SINGLE PARTICLE POTENTIAL OF A Σ HYPERON IN NUCLEAR MATTER III. DENSITY AND MOMENTUM DEPENDENCE*

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(Received June 10, 2005)

The previously derived expressions for the real part of the single particle potential of a Σ hyperon in nuclear matter, V_Σ, are applied to investigate the dependence of V_Σ on the nuclear matter density and Σ momentum. Results for V_Σ, in particular its isospin, spin, and spin-isospin dependent parts, obtained for four models of the Nijmegen baryon–baryon interaction are presented and discussed.

PACS numbers: 21.80.+a

1. Introduction

The single particle (s.p.) potential V_Σ of the Σ hyperon in nuclear matter, in particular its isospin and spin dependence, was discussed in [1], and [2]. Let us consider a Σ hyperon moving in nuclear matter of density ρ composed of $Z_\uparrow(Z_\downarrow)$ protons with spin up (down), and $N_\uparrow(N_\downarrow)$ neutrons with spin up (down). As discussed in [1, 2], the s.p. potential of a Σ^+ hyperon with spin up/down and momentum $\boldsymbol{k}_\Sigma$ has the form (in the linear approximation in the α parameters):

$$V_\Sigma(\rho;\uparrow/\downarrow,\Sigma^+,k_\Sigma) = V_0(\rho;k_\Sigma)+\tfrac{1}{2}\alpha_\tau V_\tau(\rho;k_\Sigma)\pm\tfrac{1}{4}\alpha_\sigma V_\sigma(\rho;k_\Sigma)\pm\tfrac{1}{2}\alpha_{\sigma\tau}V_{\sigma\tau}(\rho;k_\Sigma)\,, \quad (1)$$

where the proton or isospin excess parameter $\alpha_\tau = (Z_\uparrow + Z_\downarrow - N_\uparrow - N_\downarrow)/A$, the spin excess parameter $\alpha_\sigma = (Z_\uparrow + N_\uparrow - Z_\downarrow - N_\downarrow)/A$, and the spin-isospin excess parameter $\alpha_{\sigma\tau} = (Z_\uparrow + N_\downarrow - Z_\downarrow - N_\uparrow)/A$. For the assumed charge independence of the baryon–baryon interaction, the expression for $V_\Sigma(\uparrow/\downarrow,\Sigma^-,k_\Sigma)$ differs from (1) only by the sign at the τ and

* This research was partly supported by the Polish State Committee for Scientific Research (KBN) under grant 2P0387522.

$\sigma\tau$ terms, whose coefficients become $-\alpha_\tau$ and $\mp\alpha_{\sigma\tau}$. The expression for $V_\Sigma(\uparrow/\downarrow, \Sigma^0, k_\Sigma)$ differs from (1) by the absence of the τ and $\sigma\tau$ terms[1].

Results for V_0 and V_x, $x = \tau, \sigma, \sigma\tau$, presented in [1] and [2], obtained with four models of the Nijmegen baryon–baryon interaction, were calculated at the equilibrium density of nuclear matter $\rho = \rho_0 = 0.166$ fm^{-3} ($k_{\rm F} = 1.35$ fm^{-1}). However, values of V_Σ not only at densities $\rho = \rho_0$ are important in describing the behavior of Σ hyperons in various situations: in Σ nuclear production processes [the strangeness exchange (K^-, π) and associated production (π, K^+) processes], in Σ^- atoms (see, *e.g.*, [3]), and probably also in astrophysical systems like neutron stars (see, *e.g.*, [4, 5]).

In the present paper we present the results obtained for V_0 and V_x as functions of the nuclear matter density ρ. Furthermore, the dependence of V_0 and V_x on the Σ momentum k_Σ is also considered. The present discussion, similarly as the discussion in [1] and [2], is restricted to the real Σ potential.

2. Results and discussion

We apply the expressions for V_0 and V_x in terms of the effective ΣN interaction in nuclear matter, derived in [1], and [2]. We use the YNG effective ΣN interaction of Yamamoto *et al.* [6]. It is a configuration space representation of the G-matrix calculated in the low order Brueckner (LOB) theory for four models of the Nijmegen baryon–baryon interaction: model D [7], model F [8], the soft-core (SC) model [9], and the new soft-core (NSC) model of Rijken *et al.* [10].

Our results for V_0 as function of ρ obtained with models D, F, and SC of the Nijmegen baryon–baryon interaction are shown in Fig. 1. Solid, broken, and dotted curves represent results obtained respectively for Σ momentum $k_\Sigma = 0$, $k_\Sigma = 1$, and $k_\Sigma = 2$ fm^{-1}. We do not show our results obtained for V_0 with the NSC model, because they would make Fig. 1 less transparent. The NSC, $k_\Sigma = 0$ curve would almost coincide with the D, $k_\Sigma = 1$ fm^{-1} curve, and the dependence of the NSC results on k_Σ is similar to this dependence of our SC results.

Notice that at densities ρ of the order of the nuclear matter equilibrium density $\rho_0 = 0.166$ fm^{-3}, model F leads to a repulsive V_0 whereas all the remaining models lead to an attractive V_0. This repulsive character of the Σ potential has been suggested by the analyses [11, 12] of the Brookhaven (K^-, π) experiments on ^{9}Be [15] and of the KEK (π, K^+) experiments on ^{28}Si [13, 14], and also by the phenomenological analysis of Σ^- atoms [16]. The attractive character of the Σ potential at low densities (relevant in Σ^- atoms) revealed by the model F curves in Fig. 1 guarantees that it leads to

[1] We consider the case of pure central ΣN effective interaction for which the possible dependence of V_σ and $V_{\sigma\tau}$ on the direction of $\boldsymbol{k}_\Sigma$ does not appear.

the strong-interaction energy shifts in Σ^- atoms towards increased binding of the atomic levels, in agreement with the experiment (see [3]).

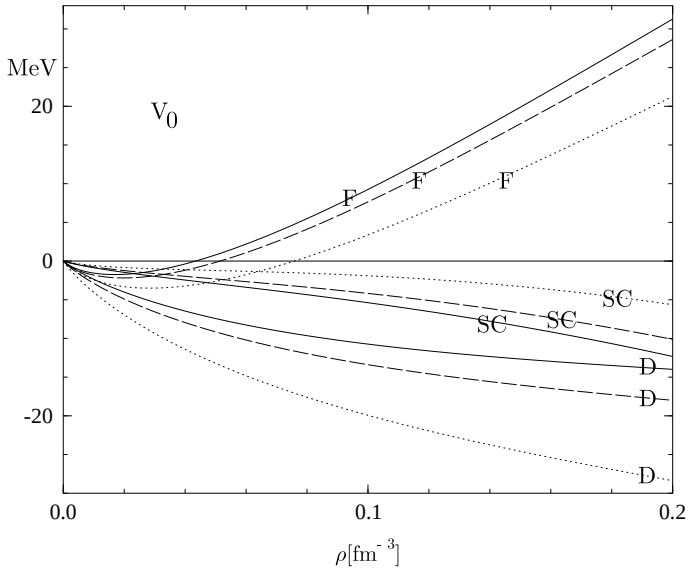

Fig. 1. Potential V_0 as function of ρ obtained with models F, D, and SC of the Nijmegen interaction. Solid curves were calculated for Σ momentum $k_\Sigma = 0$, and broken (dotted) curves for $k_\Sigma = 1(2)$ fm^{-1}.

The difference between the broken $k_\Sigma = 1$ fm^{-1} curves and the solid $k_\Sigma = 0$ curves is relatively small and is decreasing with decreasing density ρ. The average Σ momentum in the lowest observed Σ^- atomic state in Pb[2] is $\bar{k}_\Sigma \simeq 0.4$ fm^{-1}, and the density ρ relevant in Σ^- atoms is about an order of magnitude smaller than ρ_0. Consequently, assuming zero Σ momentum in calculating V_0 in Σ^- atoms, as it was done in [3], appears to be a reasonable approximation.

Our results for V_τ, V_σ, and $V_{\sigma\tau}$ as functions of ρ obtained with models D, F, SC, and NSC of the baryon–baryon interactions are shown in Figs 2–4. Solid, broken, and dotted curves represent results obtained for $k_\Sigma = 0$, $k_\Sigma = 1$, and $k_\Sigma = 2$ fm^{-1}. In Fig. 3 for V_σ, we do not show our D, $k_\Sigma = 2$ fm^{-1} results which partly coincide with our F, $k_\Sigma = 2$ fm^{-1} results.

As is seen in Figs 2–4, the dependence of $V_x, x = \tau, \sigma, \sigma\tau$, on the Σ momentum k_Σ is relatively weak, compared to the case of V_0. Consequently, the comparison of the calculated values of V_x at k_Σ with experimental data presented in [1,2], is valid also at $k_\Sigma > 0$. It appears that only the Lane potential V_τ may be directly related to existing experimental data. As pointed out

[2] In other (lighter) observed Σ^- atoms $\bar{k}_\Sigma$ is smaller than in Pb atoms.

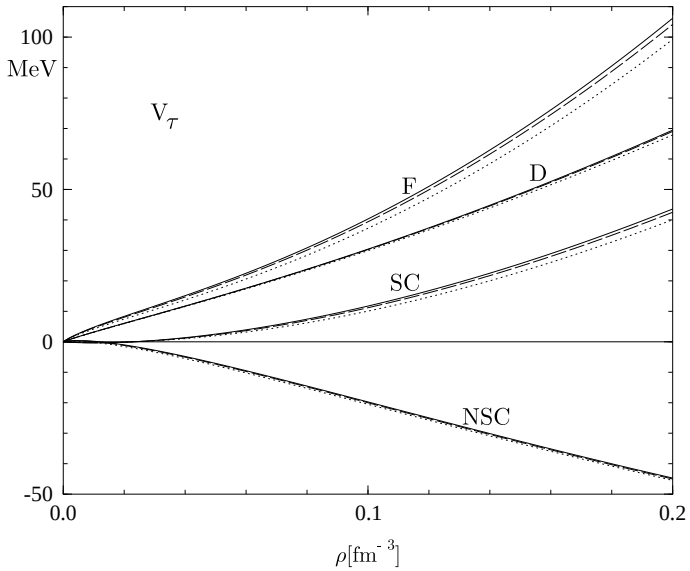

Fig. 2. Potential V_τ as function of ρ obtained with models F, D, SC, and NSC of the Nijmegen interaction. Solid curves were calculated for Σ momentum $k_\Sigma = 0$, and broken (dotted) curves for $k_\Sigma = 1(2)$ fm^{-1}.

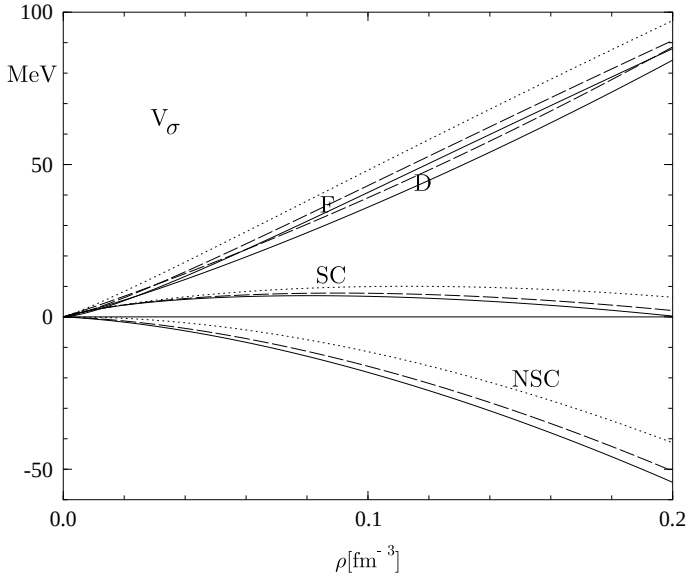

Fig. 3. Similar as Fig. 2 but for V_σ.

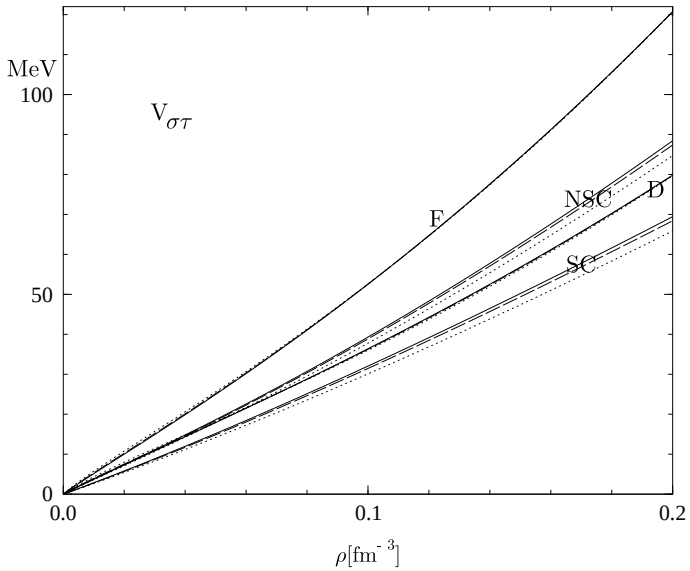

Fig. 4. Similar as Fig. 2 but for $V_{\sigma\tau}$.

in [1], a comparison between the pion spectra measured in Brookhaven [15] in the (K^-, π^+) and the (K^-, π^-) reactions on ^{9}Be suggests a strong $V_\tau > 0$, similar to our F model result. This conclusion, supported also by the analysis of Σ^- atoms [3, 16], favors model F, and eliminates the NSC model as a realistic representation of the Σ–N interaction.

Let us point out that to calculate V_x, we need the effective ΣN interaction which depends on two densities, *e.g.*, ρ_n and ρ_p in the case of V_τ. This intrinsic dependence of the effective ΣN interaction on two densities leads to a contribution to V_x, denoted in [1,2] as $V_x^{(I)}$. In our present calculations, similarly as in [1, 2], in calculating $V_x^{(I)}$, we applied the assumption (the single density approximation), that the effect of nuclear matter on the interaction between Σ and nucleons with given spin and isospin is dominated by the density of these nucleons. Although it is a plausible approximation, its accuracy is hard to estimate. At equilibrium density $\rho = \rho_0$, values of $V_x^{(I)}$ are listed in Table I of Ref. [2]. A particular case is our SC result for V_σ: without the $V_\sigma^{(I)}$ contribution V_σ would increase monotonically with increasing ρ. In the remaining cases the effect of $V_x^{(I)}$ is only quantitative. In general, the potentials V_x without the $V_x^{(I)}$ contribution are roughly proportional to the density ρ.

So far we considered the s.p. potential V_Σ defined in [1,2] in terms of the effective ΣN interaction. This s.p. potential which appears in calculating the energy of the system is often called the model potential. It should be distinguished from the s.p. potential $U_\Sigma(\boldsymbol{k}_\Sigma)$ of a Σ hyperon, defined together with its kinetic energy $\hbar^2 k_\Sigma^2/2M_\Sigma$ as the removal energy. The s.p. potential U_Σ differs from V_Σ by the rearrangement potential. The potential U_Σ may be decomposed into U_0 and U_x, $x = \tau, \sigma, \sigma\tau$, similarly as V_Σ in Eq.(1). According to the estimates in [17], we have:

$$U_0(\rho, k_\Sigma) = (1 - \bar{\kappa}(\rho))V_0(\rho, k_\Sigma), \;\; U_x(\rho, k_\Sigma) = (1 - \bar{\kappa}(\rho))V_x(\rho, k_\Sigma), \quad (2)$$

where $\bar{\kappa}(\rho)$ is the ΣN "wound integral" or correlation volume (divided by the volume per nucleon in nuclear matter) defined in [17]. If we assume a linear[3] dependence of $\bar{\kappa}(\rho)$ on ρ,

$$\bar{\kappa}(\rho) \simeq (\rho/\rho_0)\bar{\kappa}(\rho_0), \quad (3)$$

and use for $\bar{\kappa}(\rho_0)$ the value 0.15 discussed in [17], the connection between the potentials U and V takes the simple form:

$$U_0(\rho, k_\Sigma) = [1 - 0.15(\rho/\rho_0)]V_0(\rho, k_\Sigma)\,,$$

and

$$U_x(\rho, k_\Sigma) = [1 - 0.15(\rho/\rho_0)]V_x(\rho, k_\Sigma)\,.$$

Since $\kappa(\rho)$ is the small parameter of the Brueckner theory, the approximate expression (3) enables us to estimate the upper limit for the density ρ for which our results based on this theory are reliable.

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