\Sigma 0}$ and V_{P0}.

At the moment, the only available experimental results for the (π^-, K^+) reaction with Σ hyperon production are those on the silicon target at $p_\pi =$ 1.2 GeV/c reported in [13]. In Fig. 1, the data of [13] are compared with our results for $d^2\sigma/d\hat{k}_K dE_K$ obtained with three values of $V_{\Sigma 0}$: 20 MeV (curve A, attractive V_Σ), -20 MeV (curve B, repulsive V_Σ), -40 MeV (curve C, repulsive V_Σ). The B_Σ on the abscissa is the separation (binding) energy of Σ^- from the hypernuclear system produced (in the ground or excited state) with the nuclear core left in its ground state. (If the hypernuclear system produced consists of the hyperon attached to the nuclear core in a state with an excitation energy E^*, we have $-B_\Sigma = \epsilon_\Sigma + E^*$.)

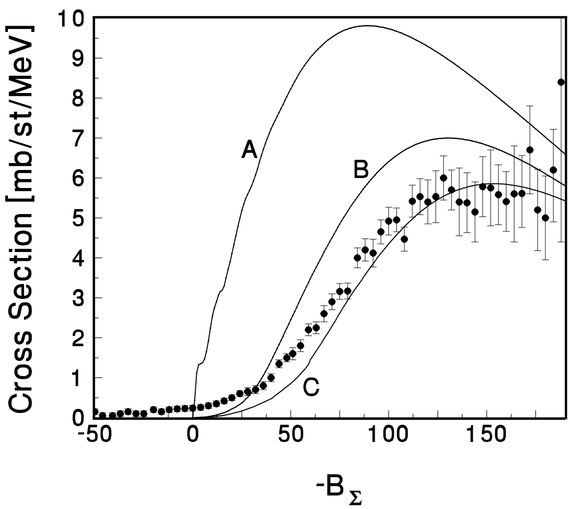

Fig. 1. Kaon spectrum from (π^-, K^+) reaction on ^{48}Si at $\theta_K = 6°$ at $p_\pi =$ 1.2 GeV/c. See text for explanation.

Our results, curves A, B, and C, were obtained with the value of $t^2 =$ 0.1257 E7 MeV^2 fm^6, fitted to the measured value of the total cross section for the free $\pi^- + P \to K + \Sigma^-$ process at pion momentum equal 1.2 GeV/c. We do not take into account the proton motion in the target Si nucleus, which would require averaging the cross section around $p_\pi^{\rm Lab} = 1.2$ GeV/c.

To obtain a sufficient accuracy, especially at higher values of $-B_\Sigma$, the summation in expression (6) was extended over values of L up to $L = 12$.

4. Discussion and conclusions

The data points in the inclusive pion spectrum in Fig. 1 at positive values of B_Σ correspond to reactions with emission of Λ hyperons (except for final Σ bound states possible only for attractive V_Σ). In the present paper, we do not consider processes with emission of Λ hyperons, and consequently, our calculated curves in Fig. 1 start at $-B_\Sigma = 0$. We calculate the cross section for the (π^-, K^+) reaction in the impulse approximation with the Σ^- wave function determined by the Schrödinger equation with a complex potential U_Σ whose imaginary part W_Σ represents the $\Sigma\Lambda$ conversion treated as an absorption. However in the inclusive (π^-, K^+) experiment, kaons may be accompanied not only by Σ^- hyperons but also by Λ hyperons produced in the $\Sigma\Lambda$ conversion[2]. Thus treating the $\Sigma\Lambda$ as absorption neglects the part of kaon spectrum accompanied by Λ emission.

One way of including this Λ emission part of kaon spectrum was shown in [14] where the Λ channel was introduced explicitly into the description. The results of [14] indicate that although Λ emission is crucial at negative and small positive values of $-B_\Sigma$, at higher values of $-B_\Sigma$ it is relatively less important, especially when the Λ–nucleus interaction in the Λ channel is taken into account. The method used in [14] is an approximate one — it applies the effective two-channel approach [18].

Another way of including the part of kaon spectrum accompanied by Λ emission is the Green function method [19], applied in [13]. After introducing approximations clearly stated in [19] (concerning , *e.g.*, the coupling between different core states), the Green function method leads to expressions for the cross sections for the (π^-, K^+) reactions accompanied by Σ emission (the Σ escape cross section $d^2\sigma_{\rm esc}/d\hat{k}_K dE_K$), and by Λ emission (the $\Sigma\Lambda$ conversion cross section $d^2\sigma_{\rm con}/d\hat{k}_K dE_K$), which depend on the effective s.p. Σ Hamiltonian obtained from the full Σ and Λ channel Hamiltonian by eliminating the Λ channel.

In actual calculations, one uses in the effective Σ Hamiltonian a phenomenological local Σ–nucleus s.p. potential. In principle, the ΣN elastic scattering may contribute to the Σ–nucleus potential (see, *e.g.*, [15]), and it could imply that the ΣN elastic scattering contributes to the conversion cross section. Let us also mention, that in this approach one cannot investigate the effect of the Λ–nucleus interaction.

The results shown in Fig. 1 clearly rule out an attractive potential V_Σ. Even if we applied an adjustable scaling factor — a procedure applied in [13] — to the curve A (which has a maximum at $-B_\Sigma = 89$ MeV), the resulting curve would differ from the experimental points.

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[2] A direct production of Λ hyperons is not possible in the (π^-, K^+) reaction.

From the results shown in Fig. 1, we conclude that the strength of the repulsive Σ potential is somewhere between 20 and 40 MeV, 20 MeV $\lesssim -V_{\Sigma 0} \lesssim$ 40 MeV. The present estimate leads to V_Σ slightly more repulsive than estimates based on the analysis of strangeness exchange reactions [4] and Σ^- atoms [2]. Nevertheless the present estimate is not inconsistent with the other estimates, especially considering the error bars at the data at high values of B_Σ.

We do not believe that considering distortion of kaons and pions would change our conclusion, as it was shown in [13] the distortion effect does not affect the spectrum shape very much.

Our present result differs from the result of the analysis in [13] which suggest a 150 MeV strength of the repulsion in V_Σ, which would be completely inconsistent with the Σ^- atomic data, and the analysis of the (K^-, π) reactions.

Our final conclusion is that the s.p. Σ potential is repulsive inside the nucleus, and the strength of the repulsion is consistent with the Nijmegen model F of the barion–barion interaction.

Appendix A

Determination of the $\pi^- + P \to K^+ + \Sigma^-$ transition matrix

We start from the expression for σ, the total cross section for the elementary process $\pi^+ + P \to K^+ + \Sigma^-$:

$$\sigma = \frac{\Omega}{v} \sum_f \frac{\delta W_{i \to f}}{\delta t} = \frac{\Omega}{v} \sum_f \frac{2\pi}{\hbar} \delta(E_f - E_i) |(f|\hat{t}|i)|^2, \tag{A.1}$$

where $\delta W_{i \to f}/\delta t$ is the probability per unit time of the transition between the initial state i of the $\pi^- + P$ system (with energy $E_i = E_\pi + E_P$) and the final state f of the $K^+ + \Sigma^-$ system (with energy $E_f = E_K + E_\Sigma$), $\hat{t}$ is the transition matrix, and v/Ω is the incoming flux (v is the relative velocity of the incoming pions and target protons, and Ω is the normalization volume). The initial and final states are plane waves: $|i) = |\boldsymbol{k}_\pi \boldsymbol{k}_P)$ and $|f) = |\boldsymbol{k}_K \boldsymbol{k}_\Sigma)$. They are normalized in Ω [3].

We approximate the transition matrix $\hat{t}$ by a constant t:

$$(f|\hat{t}|i) = t(f|i) = \frac{t}{\Omega^2} \int d\boldsymbol{r} \mathrm{e}^{-i(\boldsymbol{K}_f - \boldsymbol{K}_i)\boldsymbol{r}}, \tag{A.2}$$

where $\boldsymbol{K}_i = \boldsymbol{k}_\pi + \boldsymbol{k}_P$, and $\boldsymbol{K}_f = \boldsymbol{k}_K + \boldsymbol{k}_\Sigma$.

[3] We assume that the interaction responsible for the $\pi P \to K\Sigma$ transition does not depend on P and Σ spin which is ignored in our notation.

Since $\int d\boldsymbol{r} \mathrm{e}^{i\boldsymbol{K}r}$ vanishes for $\boldsymbol{K} \neq 0$, we have $\left|\int d\boldsymbol{r}\mathrm{e}^{-i(\boldsymbol{K}_f-\boldsymbol{K}_i)\boldsymbol{r}}\right|^2 = \int d\boldsymbol{r}\mathrm{e}^{-i(\boldsymbol{K}_f-\boldsymbol{K}_i)\boldsymbol{r}} \times \int d\boldsymbol{r} = \Omega \int d\boldsymbol{r}\mathrm{e}^{-i(\boldsymbol{K}_f-\boldsymbol{K}_i)\boldsymbol{r}}$. For $\Omega \to \infty$, we have $\int d\boldsymbol{r}\mathrm{e}^{-i(\boldsymbol{K}_f-\boldsymbol{K}_i)\boldsymbol{r}} \to (2\pi)^3\delta(\boldsymbol{K}_f - \boldsymbol{K}_i)$. Furthermore, we may replace in (A.1), $\sum_f = \sum_{\boldsymbol{k}_K\boldsymbol{k}_\Sigma}$ by $\left[\Omega/(2\pi)^3\right]^2 \int d\boldsymbol{k}_K \int d\boldsymbol{k}_\Sigma$. In this way, we obtain:

$$\frac{\sigma}{t^2} = \frac{1}{(2\pi)^2}\frac{1}{(\hbar v)} \int d\boldsymbol{k}_K \int d\boldsymbol{k}_\Sigma \delta(E_f - E_i)\delta(\boldsymbol{K}_f - \boldsymbol{K}_i). \tag{A.3}$$

As σ does not depend on the reference frame, we choose the center of mass (CMS) frame, in which the calculation of σ is simplest. By E and $\boldsymbol{p} = \hbar\boldsymbol{k}$, we denote energies and momenta in this frame. In CMS we have $\boldsymbol{K}_i = 0$, and the last δ in (A.3) becomes $\delta(\boldsymbol{K}_f) = \delta(\boldsymbol{k}_K + \boldsymbol{k}_\Sigma)$, and (A.3) takes the form:

$$\frac{\sigma}{t^2} = \frac{1}{(2\pi)^2}\frac{1}{(\hbar v)} \int d\boldsymbol{k}_\Sigma \delta(E_f - E_i) = \frac{1}{\pi}\frac{1}{\hbar v} \int dk_K k_K^2 \delta(E_f - E_i)\,, \tag{A.4}$$

where

$$E_f = E_K + E_\Sigma = \sqrt{(M_K c^2)^2 + (c\hbar k_K)^2} + \sqrt{(M_\Sigma c^2)^2 + (c\hbar k_K)^2}\,. \tag{A.5}$$

Since

$$dE_f = dE_K + dE_\Sigma = \frac{E_K + E_\Sigma}{E_K E_\Sigma}(c\hbar)^2 k_K dk_K\,, \tag{A.6}$$

we may change the integration over k_K in (A.4) into integration over E_f. Thus we get

$$\sigma = t^2 \frac{1}{\pi}\frac{1}{(\hbar c)^4}\frac{E_\pi^{\mathrm{Lab}}}{p_\pi^{\mathrm{Lab}}}\frac{p_K E_K E_\Sigma}{E_K + E_\Sigma}\bigg|_{E_f = E_i}, \tag{A.7}$$

where v has been expressed through the pion laboratory momentum and energy, $v = c^2 p_\pi^{\mathrm{Lab}}/E_\pi^{\mathrm{Lab}}$.

The CMS quantities in (A.7) may be easily expressed through the invariant s (see, *e.g.*, [20]):

$$s = (E_K + E_\Sigma)^2 = (E_\pi + E_P) = (M_P c^2)^2 + (M_\pi c^2)^2 + 2E_\pi^{\mathrm{Lab}} M_P c^2. \tag{A.8}$$

We have:

$$E_K = \frac{s + (M_K c^2)^2 - (M_\Sigma c^2)^2}{2\sqrt{s}}\,, \quad E_\Sigma = \frac{s + (M_\Sigma c^2)^2 - (M_K c^2)^2}{2\sqrt{s}}\,, \tag{A.9}$$

$$cp_K = \frac{1}{2\sqrt{s}}\sqrt{\lambda(s, (M_K c^2)^2, (M_\Sigma c^2)^2)}\,, \tag{A.10}$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

Expression (A.7) for σ together with Eqs (A.8)–(A.10) enable us to calculate σ as function of pion laboratory momentum $p_\pi^{\rm Lab}$.

To determine t, we used the total cross section σ measured by Good and Kofler [21] at $p_\pi^{\rm Lab} = 1.12$–1.23 GeV/c. At each of the pion momentum applied in [21], we determined t so that our calculated value of σ agreed with the measured value. Our results for t^2 are shown in Table I together with values of σ measured in [21].

TABLE I

Total cross section σ for the $\pi^- P \to K^+ \Sigma^-$ reaction measured in [21] and our results for t^2.

$p_\pi^{\rm Lab}$ [MeV/c]	σ [fm^2]	t^2 [MeV^2 fm^6]
1125	21.8±1.1	0.1631±0.0082 E7
1225	23.5±1.7	0.1132±0.0082 E7
1275	20.9±1.4	0.0869±0.0058 E7
1325	24.5±2.1	0.0901±0.0077 E7

For pion momentum $p_\pi^{\rm Lab} = 1.2$ GeV/c, at which the KEK experiment [13] was performed, we get — by interpolating the t^2 values in Table I — the result: $t^2 = 0.1257$ E7 MeV^2 fm^6.

REFERENCES

[1] C. J. Batty, E. Friedman, A. Gal, *Phys. Rep.* **287**, 385 (1997).

[2] J. Dąbrowski, J. Rożynek, G.S. Anagnostatos, *Eur. Phys. J.* **A14**, 125 (2002).

[3] C.B. Dover, D.J. Millener, A. Gal, *Phys. Rep.* **184**, 1 (1989).

[4] J. Dąbrowski, J. Rożynek, *Acta Phys. Pol. B* **29**, 2147 (1998).

[5] N.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev.* **D12**, 744 (1975); **D15**, 2547 (1977).

[6] N.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev.* **D20**, 1663 (1979).

[7] P.M.M. Maessen, T.A. Rijken, J.J. de Swart, *Phys. Rev.* **C40**, 2226 (1989); *Nucl. Phys.* **A547**, 245c (1992).

[8] T.A. Rijken, V.G.J. Stoks, Y. Yamamoto, *Phys. Rev.* **C59**, 21 (1999).

[9] J. Dąbrowski, J. Rożynek, *Phys. Rev.* **C23**, 1706 (1981).

[10] Y. Yamamoto, H. Bandō, *Progr. Theor. Phys., Suppl.* **81**, 9 (1985).

[11] Y. Yamamoto, T. Motoba, H. Himeno, K. Ikeda, S. Nagata, *Progr. Theor. Phys. Suppl.* **117**, 361 (1994).

[12] J. Dąbrowski, *Phys. Rev.* **C60**, 025205 (1999).

[13] H. Noumi *et al.*, *Phys. Rev. Lett.* **89**, 072301 (2002).

[14] J. Dąbrowski, J. Rożynek, *Acta Phys. Pol. B* **27**, 985 (1996).

[15] J. Dąbrowski, J. Rożynek, *Acta Phys. Pol. B* **14**, 439 (1983).

[16] J. Dąbrowski, *Phys. Lett.* **B139**, 7 (1984).

[17] G. Jacob, Th.A.J. Maris, *Rev. Mod. Phys.* **45**, 6 (1973).

[18] P.H. Timmers *et al.*, *Phys. Rev.* **D29**, 1928 (1984); M. Kohno, R. Hausman, P. Siegel, W. Weise *Nucl. Phys.* **A470**, 609 (1987).

[19] O. Morimatsu, K. Yazaki, *Nucl. Phys.* **A435**, 727 (1985); *Nucl. Phys.* **A483**, 493 (1988).

[20] G. Källén, *Elementary Particle Physics*, Addison-Wesley, Reading, Mass. 1964.

[21] M.L. Good, R.R. Kofler, *Phys. Rev.* **183**, 1142 (1969).