

# ENTROPY ANALYSIS IN $\pi^+p$ AND $K^+p$ COLLISIONS AT $\sqrt{s} = 22$ GeV

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The entropy properties are analyzed by Ma's coincidence method in  $\pi^+p$  and  $K^+p$  collisions of the NA22 experiment at 250 GeV/ $c$  incident momentum. By using the Rényi entropies, we test the scaling law and additivity properties in rapidity space. The behavior of the Rényi entropies as a function of the average number of particles is investigated. The results are compared with those from the PYTHIA Monte Carlo event generator.

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## 1. Introduction

It was suggested recently that the event coincidence probability method of measuring entropy, originally proposed by Ma [1], is well suited for the analysis of local properties in multiparticle systems produced in high energy collisions [2–8]. In heavy ion collisions it can be used in the search for formation of a quark gluon plasma (QGP) [2], but the method is equally applicable to hadron–hadron collisions [5]. It also proved effective for the study of systems of particles created in multiplicative branching processes [8] as present in high-energy QCD jet fragmentation. Finally, entropy measurements offer an additional tool to analyze event-to-event fluctuations and particle correlations [4, 6].

The existence of a QGP at high energy density is a prediction of QCD. Many theorists and experimentalists are searching for possible signals of the formation of such a new state of matter. The primary goal of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory is to create and to study this deconfined state. Systematic measurements of the local entropy at RHIC may provide direct information about the internal degrees of freedom of the QGP state and its evolution. It is thus very important to first see how the proposed entropy measures behave in hadron–hadron collisions.

In this note, we present the results of a study of entropy in  $\pi^+p$  and  $K^+p$  collisions at 250 GeV/ $c$  incident momentum ( $\sqrt{s} = 22$  GeV) collected by the NA22 experiment. We investigate the dependence on discretization of the system, the effects of particle correlations by testing scaling and additivity properties of the entropy measures, as well as the multiplicity dependence of the Rényi entropies in rapidity space. We also compare the results of the NA22 data to those of the Monte Carlo event generator PYTHIA 5.7 [9] used for inelastic, non-single-diffractive pion–proton collisions with Bose–Einstein Correlations (BEC) included, as well as to a simple random production model. We end with a short conclusion and outlook.

## 2. Procedure and variables

The original definitions of the standard Shannon entropy and the Rényi entropies are:

$$S = - \sum_j p_j \ln p_j, \quad H_k = \sum_j (p_j)^k. \quad (1)$$

Here,  $p_j = n_j/N$  denotes the probability to obtain a specific configuration of the system, where  $n_j$  is the number of events in such a configuration and  $N$  is the total number of events. The sum runs over all possible configurations.

Following Ma's method [1] and as explained in [4], the first step in the entropy measurement is to determine coincidence probabilities. For every event, a certain phase space region is divided into  $M$  bins of equal size. An event is then characterized by the number of particles,  $m_i$ , in each bin, *i.e.*, by a set of integer numbers  $s \equiv \{m_i\}$ , where  $i = 1, \dots, M$ . After counting how many times,  $n_s$ , the set  $s$  appears in the whole event sample, one can determine the sums

$$N_k = \sum_s n_s (n_s - 1) \dots (n_s - k + 1). \quad (2)$$

This gives the total numbers of observed coincidences of  $k$  configurations.

The coincidence probability of  $k$  configurations is then given by [1]

$$C_k = \frac{N_k}{N(N-1) \dots (N-k+1)}, \quad (3)$$

where  $N$ , as above, is the total number of events in the sample. Only states with  $n_s \geq k$  contribute to  $C_k$ .

From the coincidence probabilities, one calculates the Rényi entropies defined as [10]

$$H_k \equiv -\frac{\ln C_k}{k-1}. \quad (4)$$

The Shannon entropy  $S$  is formally equal to the limit of  $H_k$  as  $k \rightarrow 1$ . So it can only be obtained by an extrapolation method. One possibility we will investigate in some detail is to take [3]

$$H_k = a \frac{\ln k}{k-1} + a_0 + a_1(k-1) + a_2(k-1)^2 + \dots \quad (5)$$

These expansions may be used as a system of linear equations to determine  $a$  and  $a_i$ , with as many terms as the number of  $H_k$ 's available. The Shannon entropy is then calculated as  $S = a + a_0$ . It is instructive to see if the value of  $S$  depends on the number of Rényi entropies used. In general, it turns out that already the value obtained from two terms,  $S[H_2, H_3]$ , and that from three terms,  $S[H_2, H_3, H_4]$ , nearly coincide.

One of the most attractive features of Ma's coincidence method compared to the standard method of using Eq. (1) directly is that, as seen from Eq. (3), the statistical error decreases very fast with increasing number of configurations. Moreover, it has been demonstrated [5] that the extrapolation method yields more stable results than the standard method.

For a system close to thermal equilibrium, and if the phase space subdivision is sufficiently fine-grained, the scaling relation

$$H_k(lM) = H_k(M) + \ln l \Rightarrow S(lM) = S(M) + \ln l \quad (6)$$

holds. Here,  $M$  and  $lM$  are the numbers of bins in two different discretizations.

Another feature is additivity: the entropies measured in a region  $R$  which is the union of two non-overlapping and independent regions  $R_1$  and  $R_2$  satisfy

$$\begin{aligned} H_k(R) &= H_k(R_1) + H_k(R_2) \\ \Rightarrow S(R) &= S(R_1) + S(R_2). \end{aligned} \quad (7)$$

We will check these two important properties below.

### 3. Data sample

The NA22 experiment made use of the European Hybrid Spectrometer (EHS) in combination with the Rapid Cycling Bubble Chamber (RCBC). Details on the setup and on the reconstruction of the data can be found in [11, 12].

Charged-particle momenta are measured over the full solid angle with an average resolution varying from 1–2% for tracks reconstructed in RCBC and 1–2.5% for tracks reconstructed in the first lever arm, to 1.5% for tracks reconstructed in the full spectrometer. Ionization information is used to identify and exclude protons up to 1.2 GeV/ $c$  and electrons (positrons) up to 200 MeV/ $c$ . All unidentified tracks are given the pion mass.

In our analysis, events are accepted when the measured and reconstructed charged-particle multiplicity are the same, no electron is detected among the secondary tracks and the number of badly reconstructed tracks is 0. The selection of the data is described in detail in [13]. After all necessary rejections, a total of 44,524 inelastic, non-single-diffractive events is obtained. Acceptance losses are corrected by a multiplicity-dependent event weighting procedure.

### 4. Results and discussions

In Fig. 1, we show the Shannon entropy and the Rényi entropies  $H_2$  to  $H_4$  as obtained from an  $M = 9$ -fold division of central cms rapidity windows of size  $\Delta y$  ranging from 1 to 6 units. The Shannon entropy is calculated by extrapolation according to Eq. (5). The solid symbols show the NA22 data, the open symbols the PYTHIA predictions. All Rényi and Shannon

entropies increase as the rapidity window widens. The values of  $H_k$  decrease with increasing  $k$ . The figure further shows that the two-term extrapolated Shannon entropy  $S[H_2, H_3]$  agrees well with the three-term extrapolation  $S[H_2, H_3, H_4]$ . For the  $H_k$ , PYTHIA tends to be flatter than the data. This trend increases with increasing order  $k$ , so that the extrapolation procedure leads to a Shannon entropy  $S$  which rises more steeply than the data.

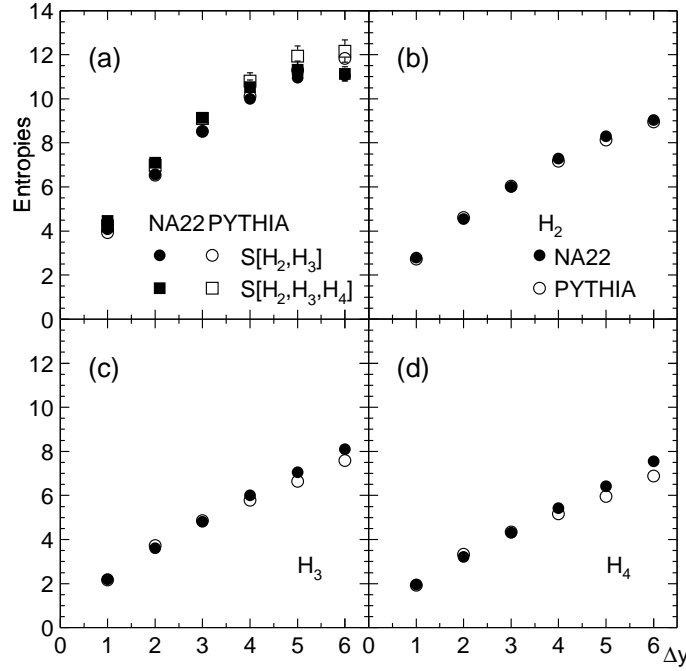


Fig. 1. Shannon and Rényi entropies obtained from an  $M = 9$ -fold division of central rapidity windows of size  $\Delta y$  ranging from 1 to 6 units.

It may be worthwhile to point out that, in general, the extrapolation procedure for the Shannon entropy is not unique and thus will introduce an additional uncertainty [3, 4]. For example, using a polynomial expansion (Eq. (20) of Ref. [3]) for  $\Delta y = 6$  results in a value which is about 10% smaller for  $S[H_2, H_3]$  and 7% smaller for  $S[H_2, H_3, H_4]$ . The Rényi entropies are of great interest by themselves and provide valuable information about the system without the need for any extrapolation. In the following, we therefore concentrate on the Rényi entropies, in particular on  $H_2$ .

If scaling according to Eq. (6) holds, one should observe a linear relation if  $H_k(M)$  is plotted as a function of  $-\ln \delta y$ , where  $\delta y = \Delta y/M$  is the bin size.

We first test scaling of  $H_2$  in central rapidity windows in Fig. 2. The solid circles, squares and triangles represent the NA22 data for  $|y| < 0.5, 1.0, 2.0$ , respectively. They all are flattening with increasing  $-\ln \delta y$ , *i.e.*, the scaling law does not hold for the NA22 data. This implies that there are strong particle correlations in the process under investigation. The open symbols in Fig. 2 represent the PYTHIA results with a Bose–Einstein correlation strength  $\lambda = 1.0$  [9]. They do not fully agree with the data. We have verified that this discrepancy is not sensitive to the actual value of  $\lambda$  used.

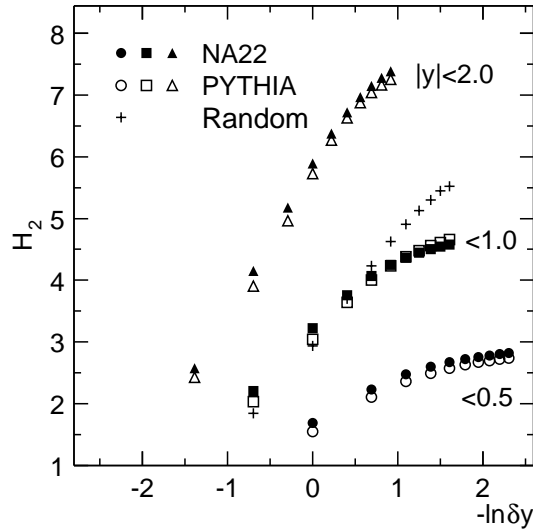


Fig. 2. The dependence on the dividing bin sizes of the second Rényi entropy calculated in central rapidity windows.

The cross symbols for  $|y| < 1.0$  in Fig. 2 show the results from a random model generated as follows: take the multiplicity distribution in the window  $|y| < 1.0$  from the data, but the multiplicity fluctuations in each  $\delta y$  bin to be Poissonian with a mean as in the data. In this model, there are no correlations between particles, so it should give a nearly straight-line relationship, as indeed shown by the figure.

Due to the asymmetry of the proton and meson fragmentation regions in the NA22 experiment, we plot forward and backward hemispheres separately in different non-central windows of fixed size  $\Delta y = 1$  in Fig. 3(a) and (b), respectively. We see that, again, the  $H_2$  values are flattening with increasing  $-\ln \delta y$ , so that no linear relationship is found. PYTHIA has the same trend, but overestimates the data in the peripheral regions. We also note that  $H_2$  becomes smaller for more peripheral windows, due to decreasing particle density, *i.e.*, decreasing coincidence probability.

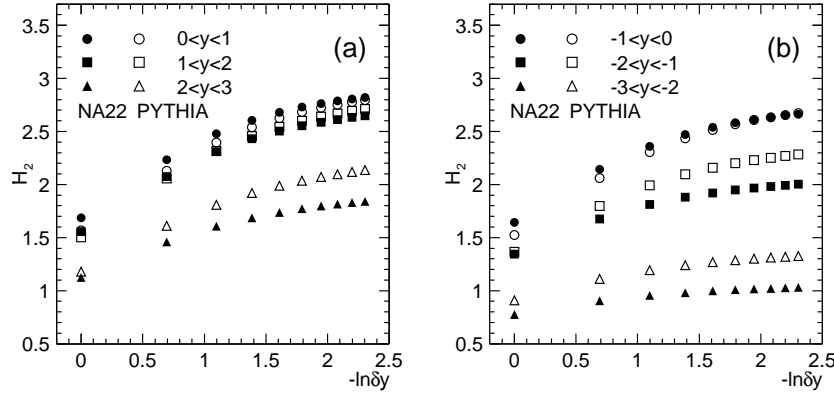


Fig. 3. The dependence on the dividing bin sizes of the second Rényi entropy calculated in (a) forward and (b) backward hemispheres in different non-central rapidity windows.

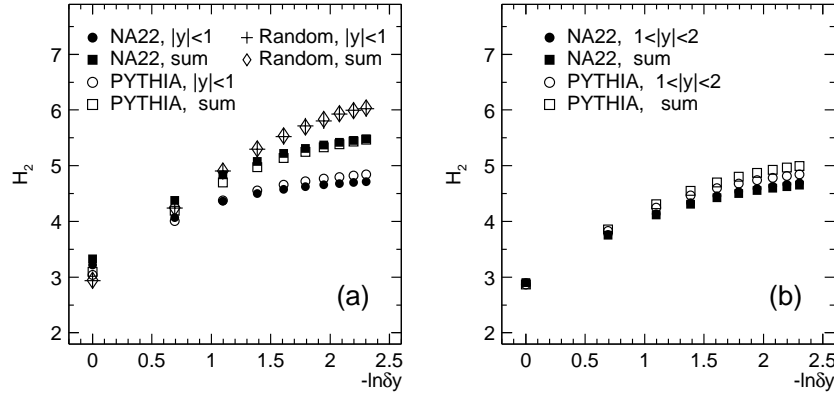


Fig. 4. The second Rényi entropy as a function of  $-\ln \delta y$  measured in two (a) adjacent and (b) separate intervals.

In order to test additivity, Fig. 4(a) shows  $H_2$  obtained in two ways: one is a direct measurement in the rapidity window  $|y| < 1$  (solid circles), the other the sum of the values obtained from the adjacent ranges  $-1 < y < 0$  and  $0 < y < 1$  (solid squares). A clear difference is observed between these two results. The  $H_2$  obtained from summation is larger than the one calculated directly, and the difference increases with increasing  $-\ln \delta y$ . So, additivity of  $H_2$  is not observed here. The reason is that strong correlations exist between the particles belonging to these two adjacent  $y$  ranges. Indeed, no violation of additivity is observed for the random model (crosses and diamonds in Fig. 4(a)), thus confirming the role of correlations in the data.

Fig. 4(b) shows the results from two regions separated by two rapidity units. In this case, the difference between the direct measurement in the window  $1 < |y| < 2$  (solid circles) and the sum of the two non-adjacent windows  $-2 < y < -1$  and  $1 < y < 2$  (solid squares) is very small. PYTHIA (open symbols) shows the same trend but again overestimates the values in the non-central rapidity regions.

The additivity observed here is consistent with the experimental observation that correlations between particles belonging to widely separated rapidity intervals are small [14]. However, at much higher energies, strong long-range correlations are observed [15] which should lead to a breaking of the additivity property. This was confirmed in PYTHIA model calculations for proton-proton collision at  $\sqrt{s} = 1800$  GeV [5].

As advocated in [6], the dependence of Rényi entropies on particle multiplicity  $n$  carries important information on the produced system. An entropy proportional to  $n$  is indicative of an equilibrated system with no strong long-range correlations. On the other hand, for non-equilibrated systems or a superposition of sub-systems with different properties, proportionality to  $\ln n$  is expected.

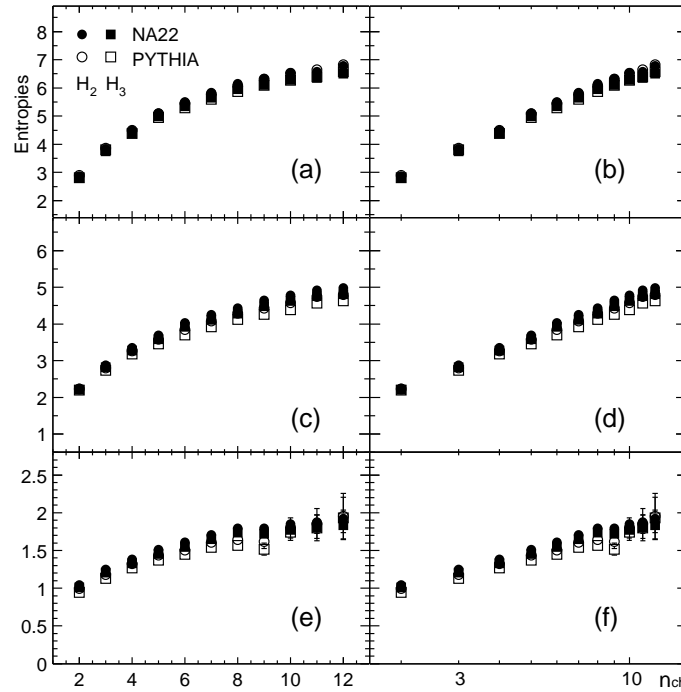


Fig. 5. The dependence of the second and third Rényi entropies on multiplicity in linear (left) and logarithmic (right) scale, with (a)(b)  $M = 6$ -fold division in the rapidity window  $|y| < 3$ , (c)(d)  $M = 4$  in  $|y| < 2$  and (e)(f)  $M = 2$  in  $|y| < 1$ .



TABLE I

Fit values of Rényi entropies in different rapidity windows.

		$c_1$	$c_2$	$\chi^2/\text{NDF}$
$ y  < 3, M = 6$	$H_2$	$1.54 \pm 0.06$	$2.18 \pm 0.04$	46.23/8
	$H_3$	$1.60 \pm 0.06$	$2.06 \pm 0.04$	49.15/8
$ y  < 2, M = 4$	$H_2$	$1.17 \pm 0.06$	$1.57 \pm 0.03$	5.77/8
	$H_3$	$1.22 \pm 0.05$	$1.48 \pm 0.03$	7.14/8
$ y  < 1, M = 2$	$H_2$	$0.68 \pm 0.06$	$0.52 \pm 0.04$	1.79/8
	$H_3$	$0.65 \pm 0.05$	$0.50 \pm 0.03$	4.22/8

In Fig. 5, we show the charged-particle multiplicity dependence of  $H_2$  (solid circles) and  $H_3$  (solid squares) for three different rapidity windows, where the number of divisions  $M$  is chosen such that  $\delta y$  is the same for each rapidity window. Table I shows the fit results for the logarithmic function

$$y = c_1 + c_2 \ln n_{\text{ch}}. \quad (8)$$

In the central region, a logarithmic rather than a linear relation is observed. This again confirms that there is no thermal equilibrium in the system. PYTHIA (open symbols) agrees quite well with the data.

## 5. Conclusions and outlook

We have analyzed the entropy properties of multiparticle production in  $\pi^+p$  and  $K^+p$  collisions at 250 GeV/ $c$ . By using Rényi entropies, we find that both the scaling and the additivity properties are not generally valid and that the multiplicity dependence shows a logarithmic rather than a linear relation. All of these confirm the expectation that thermal equilibrium is not reached in hadron–hadron collisions at  $\sqrt{s} = 22$  GeV.

The PYTHIA Monte Carlo model agrees, in general, quite well with the data. However, significant deviations exist. In particular, the model overestimates the values in the peripheral rapidity regions, presumably due to too weak correlations. This shows that Rényi entropies provide a new sensitive measure of multiparticle correlations.

It would be interesting to investigate the entropy properties of high-temperature, high-density systems, which may create the long expected QGP. RHIC has already collected data on gold–gold collisions at 130 and 200 GeV and  $pp$  collisions at 200 GeV. The results presented here should provide a valuable guide to the interpretation of these measurements.

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