MOMENTS OF THE PARTICLE PHASE-SPACE DENSITY AT FREEZE-OUT AND COINCIDENCE PROBABILITIES

A. Bialas[a,b], W. Czyż[a], K. Zalewski[a,b]

[a]M. Smoluchowski Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Kraków, Poland
and
[b]The H. Niewodniczanski Institute of Nuclear Physics
Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland

bialas@th.if.uj.edu.pl
zalewski@th.if.uj.edu.pl

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It is pointed out that the moments of phase-space particle density at freeze-out can be determined from the coincidence probabilities of the events observed in multiparticle production. A method to measure the coincidence probabilities is described and its validity examined.

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Recently we have proposed [1] a method for measuring the average of the particle density in multiparticle phase-space. Generalizing the approach used by Bertsch [2] for single particle phase-space densities, we defined the average density for a final state consisting of M particles as

$$\langle D_M \rangle = M \int dX dK \; W^2(\boldsymbol{K}, \boldsymbol{X}) \, . \tag{1}$$

Here $W(\boldsymbol{K}, \boldsymbol{X})$ is the M-particle distribution function and the integrations are over the $3M$ components of the position vectors and over the $3M$ components of the momentum vectors of the particles. Following Bertsch we replaced the emission function depending on the four-vectors K and X by a function depending on the three-vectors $\boldsymbol{K}$ and $\boldsymbol{X}$ at some representative time t_0 which does not need to be specified. This density distribution represents the situation at freeze-out.

In the present paper the method of measurement is generalized to arbitrary integer moments (l) of the phase-space density

$$\langle D_M^l \rangle = M^l \int dX dK \; W^{l+1}(\boldsymbol{K}, \boldsymbol{X}) \,. \tag{2}$$

As our argument follows closely that of [1], we shall be very brief. For details the reader can consult [1].

The method uses the coincidence probabilities defined as follows. An M-particle final state could be characterized by specifying the momenta of the M particles $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_M$. Then, however, with continuous p's, no two observed events would be identical. It has been proposed instead [3–7], to introduce momentum bins of finite width and to consider two events as identical, if they have the same distribution of particles among the bins. Let us denote the total number of events by N and the number of states which occurred l times by N_l. If a state occurs $k > l$ times it contributes to N_l its weight $\binom{k}{l}$. Then the number of l-fold coincidences is defined by

$$C_M^{\rm exp}(l) = \binom{N}{l}^{-1} N_l \,. \tag{3}$$

$C_M^{\rm exp}(l)$ is the ratio of the number of sets of l events with equivalent final states to the total number of sets of l events. Thus, for large N, it is the probability of an l-fold coincidence. The special case $l = 2$ had been considered, in a different context, by Ma [8]. The coincidence probability depends, of course, on the binning. For a given binning it is in principle not difficult to find from a data sample the coincidence probabilities. In practice the limiting factor is statistics.

We will now show that for suitably defined bins the *effective* coincidence probabilities

$$\hat{C}_M(l+1) \equiv \left(\frac{(2\pi)^{3M}}{M} \right)^l \left\langle D_M^l \right\rangle \,, \tag{4}$$

can be approximated by $C_M^{\rm exp}(l+1)$[1].

To see that, we first express the measured l-fold coincidences (3) by the $3M$-dimensional distribution of momenta

$$w(\boldsymbol{K}) = \int dX \; W(\boldsymbol{K}, \boldsymbol{X}) \,. \tag{5}$$

Let us denote the $3M$-dimensional momentum bins by $j = 1, \ldots, J$ and their volumes by ω_j. Then the probability that an event corresponds to a point in bin j is

[1] The power of 2π in (4) appears because for M free particles there is one quantum state per volume $(2\pi)^{3M}$ in phase-space.

$$P_j = \int_{\omega_j} dK \; w(\boldsymbol{K}) \tag{6}$$

and the average density in bin j

$$w_j = \frac{P_j}{\omega_j} \, . \tag{7}$$

The probability that l events chosen at random correspond each to a point in bin j is P_j^l. Therefore, the probability that l events chosen at random are the same is

$$C_M^{\exp}(l) = \sum_j P_j^l = \sum_j (\omega_j)^l (w_j)^l \, . \tag{8}$$

Next, we observe that, as seen from (2)

$$\left\langle D^l \right\rangle = M^l \sum_j \int_{\omega_j} dK \int dX \; W^{l+1}(\boldsymbol{K}, \boldsymbol{X}) \, . \tag{9}$$

In order to proceed further, following [1], we restrict ourselves to the phase-space distributions of the general form

$$W(\boldsymbol{K}, \boldsymbol{X}) = \frac{1}{(L_x L_y L_z)^M} \, G\left(\frac{\boldsymbol{X}}{L}\right) w(\boldsymbol{K}) \, . \tag{10}$$

Here $\boldsymbol{X}/L$ is a $3M$-dimensional vector with components $(X_1 - \overline{X}_1)/L_x, \ldots,$ $(Z_1 - \overline{Z}_M)/L_z$. The parameters $L_x, L_y, L_z, \overline{X}_1, \ldots, \overline{Z}_M$ are in general functions of $\boldsymbol{K}$. They could also be different for different kinds of particles. Function $G(u)$ satisfies the conditions

$$\int du \; G(u) = 1 \, , \qquad \int du \; uG(u) = 0 \, , \qquad \int du \; u^2 G(u) = 1 \, , \tag{11}$$

where all the integrations are $3M$-fold. These relations imply

$$\int dX \; G\left(\frac{\boldsymbol{X}}{L}\right) \;=\; (L_x L_y L_z)^M \, , \tag{12}$$

$$\langle X_1 \rangle \;=\; \overline{X}_1, \ldots, \langle Z_M \rangle = \overline{Z}_M \, , \tag{13}$$

$$\sigma^2(X_i) = L_x^2 \, , \qquad \sigma^2(Y_i) \;=\; L_y^2 \, , \qquad \sigma^2(Z_i) = L_z^2 \, , \tag{14}$$

for $i = 1, \ldots, M$. The first condition ensures that $w(\boldsymbol{K})$ is the observed $3M$-dimensional momentum distribution. The other two yield the physical interpretation of the parameters $\overline{X}_i, \overline{Y}_i, \overline{Z}_i, L_x, L_y, L_z$.

Using the Ansatz (10) and the definition (4) one finds

$$\hat{C}_M(l+1) = (2\pi g_{l+1})^{3Ml} \sum_j \left(\prod_{m=1}^{M} (L_x L_y L_z)_j^{(m)} \right)^{-l} \int_{\omega_j} dK \; w^{l+1}(\boldsymbol{K}) \,, \quad (15)$$

with $(g_{l+1})^{3Ml} \equiv \int du \; G^{l+1}(u)$.

Comparing this formula with formula (8) it is seen that if the bin sizes are

$$\omega_j = \prod_{m=1}^{M} \frac{(2\pi g_l)^3}{(L_x L_y L_z)_j^{(m)}} \,, \quad (16)$$

we obtain

$$\hat{C}_M(l) = C_M^{\exp}(l) \frac{\sum_j \frac{1}{\omega_j} \int_{\omega_j} dK \, [w(\boldsymbol{K})]^l}{\sum_j \left[\frac{1}{\omega_j} \int_{\omega_j} dK w(\boldsymbol{K}) \right]^l} \,. \quad (17)$$

Eq. (16) is rather general and can be applied to an entirely arbitrary discretization procedure. In the simple (but probably most practical) case when $L_x L_y L_z$ does not depend on $\vec{K}$, one can choose bins of constant lengths $\Delta_x, \Delta_y, \Delta_z$ along the axes K_x, K_y, K_z, respectively. Then the condition for the size of the bin is

$$\Delta_x \Delta_y \Delta_z = \frac{(2\pi g_l)^3}{L_x L_y L_z} \quad (18)$$

and $\omega_j = [\Delta_x \Delta_y \Delta_z]^M$.

These results, being very similar to that of [1], invite several comments which were elaborated there at length. Here we only repeat the main points.

As a first approximation we may omit the ratio of averages in the square bracket, giving $\hat{C}_M(l) = C_M^{\exp}(l)$. This approximation becomes exact when function $w(\boldsymbol{K})$ is constant within each bin j. The condition is on the volume of the bin, but does not constrain its shape. This freedom can be used to improve the approximation. Bins should be chosen narrow in directions where function $w(\boldsymbol{K})$ changes rapidly and broad in directions with little or no variation of $w(\boldsymbol{K})$. For given variability of $w(\boldsymbol{K})$ our approximation improves when the bins become smaller, *i.e.* when the product $L_x L_y L_z$ increases. Thus we expect the method to work much better for central heavy ion collisions than *e.g.* for e^+e^- annihilations. With very good statistics, or a reliable Monte Carlo event generator, one could estimate the term, which here we replaced by unity, and thus improve the approximation. In order to find the bin size it is necessary to know the volume in coordinate space $(L_x L_y L_z)$ and the integral $\int du\ G^l(u)$. Information about the volume may be provided by interferometric measurements. The integral is not very sensitive to the shape of the distribution. For a rectangular box $2\pi g_l = \pi/\sqrt{3}$. For Gaussians $2\pi g_l = \sqrt{2\pi}/[\sqrt{l}]^{1/(l-1)}$.

In conclusion, we have generalized the results of [1] to measurements of the higher moments of the phase-space density of particles produced in high-energy collisions. It turns out that these moments can be approximated by the measured coincidence probabilities of the appropriately discretized events. The optimal discretization is suggested and shown to depend critically on the volume of the system in configuration space. Thus the actual measurements would require this additional information to be effective.

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REFERENCES

[1] A. Bialas, W. Czyż, K. Zalewski, `hep-ph/0506233`.

[2] G.F. Bertsch, *Phys. Rev. Lett.* **72**, 2349 (1994); Erratum **77**, 789 (1996); D.A. Brown, S.Y. Panitkin, G.F. Bertsch, *Phys. Rev.* **C62**, 014904 (2000).

[3] A. Bialas, W. Czyż, J. Wosiek, *Acta Phys. Pol. B* **30**, 107 (1999).

[4] A. Bialas, W. Czyż, *Phys. Rev.* **D61**, 074021 (2000).

[5] A. Bialas, W. Czyż, *Acta Phys. Pol. B* **31**, 687 (2000).

[6] A. Bialas, W. Czyż, *Acta Phys. Pol. B* **31**, 2803 (2000).

[7] A. Bialas, W. Czyż, *Acta Phys. Pol. B* **34**, 3363 (2003).

[8] S.K. Ma, *J. Stat. Phys.* **26**, 221 (1981); S.K. Ma, *Statistical Physics*, World Scientific, Singapore 1985.