

MIXING IN QUANTUM FIELD THEORY*

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We report on recent results on the particle mixing and oscillations in Quantum Field Theory, in particular we discuss the proper definition of flavor charge and states in field mixing.

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1. Introduction

In the context of Quantum Field Theory (QFT), the non-perturbative vacuum structure associated with the field mixing [1–14] leads to a modification of flavor oscillation formulas [2, 7, 9–11], exhibiting new features with respect to the usual ones in (Quantum Mechanics) QM [15–18]. Moreover, the non-perturbative field theory effects may contribute in a crucial way in other physical contexts as shown in Ref. [19], where the neutrino mixing contribution to the dark energy is computed.

In this report we present the main results on the neutrino mixing and oscillations in Quantum Field Theory. In Sec. 2, we introduce the formalism of the fermion mixing in QFT. Sec. 3, is devoted to the study of the currents and charges for the flavor mixing and to the discussion of the proper definition of flavor charges and states. In Sec. 4, we discuss neutrino oscillations and in Sec. 5, we compute the neutrino mixing contribution to the dark energy. Sec. 6, is devoted to the conclusions.

2. Fermion mixing

For simplicity, we consider two Dirac neutrino fields. The Pontecorvo mixing transformations are [15]

$$\begin{aligned}\nu_e(x) &= \nu_1(x) \cos \theta + \nu_2(x) \sin \theta, \\ \nu_\mu(x) &= -\nu_1(x) \sin \theta + \nu_2(x) \cos \theta,\end{aligned}\tag{1}$$

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where $\nu_e(x)$ and $\nu_\mu(x)$ are the fields with definite flavors, θ is the mixing angle and ν_1 and ν_2 are the fields with definite masses $m_1 \neq m_2$. Explicitly ν_1 and ν_2 are given by

$$\nu_i(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}, r} \left[u_{\mathbf{k}, i}^r(t) \alpha_{\mathbf{k}, i}^r + v_{-\mathbf{k}, i}^r(t) \beta_{-\mathbf{k}, i}^{r\dagger} \right] e^{i\mathbf{k} \cdot \mathbf{x}}, \quad i = 1, 2, \quad (2)$$

with $u_{\mathbf{k}, i}^r(t) = e^{-i\omega_{\mathbf{k}, i} t} u_{\mathbf{k}, i}^r(0)$, $v_{\mathbf{k}, i}^r(t) = e^{i\omega_{\mathbf{k}, i} t} v_{\mathbf{k}, i}^r(0)$ and $\omega_{\mathbf{k}, i} = \sqrt{\mathbf{k}^2 + m_i^2}$. The vacuum for the α_i and β_i operators is denoted by $|0\rangle_{1,2}$: $\alpha_{\mathbf{k}, i}^r |0\rangle_{1,2} = \beta_{\mathbf{k}, i}^{r\dagger} |0\rangle_{1,2} = 0$. The anticommutation relations are the usual ones (see [1]). The orthonormality and completeness relations are: $u_{\mathbf{k}, i}^{r\dagger} u_{\mathbf{k}, i}^s = v v_{\mathbf{k}, i}^{r\dagger} v_{\mathbf{k}, i}^s = \delta_{rs}$, $u_{\mathbf{k}, i}^{r\dagger} v_{-\mathbf{k}, i}^s = v_{-\mathbf{k}, i}^{r\dagger} u_{\mathbf{k}, i}^s = 0$ and $\sum_r (u_{\mathbf{k}, i}^r u_{\mathbf{k}, i}^{r\dagger} + v_{-\mathbf{k}, i}^r v_{-\mathbf{k}, i}^{r\dagger}) = 1$.

The flavor fields can be expanded as (we use $(\sigma, i) = (e, 1), (\mu, 2)$):

$$\begin{aligned} \nu_\sigma(x) &= G_\theta^{-1}(t) \nu_i(x) G_\theta(t) \\ &= \frac{1}{\sqrt{V}} \sum_{\mathbf{k}, r} \left[u_{\mathbf{k}, i}^r(t) \alpha_{\mathbf{k}, \nu_\sigma}^r(t) + v_{-\mathbf{k}, i}^r(t) \beta_{-\mathbf{k}, \nu_\sigma}^{r\dagger}(t) \right] e^{i\mathbf{k} \cdot \mathbf{x}}, \end{aligned} \quad (3)$$

where $G_\theta(t)$ is the generator of the mixing transformations (1) given by

$$G_\theta(t) = \exp \left[\theta \int d^3 \mathbf{x} \left(\nu_1^\dagger(x) \nu_2(x) - \nu_2^\dagger(x) \nu_1(x) \right) \right]. \quad (4)$$

The flavor fields ν_e and ν_μ in Eq. (3) are expanded in the same basis of ν_1 and ν_2 . By introducing the operators

$$S_+(t) \equiv \int d^3 \mathbf{x} \nu_1^\dagger(x) \nu_2(x), \quad S_-(t) \equiv \int d^3 \mathbf{x} \nu_2^\dagger(x) \nu_1(x) = (S_+)^\dagger, \quad (5)$$

$G_\theta(t)$ can be written as $G_\theta(t) = \exp[\theta(S_+ - S_-)]$.

Introducing S_3 and the Casimir operator S_0 (proportional to the total charge) as follows

$$S_3 \equiv \frac{1}{2} \int d^3 \mathbf{x} \left(\nu_1^\dagger(x) \nu_1(x) - \nu_2^\dagger(x) \nu_2(x) \right), \quad (6)$$

$$S_0 \equiv \frac{1}{2} \int d^3 \mathbf{x} \left(\nu_1^\dagger(x) \nu_1(x) + \nu_2^\dagger(x) \nu_2(x) \right), \quad (7)$$

the $\text{su}(2)$ algebra is closed: $[S_+(t), S_-(t)] = 2S_3$, $[S_3, S_\pm(t)] = \pm S_\pm(t)$, $[S_0, S_3] = [S_0, S_\pm(t)] = 0$. Then the generator $G_\theta(t)$ is an element of the $\text{SU}(2)$ group.

The flavor annihilation operators and the flavor vacuum are defined as

$$\begin{pmatrix} \alpha_{\mathbf{k},\nu_\sigma}^r(t) \\ \beta_{-\mathbf{k},\nu_\sigma}^{r\dagger}(t) \end{pmatrix} = G_\theta^{-1}(t) \begin{pmatrix} \alpha_{\mathbf{k},i}^r \\ \beta_{-\mathbf{k},i}^{r\dagger} \end{pmatrix} G_\theta(t). \tag{8}$$

$$|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t) |0\rangle_{1,2}. \tag{9}$$

$|0(t)\rangle_{e,\mu}$ is an SU(2) generalized coherent state and it turns out to be orthogonal to the vacuum for the mass eigenstates $|0\rangle_{1,2}$ in the infinite volume limit. Note the time dependence of $|0(t)\rangle_{e,\mu}$. In the following we will denote the flavor vacuum state at the reference time $t = 0$ as $|0\rangle_{e,\mu}$.

The explicit expression of the flavor annihilation/creation operators for $\mathbf{k} = (0, 0, |\mathbf{k}|)$ is:

$$\begin{aligned} \alpha_{\mathbf{k},\nu_e}^r(t) &= \cos \theta \alpha_{\mathbf{k},1}^r + \sin \theta \left(U_{\mathbf{k}}^*(t) \alpha_{\mathbf{k},2}^r + \varepsilon^r V_{\mathbf{k}}(t) \beta_{-\mathbf{k},2}^{r\dagger} \right), \\ \alpha_{\mathbf{k},\nu_\mu}^r(t) &= \cos \theta \alpha_{\mathbf{k},2}^r - \sin \theta \left(U_{\mathbf{k}}(t) \alpha_{\mathbf{k},1}^r - \varepsilon^r V_{\mathbf{k}}(t) \beta_{-\mathbf{k},1}^{r\dagger} \right), \\ \beta_{-\mathbf{k},\nu_e}^r(t) &= \cos \theta \beta_{-\mathbf{k},1}^r + \sin \theta \left(U_{\mathbf{k}}^*(t) \beta_{-\mathbf{k},2}^r - \varepsilon^r V_{\mathbf{k}}(t) \alpha_{\mathbf{k},2}^{r\dagger} \right), \\ \beta_{-\mathbf{k},\nu_\mu}^r(t) &= \cos \theta \beta_{-\mathbf{k},2}^r - \sin \theta \left(U_{\mathbf{k}}(t) \beta_{-\mathbf{k},1}^r + \varepsilon^r V_{\mathbf{k}}(t) \alpha_{\mathbf{k},1}^{r\dagger} \right), \end{aligned} \tag{10}$$

with $\varepsilon^r = (-1)^r$ and

$$U_{\mathbf{k}}(t) \equiv u_{\mathbf{k},2}^{r\dagger}(t) u_{\mathbf{k},1}^r(t) = v_{-\mathbf{k},1}^{r\dagger}(t) v_{-\mathbf{k},2}^r(t), \tag{11}$$

$$V_{\mathbf{k}}(t) \equiv \varepsilon^r u_{\mathbf{k},1}^{r\dagger}(t) v_{-\mathbf{k},2}^r(t) = -\varepsilon^r u_{\mathbf{k},2}^{r\dagger}(t) v_{-\mathbf{k},1}^r(t). \tag{12}$$

We have:

$$U_{\mathbf{k}}(t) = |U_{\mathbf{k}}| e^{i(\omega_{k,2} - \omega_{k,1})t}, \quad V_{\mathbf{k}}(t) = |V_{\mathbf{k}}| e^{i(\omega_{k,2} + \omega_{k,1})t}, \tag{13}$$

$$\begin{aligned} |U_{\mathbf{k}}| &= \left(\frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{\frac{1}{2}} \left(\frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{\frac{1}{2}} \left(1 + \frac{|\mathbf{k}|^2}{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)} \right), \\ |V_{\mathbf{k}}| &= \left(\frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{\frac{1}{2}} \left(\frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{\frac{1}{2}} \left(\frac{|\mathbf{k}|}{(\omega_{k,2} + m_2)} - \frac{|\mathbf{k}|}{(\omega_{k,1} + m_1)} \right), \end{aligned} \tag{14}$$

$$|U_{\mathbf{k}}|^2 + |V_{\mathbf{k}}|^2 = 1. \tag{15}$$

The condensation density is given by

$${}_{e,\mu}\langle 0 | \alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r | 0 \rangle_{e,\mu} = {}_{e,\mu}\langle 0 | \beta_{\mathbf{k},i}^{r\dagger} \beta_{\mathbf{k},i}^r | 0 \rangle_{e,\mu} = \sin^2 \theta |V_{\mathbf{k}}|^2, \quad i = 1, 2. \tag{16}$$

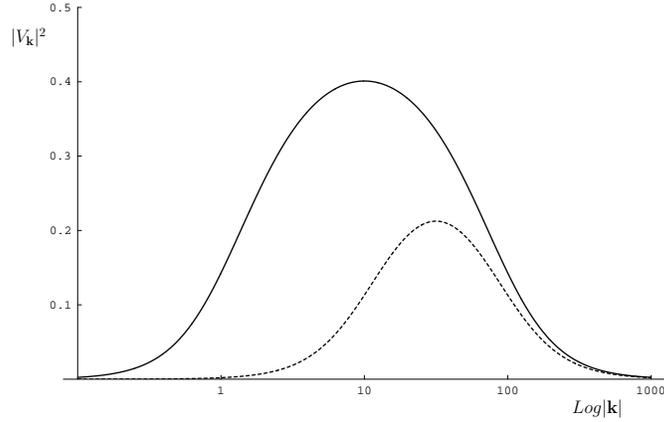


Fig. 1. The fermion condensation density $|V_{\mathbf{k}}|^2$ as a function of $|\mathbf{k}|$ for $m_1 = 1$, $m_2 = 100$ (solid line) and $m_1 = 10$, $m_2 = 100$ (dashed line).

3. Flavor charges and states

In this Section we report the analysis of the transformations acting on a doublet of free fields with different masses and discuss the proper definition of flavor charges and states.

Let us consider the Lagrangian describing two free Dirac fields with masses m_1 and m_2 :

$$\mathcal{L}(x) = \bar{\nu}_m(x) (i \not{\partial} - \mathbf{M}_d) \nu_m(x), \quad (17)$$

where $\nu_m^T = (\nu_1, \nu_2)$ and $\mathbf{M}_d = \text{diag}(m_1, m_2)$.

The Lagrangian $\mathcal{L}(x)$ is invariant under global U(1) phase transformations of the type $\nu'_m(x) = e^{i\alpha} \nu_m(x)$, as a result, we have the conservation of the Noether charge $Q = \int I^0(x) d^3\mathbf{x}$ (with $I^\mu(x) = \bar{\nu}_m(x) \gamma^\mu \nu_m(x)$) which is indeed the total charge of the system, *i.e.* the total lepton number.

Consider then the global SU(2) transformation [3]:

$$\nu'_m(x) = e^{i\alpha_j \cdot \tau_j} \nu_m(x), \quad j = 1, 2, 3. \quad (18)$$

with α_j real constants, $\tau_j = \sigma_j/2$ with σ_j being the Pauli matrices.

Since $m_1 \neq m_2$, the Lagrangian is not invariant under the transformations (18) and, by use of the equations of motion, we obtain the variation of the Lagrangian:

$$\delta\mathcal{L} = i\alpha_j \bar{\nu}_m(x) [\tau_j, \mathbf{M}_d] \nu_m(x) = -\alpha_j \partial_\mu J_{m,j}^\mu(x), \quad (19)$$

where the currents are given by:

$$J_{m,j}^\mu(x) = \bar{\nu}_m(x) \gamma^\mu \tau_j \nu_m(x), \quad j = 1, 2, 3. \quad (20)$$

The related charges $Q_{m,j}(t) = \int d^3\mathbf{x} J_{m,j}^0(x)$, satisfy the su(2) algebra: $[Q_{m,i}(t), Q_{m,j}(t)] = i\varepsilon_{ijk}Q_{m,k}(t)$.

Note that the Casimir operator is proportional to the total (conserved) charge: $Q_{m,0} = 1/2Q$.

Also $Q_{m,3}$ is conserved, due to the fact that the mass matrix M_d is diagonal and this implies the conservation of charge separately for ν_1 and ν_2 . The U(1) Noether charges associated with ν_1 and ν_2 can be then expressed as

$$Q_1 \equiv \frac{1}{2}Q + Q_{m,3}, \quad Q_2 \equiv \frac{1}{2}Q - Q_{m,3}. \tag{21}$$

with Q total (conserved) charge. The normal ordered charge operators are:

$$:Q_i: \equiv \int d^3\mathbf{x} : \nu_i^\dagger(x) \nu_i(x) := \sum_r \int d^3\mathbf{k} \left(\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r - \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^r \right), \tag{22}$$

where $i = 1, 2$ and the $: \dots :$ denotes normal ordering with respect to the vacuum $|0\rangle_{1,2}$.

The neutrino states with definite masses defined as

$$|\nu_{\mathbf{k},i}^r\rangle = \alpha_{\mathbf{k},i}^{r\dagger}|0\rangle_{1,2}, \quad i = 1, 2, \tag{23}$$

are then eigenstates of Q_1 and Q_2 , which can be identified with the lepton charges in the absence of mixing.

Let us now consider the Lagrangian written in the flavor basis

$$\mathcal{L}(x) = \bar{\nu}_f(x) (i \not{\partial} - M) \nu_f(x), \tag{24}$$

where $\nu_f^T = (\nu_e, \nu_\mu)$ and $M = \begin{pmatrix} m_e & m_{e\mu} \\ m_{e\mu} & m_\mu \end{pmatrix}$.

The variation of the Lagrangian (24) under the SU(2) transformation

$$\nu'_f(x) = e^{i\alpha_j \cdot \tau_j} \nu_f(x), \quad j = 1, 2, 3, \tag{25}$$

is given by

$$\delta\mathcal{L}(x) = i\alpha_j \bar{\nu}_f(x) [\tau_j, M] \nu_f(x) = -\alpha_j \partial_\mu J_{f,j}^\mu(x), \tag{26}$$

$$J_{f,j}^\mu(x) = \bar{\nu}_f(x) \gamma^\mu \tau_j \nu_f(x), \quad j = 1, 2, 3. \tag{27}$$

Again, the charges $Q_{f,j}(t) = \int d^3\mathbf{x} J_{f,j}^0(x)$ close the su(2) algebra.

Note that, because of the off-diagonal (mixing) terms in M , $Q_{f,3}(t)$ is time dependent. This implies an exchange of charge between ν_e and ν_μ ,

resulting in the phenomenon of neutrino oscillations. The flavor charges for mixed fields are defined as [3]

$$Q_{\nu_e}(t) = \frac{1}{2}Q + Q_{f,3}(t), \quad Q_{\nu_\mu}(t) = \frac{1}{2}Q - Q_{f,3}(t). \quad (28)$$

The normal ordered charge operators are

$$\begin{aligned} :: Q_{\nu_\sigma}(t) :: &\equiv \int d^3\mathbf{x} :: \nu_\sigma^\dagger(x) \nu_\sigma(x) :: \\ &= \sum_r \int d^3\mathbf{k} \left(\alpha_{\mathbf{k},\nu_\sigma}^{r\dagger}(t) \alpha_{\mathbf{k},\nu_\sigma}^r(t) - \beta_{-\mathbf{k},\nu_\sigma}^{r\dagger}(t) \beta_{-\mathbf{k},\nu_\sigma}^r(t) \right), \end{aligned} \quad (29)$$

where $\sigma = e, \mu$, and $:: \dots ::$ denotes normal ordering with respect to $|0\rangle_{e,\mu}$. The definition for any operator A , is

$$:: A :: \equiv A - {}_{e,\mu}\langle 0|A|0\rangle_{e,\mu}. \quad (30)$$

Note that $:: Q_{\nu_\sigma}(t) :: = G_\theta^{-1}(t) : Q_j : G_\theta(t)$, with $(\sigma, j) = (e, 1), (\mu, 2)$ and

$$:: Q :: = :: Q_{\nu_e}(t) :: + :: Q_{\nu_\mu}(t) :: = : Q_1 : + : Q_2 : = : Q : . \quad (31)$$

The flavor states are defined as eigenstates of the flavor charges Q_{ν_σ} at a reference time $t = 0$

$$|\nu_{\mathbf{k},\sigma}^r\rangle \equiv \alpha_{\mathbf{k},\nu_\sigma}^{r\dagger}(0)|0(0)\rangle_{e,\mu}, \quad \sigma = e, \mu \quad (32)$$

and similar ones for antiparticles. We have

$$\begin{aligned} :: Q_{\nu_e}(0) :: |\nu_{\mathbf{k},e}^r\rangle &= |\nu_{\mathbf{k},e}^r\rangle, \\ :: Q_{\nu_\mu}(0) :: |\nu_{\mathbf{k},\mu}^r\rangle &= |\nu_{\mathbf{k},\mu}^r\rangle, \end{aligned} \quad (33)$$

$$\begin{aligned} :: Q_{\nu_e}(0) :: |\nu_{\mathbf{k},\mu}^r\rangle &= :: Q_{\nu_\mu}(0) :: |\nu_{\mathbf{k},e}^r\rangle = 0, \\ :: Q_{\nu_\sigma}(0) :: |0\rangle_{e,\mu} &= 0. \end{aligned} \quad (34)$$

These results are not trivial since the usual Pontecorvo states [15]

$$|\nu_{\mathbf{k},e}^r\rangle_{\text{P}} = \cos\theta |\nu_{\mathbf{k},1}^r\rangle + \sin\theta |\nu_{\mathbf{k},2}^r\rangle, \quad (35)$$

$$|\nu_{\mathbf{k},\mu}^r\rangle_{\text{P}} = -\sin\theta |\nu_{\mathbf{k},1}^r\rangle + \cos\theta |\nu_{\mathbf{k},2}^r\rangle, \quad (36)$$

are *not* eigenstates of the flavor charges [20]. Indeed the expectation values of the flavor charges on the Pontecorvo states are

$$\begin{aligned} {}_{\text{P}}\langle \nu_{\mathbf{k},e}^r | :: Q_{\nu_e}(0) :: | \nu_{\mathbf{k},e}^r \rangle_{\text{P}} &= \cos^4\theta + \sin^4\theta \\ &+ 2|U_{\mathbf{k}}| \sin^2\theta \cos^2\theta + \sum_r \int d^3\mathbf{k}, \end{aligned} \quad (37)$$

and

$${}_{1,2}\langle 0 | : Q_{\nu_e}(0) : | 0 \rangle_{1,2} = \sum_r \int d^3\mathbf{k}, \tag{38}$$

which are both infinite.

Although the infinities in Eqs. (37) and (38) may be removed by normal ordering with respect to the mass vacuum, we have that

$${}_{1,2}\langle 0 | (: Q_{\nu_e}(0) :)^2 | 0 \rangle_{1,2} = 4 \sin^2 \theta \cos^2 \theta \int d^3\mathbf{k} |V_{\mathbf{k}}|^2, \tag{39}$$

$$\begin{aligned} {}_P \langle \nu_{\mathbf{k},e}^r | (: Q_{\nu_e}(0) :)^2 | \nu_{\mathbf{k},e}^r \rangle_P &= \cos^6 \theta + \sin^6 \theta \\ &+ \sin^2 \theta \cos^2 \theta \left[2|U_{\mathbf{k}}| + |U_{\mathbf{k}}|^2 + 4 \int d^3\mathbf{k} |V_{\mathbf{k}}|^2 \right], \end{aligned} \tag{40}$$

are both infinite, making the corresponding quantum fluctuations divergent. Thus, the correct flavor state and normal ordered operators are those defined in Eqs. (32) and (30), respectively.

4. Neutrino oscillations

In the standard QM treatment [15], the Pontecorvo states (35)–(36) are usually assumed to be produced in a charged current weak interaction process, together with the respective charged (anti-) leptons. However, as we have shown above, such states are not eigenstates of the (neutrino) lepton charges when mixing is present.

In other words, using such states produces a *violation of lepton charge conservation* both in the production and in the detection vertices. Indeed, in presence of mixing, the lepton charge is violated (for a given family), but this violation occurs during time evolution (flavor oscillations), whereas the lepton number is conserved in a charged current vertex due to the form of the weak interaction.

To be more specific, let us define the following quantities

$$\begin{aligned} A_0 &\equiv {}_P \langle \nu_{\mathbf{k},e}^r | : Q_{\nu_e}(0) : | \nu_{\mathbf{k},e}^r \rangle_P \\ &= \cos^4 \theta + \sin^4 \theta + 2|U_{\mathbf{k}}| \sin^2 \theta \cos^2 \theta < 1, \end{aligned} \tag{41}$$

$$\begin{aligned} 1 - A_0 &\equiv {}_P \langle \nu_{\mathbf{k},e}^r | : Q_{\nu_\mu}(0) : | \nu_{\mathbf{k},e}^r \rangle_P \\ &= 2 \sin^2 \theta \cos^2 \theta - 2|U_{\mathbf{k}}| \sin^2 \theta \cos^2 \theta > 0, \end{aligned} \tag{42}$$

for any $\theta \neq 0$, $\mathbf{k} \neq 0$ and for $m_1 \neq m_2$. We have

$${}_P\langle \nu_{\mathbf{k},e}^r | : Q_{\nu_e}(0) : | \nu_{\mathbf{k},e}^r \rangle_P + {}_P\langle \nu_{\mathbf{k},e}^r | : Q_{\nu_\mu}(0) : | \nu_{\mathbf{k},e}^r \rangle_P = 1. \quad (43)$$

Let us now consider an ideal experiment in which neutrinos are created and detected by means of some charged weak interaction process. What is measured in the experiment is the number of charged leptons, say (anti-) electrons, both in the source and in the detector. Indicating with N_e^S such a number at the neutrino source and with $N_e^D(t)$ the number at the detector, in the usual treatment one assumes that $N_{\nu_e}^S = N_e^S$ and $N_{\nu_e}^D(t) = N_e^D(t)$, where $N_{\nu_e}^S$ and $N_{\nu_e}^D(t)$ are the neutrinos produced in the source and in the detector, respectively.

The usual Pontecorvo formulas are then given by

$$\frac{N_e^D(t)}{N_e^S} = \frac{N_{\nu_e}^D(t)}{N_{\nu_e}^S} = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta\omega}{2} t \right) = 1 - P(t), \quad (44)$$

$$\frac{N_\mu^D(t)}{N_e^S} = \frac{N_{\nu_\mu}^D(t)}{N_{\nu_e}^S} = \sin^2 2\theta \sin^2 \left(\frac{\Delta\omega}{2} t \right) = P(t). \quad (45)$$

However, considering Eqs. (41), (42), we know that Pontecorvo states violate the lepton charge. Thus, if we assume that the source produces the Pontecorvo states (35), (36) the conservation of leptonic charge both in the production and in the detection vertices implies that only a fraction of the electron neutrinos produced, is accompanied by an anti-electron: we denote these quantities with a tilde. We have then

$$\tilde{N}_e^S = A_0 N_{\nu_e}^S, \quad (46)$$

$$\tilde{N}_e^D(t) = A_0 N_{\nu_e}^D(t) + (1 - A_0) N_{\nu_\mu}^D(t). \quad (47)$$

Thus the oscillation formula becomes

$$\frac{\tilde{N}_e^D(t)}{\tilde{N}_e^S} = \frac{A_0 N_{\nu_e}^D(t) + (1 - A_0) N_{\nu_\mu}^D(t)}{A_0 N_{\nu_e}^S} = 1 - \frac{2A_0 - 1}{A_0} P(t). \quad (48)$$

Eq. (48) is clearly different from the usual Pontecorvo formula (44) which is, however, recovered in the relativistic limit. Indeed, for $|\mathbf{k}| \gg \sqrt{m_1 m_2}$ we have $|U_{\mathbf{k}}| \rightarrow 1$ and $A_0 = 1$.

5. Neutrino mixing contribution to the dark energy

In this section, we show that the non-perturbative vacuum structure associated with neutrino mixing leads to a non zero contribution to the value of the dark energy. In order to compute the vacuum energy density

$\langle \rho_{\text{vac}} \rangle$ we use the (0,0) component of the energy-momentum tensor density $\mathcal{T}_{00}(x)$ in flat space time

$$: \mathcal{T}_{00}(x) := \frac{i}{2} : \left(\bar{\nu}_m(x) \gamma_0 \overleftrightarrow{\partial}_0 \nu_m(x) \right) : \tag{49}$$

In terms of the annihilation and creation operators of fields ν_1 and ν_2 , the (0,0) component of the energy-momentum tensor $T_{00} = \int d^3x \mathcal{T}_{00}(x)$ is

$$: T_{00}^{(i)} := \sum_r \int d^3\mathbf{k} \omega_{k,i} \left(\alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r + \beta_{-\mathbf{k},i}^{r\dagger} \beta_{-\mathbf{k},i}^r \right), \tag{50}$$

with $i = 1, 2$. Note that $T_{00}^{(i)}$ is time independent.

The expectation value of $T_{00}^{(i)}$ in the flavor vacuum $|0\rangle_{e,\mu}$, gives the contribution $\langle \rho_{\text{vac}}^{\text{mix}} \rangle$ of the neutrino mixing to the vacuum energy density

$${}_{e,\mu} \langle 0 | \sum_i : T_{00}^{(i)}(0) : |0\rangle_{e,\mu} = \langle \rho_{\text{vac}}^{\text{mix}} \rangle \eta_{00}. \tag{51}$$

Since ${}_{e,\mu} \langle 0 | : T_{00}^{(i)} : |0\rangle_{e,\mu} = {}_{e,\mu} \langle 0(t) | : T_{00}^{(i)} : |0(t)\rangle_{e,\mu}$ for any t , we obtain

$$\begin{aligned} \langle \rho_{\text{vac}}^{\text{mix}} \rangle &= \sum_{i,r} \int d^3\mathbf{k} \omega_{k,i} \left({}_{e,\mu} \langle 0 | \alpha_{\mathbf{k},i}^{r\dagger} \alpha_{\mathbf{k},i}^r |0\rangle_{e,\mu} + {}_{e,\mu} \langle 0 | \beta_{\mathbf{k},i}^{r\dagger} \beta_{\mathbf{k},i}^r |0\rangle_{e,\mu} \right) \\ &= 8 \sin^2 \theta \int d^3\mathbf{k} (\omega_{k,1} + \omega_{k,2}) |V_{\mathbf{k}}|^2, \end{aligned} \tag{52}$$

i.e.

$$\langle \rho_{\text{vac}}^{\text{mix}} \rangle = 32\pi^2 \sin^2 \theta \int_0^K dk k^2 (\omega_{k,1} + \omega_{k,2}) |V_{\mathbf{k}}|^2, \tag{53}$$

where the cut-off K has been introduced. Choosing the cut-off proportional to the natural scale appearing in the mixing phenomenon, $K \simeq \sqrt{m_1 m_2}$ or the cut-off scale given by the sum of the two neutrino masses, $K = m_1 + m_2$ [21], we have $\langle \rho_{\text{vac}}^{\text{mix}} \rangle = 0.4 \times 10^{-47} \text{ GeV}^4$, which is in agreement with the estimated value of the dark energy.

6. Conclusions

In this report we have discussed some aspects of the neutrino mixing and oscillations in the context of Quantum Field Theory.

We have reported the study of the algebraic structures of field mixing and discussed the proper definition of flavor charge and states.

Moreover, we have shown that the non-perturbative field theory effects may contribute in a specific way in other physical phenomena, in particular the neutrino mixing may contribute to the value of the dark energy exactly because of the non-perturbative effects.

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