

## ON INFLATION AND TORSION IN COSMOLOGY

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(Received April 14, 2005)

In a recent letter by H. Davoudiasl, R. Kitano, T. Li and H. Murayama “The new Minimal Standard Model” (NMSM) was constructed which incorporates new physics beyond the Minimal Standard Model (MSM) of particle physics. The authors follow the principle of minimal particle content and therefore adopt the viewpoint of particle physicists. It is shown that a generalisation of the geometric structure of spacetime can also be used to explain physics beyond the MSM. It is explicitly shown that for example inflation, *i.e.* an exponentially expanding universe, can easily be explained within the framework of Einstein–Cartan theory.

PACS numbers: 04.50.+h, 98.80.Jk

## 1. Introduction

There are many ideas how physics beyond the Minimal Standard Model may be explained, however none of them so far was able to give a consistent output that could explain consistently all experimental results of particle physics and cosmology. In contrast to these modern approaches the authors of [2] adopt a conservative particle physicist’s point of view and include the minimal number of new degrees of freedom to formulate the NMSM that can explain Dark Energy, non-baryonic Dark Matter *etc.*

From a geometrical point of view it may be preferable to allow more general geometric structures rather than increasing the number of required particles. Therefore the guiding principle of this note may be called the principle of minimal geometry content.

The cosmological principle states that the universe is spatially homogeneous and isotropic. Mathematically speaking the four-dimensional (4d)

spacetime  $(\mathcal{M}, g)$  is foliated by 3d spacelike hypersurfaces of constant time which are the orbits of a Lie group  $G$  acting on  $\mathcal{M}$  with isometry group  $\text{SO}(3)$ . All fields are invariant under the action of  $G$ . The cosmological principle implies

$$\mathcal{L}_{\xi^m} g_{\mu\nu} = 0, \quad \text{and} \quad \mathcal{L}_{\xi^m} T^\lambda{}_{\mu\nu} = 0, \quad (1)$$

where  $\xi^m$  are the six Killing vectors (labelled by  $m$ ) generating the spacetime isometries.  $g_{\mu\nu}$  denotes the metric tensor and  $T^\lambda{}_{\mu\nu}$  stands for the torsion tensor, Greek indices label the holonomic components.

By imposing the restrictions (1), the metric tensor is of Robertson–Walker type

$$ds^2 = -dt^2 + \left( \frac{a(t)}{1 - \frac{k}{4}r^2} \right)^2 (dx^2 + dy^2 + dz^2) = \eta_{ij} e^i \otimes e^j, \quad (2)$$

where  $r^2 = x^2 + y^2 + z^2$  and where the 3-space is spherical for  $k = 1$ , flat for  $k = 0$  and hyperbolic for  $k = -1$ . The vielbein 1-forms in (2) read

$$e^t = dt, \quad e^x = \frac{a(t)}{1 - \frac{k}{4}r^2} dx, \quad e^y = \frac{a(t)}{1 - \frac{k}{4}r^2} dy, \quad e^z = \frac{a(t)}{1 - \frac{k}{4}r^2} dz, \quad (3)$$

where Latin indices label the anholonomic components.

When the restrictions (1) are imposed on the torsion tensor [8], the (non-vanishing) allowed components are

$$T_{xxt} = T_{yyt} = T_{zzt} = h(t), \quad (4)$$

$$T_{xyz} = T_{zxy} = T_{yzx} = f(t), \quad (5)$$

where we closely follow the notation of [5].

## 2. Einstein–Cartan theory in cosmology

In the following it is shown that inflation can be explained without introducing additional fields but considering a spacetime with torsion. The simplest theory of this type is Einstein–Cartan theory which is derived from the Einstein–Hilbert action by varying the vielbein and the spin-connection independently. Then the field equations are [6]

$$R^i{}_j - \frac{1}{2} R \delta^i{}_j + \Lambda \delta^i{}_j = 8\pi t^i{}_j, \quad (6)$$

$$T^i{}_{jk} - \delta^i{}_j T^l{}_{lk} - \delta^i{}_k T^l{}_{jl} = 8\pi s^i{}_{jk}, \quad (7)$$

where  $t^i{}_j$  is the canonical energy-momentum tensor and  $s^i{}_{jk}$  is the tensor of spin.

By taking the cosmological principle into account the field equations (6) of Einstein–Cartan theory simplify to

$$3 \left( \left( h + \frac{\dot{a}}{a} \right)^2 + \frac{k}{a} - \frac{1}{4} f^2 \right) - \Lambda = 8\pi\rho, \tag{8}$$

$$- \left( \left( h + \frac{\dot{a}}{a} \right)^2 + \frac{k}{a} - \frac{1}{4} f^2 \right) - 2 \left( \left( h + \frac{\dot{a}}{a} \right) + \frac{\dot{a}}{a} \left( h + \frac{\dot{a}}{a} \right) \right) + \Lambda = 8\pi P. \tag{9}$$

The torsion field equations (7) become

$$f = 8\pi s, \quad s(t) = S_{xyz} = S_{zxy} = S_{yzx}, \tag{10}$$

$$-2h = 8\pi q, \quad q(t) = S_{xxt} = S_{yyt} = S_{zzt}. \tag{11}$$

If no torsion source is present  $s = q = 0$ , the algebraic equations of motion imply the vanishing of the torsion tensor  $f = h = 0$ . Without torsion, the field equations (8) and (9) reduce to the standard Friedman equations of cosmology.

Let us have a closer look at the field equations (8)–(11) in case of  $q = h = 0$ , *i.e.* only the skew-symmetric part of the torsion tensor, *cf.* [7]. Then the field equations simplify to

$$3 \left( \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a} - \frac{1}{4} f^2 \right) - \Lambda = 8\pi\rho, \tag{12}$$

$$- \left( \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a} - \frac{1}{4} f^2 \right) - 2 \left( \left( \frac{\dot{a}}{a} \right) + \left( \frac{\dot{a}}{a} \right)^2 \right) + \Lambda = 8\pi P, \tag{13}$$

$$f = 8\pi s, \tag{14}$$

which implies the following conservation equation

$$\frac{\dot{\rho}}{3} + \frac{\dot{a}}{a}(\rho + P) + \frac{s}{2} \left( \dot{f} + \frac{\dot{a}}{a} f \right) = 0. \tag{15}$$

With (14) the two remaining independent field equations can be reformulated to give

$$3 \left( \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a} \right) = 8\pi\rho_{\text{eff}} = 8\pi\rho + \Lambda + \frac{3}{4}(8\pi s)^2, \tag{16}$$

$$-2\frac{\ddot{a}}{a} - \left( \frac{\dot{a}}{a} \right)^2 - \frac{k}{a} = 8\pi P_{\text{eff}} = 8\pi P - \Lambda - \frac{1}{4}(8\pi s)^2. \tag{17}$$

In (16) and (17) the matter dominated era of cosmology is defined by  $P = 0$  and  $\rho = \rho_m$  where in addition it is assumed that the torsion contribution is sufficiently small, which is indeed very reasonable as shall be seen. The radiation dominated era is defined by the equation of state  $P = \rho/3$  and  $\rho = \rho_r$ , again with a sufficiently small torsion contribution. For sake of simplicity we assume the following setup for the *torsion dominated era*, in which the universe is exponentially increasing: Assume that torsion in (16) and (17) is the leading contribution, such that one may neglect the others. In the early time of the universe the particle density was high and therefore the probability of having some non-vanishing macroscopic spin is the higher the denser the matter distribution is. On the other hand it is reasonable that the averaged spin density is exponentially decreasing with time,  $s \propto \exp(-t/\tau)$ , where  $\tau$  is a characteristic time scale. Putting this into (15) yields

$$\frac{\dot{s}}{s} = -\frac{1}{\tau} = -\frac{\dot{a}}{a}, \quad (18)$$

which simply implies that the scale factor  $a$  is an exponentially increasing function of time,  $a \propto \exp(t/\tau)$  if the torsion function is exponentially decreasing and if the torsion contribution is the leading one.

Hence a physically intuitive assumption on the behaviour of torsion can explain the inflation era of cosmology without introducing further particles. Since the torsion is rapidly decreasing, its contribution to (16) and (17) will indeed be sufficiently small after the short period of inflation. This implies that today's cosmological measurements possibly should detect some small non-vanishing torsion contribution, (see *e.g.* [3]). This torsion remnant could then be used to solve the sign problem of the cosmological constant, as was shown by the author in [1].

It is neither the author's aim to criticise the motivation and derivation of the NMSM nor to criticise the successful way that lead to the MSM. We try to show that other, equally conservative, approaches may also work. It should be emphasised that the consideration of torsion is nearly as old as general relativity itself (see *e.g.* [4] for a historical review). Thus the guiding principle of minimal geometry content might be as successful as the minimal particle content principle. Only the experiment will decide which of these two principles is the one describing nature correctly.

I wish to thank Herbert Balasin and Wolfgang Kummer for valuable comments. Moreover, I wish to thank Dominik J. Schwarz for the useful discussion. The work was supported by the Junior Research Fellowship of The Erwin Schrödinger International Institute for Mathematical Physics.

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