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LORENTZ INVARIANCE AND NEUTRINO OSCILLATIONS*

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We discuss the issue of Lorentz invariance for neutrino oscillations by resorting to the properties of flavor charges and currents. It turns out that oscillation formulas are sensitive to the effects of a boost on the source (detector), resulting in a possibly measurable effect.

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1. Introduction

Neutrino mixing and oscillations [1] are nowadays regarded as one of the most promising fields where to search for new Physics. On one hand, indeed, their experimental discovery [2] represents the first step beyond the Standard Model of Particle Physics; on the other, the peculiar nature of these particles, poses a whole series of questions [3] which are likely to require radically new ideas in order to be answered.

As a matter of fact, it is only recently [4] that a rich non-perturbative structure of the vacuum for the mixed fields was discovered, in the attempt of a (canonical) field theoretical treatment of particle mixing. This result has led to a series of developments [5–13], including the refinement of oscillation formulas, both for fermions and for bosons, exhibiting corrections with respect to the usual Quantum Mechanical ones.

Also at a more conceptual level, the above discovery has consequences, since the inequivalence of the flavor and mass representations implies that the fundamental entities are the flavor states, rather than the mass states, as usually assumed. Thus it is natural to ask what is the meaning of Lorentz invariance for such states which exhibit non-standard dispersion relations [14].

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In this paper we consider the same problem under a different angle, namely the Lorentz invariance properties of the oscillation formulas. Such an analysis has been carried out in the context of the usual QM framework by Giunti *et al.* [15, 16]: it was proved that the usual oscillation formulas are indeed Lorentz invariant, at least in the relativistic limit.

We consider this problem in the context of QFT and show that the flavor charges for mixed fields are indeed not Lorentz invariant. It follows that, in the case of source and detector in relative motion with each other, the oscillation formulas gain a correction term, which however vanishes in the extreme relativistic limit, in agreement with the QM result of Ref. [16].

We review in Section 2 some elements of QFT treatment for mixed fields which are necessary for the discussion of Section 3, where we derive the oscillation formula in the case of a boosted source (detector). Some useful relations are given in Appendix.

2. Neutrino oscillations in QFT

In Ref. [6], it was shown that flavor oscillations can be consistently described in QFT by considering expectation values of the flavor charges on states of the flavor Hilbert space. The oscillation formulas thus obtained reproduce the usual QM formulas in the relativistic limit.

More general formulas including the full space-time dependence, can be derived by using the 4-component flavor currents and wave packets, as shown in Ref. [7].

To be specific, let us consider the case of two flavor mixing with Dirac neutrinos and introduce the following Lagrangian

$$\mathcal{L}(x) = \bar{\Psi}_f(x)\,(i\,\partial\!\!\!/ - M)\,\Psi_f(x)\,, \tag{1}$$

where $\Psi_f^{\mathrm{T}} = (\nu_e, \nu_\mu)$ and $M = \begin{pmatrix} m_e & m_{e\mu} \\ m_{e\mu} & m_\mu \end{pmatrix}$. The mixing transformations

$$\begin{aligned} \nu_e(x) &= \cos\theta\;\nu_1(x) + \sin\theta\;\nu_2(x)\,, \\ \nu_\mu(x) &= -\sin\theta\;\nu_1(x) + \cos\theta\;\nu_2(x) \end{aligned} \tag{2}$$

with θ being the mixing angle, reduce the quadratic form of Eq. (1) to the Lagrangian for two free Dirac fields, with masses m_1 and m_2:

$$\mathcal{L}(x) = \bar{\Psi}_m(x)\,(i\,\partial\!\!\!/ - M_{\mathrm{d}})\,\Psi_m(x)\,, \tag{3}$$

where $\Psi_m^{\mathrm{T}} = (\nu_1, \nu_2)$ and $M_{\mathrm{d}} = \mathrm{diag}(m_1, m_2)$. One also has $m_e = m_1\cos^2\theta + m_2\sin^2\theta$, $m_\mu = m_1\sin^2\theta + m_2\cos^2\theta$, $m_{e\mu} = (m_2 - m_1)\sin\theta\cos\theta$. Without loss of generality we take θ ranging from 0 to $\frac{\pi}{4}$ (maximal mixing) and $m_2 > m_1$.

The above mixing transformations can be also implemented in the following way [4]:

$$\nu_\sigma(x) \;\equiv\; G^{-1}_\theta(t)\,\nu_i(x)\,G_\theta(t)\,, \tag{4}$$

$$G_\theta(t) \;=\; \exp\left[\theta \int d^3\boldsymbol{x}\left(\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)\right)\right]. \tag{5}$$

with $(\sigma, i) = (e, 1), (\mu, 2)$ and $t \equiv x^0$. The generator $G_\theta(t)$ allows to define flavor ladder operators and the flavor vacuum as [4]:

$$\alpha^r_{\boldsymbol{k},\sigma}(t) \;\equiv\; G^{-1}_\theta(t)\alpha^r_{\boldsymbol{k},i}(t)G_\theta(t)\,; \quad \beta^{r\dagger}_{-\boldsymbol{k},\sigma}(t) \equiv G^{-1}_\theta(t)\beta^{r\dagger}_{-\boldsymbol{k},i}(t)G_\theta(t)\,, \tag{6}$$

$$|0(t)\rangle_{e,\mu} \;\equiv\; G^{-1}_\theta(t)\;|0\rangle_{1,2}\,. \tag{7}$$

with $(\sigma, i) = (e, 1), (\mu, 2)$.

Following the discussion of Ref. [6], we study the symmetry properties of the above Lagrangian. $\mathcal{L}$ is clearly invariant under U(1). We then consider the SU(2) transformation:

$$\Psi'_f(x) \;=\; e^{i\alpha_j\tau_j}\,\Psi_f(x), \tag{8}$$

$$\delta\mathcal{L}(x) \;=\; i\alpha_j\,\bar{\Psi}_f(x)\,[\tau_j, M]\,\Psi_f(x) \;=\; -\alpha_j\,\partial_\mu J^\mu_{f,j}(x)\,, \tag{9}$$

$$J^\mu_{f,j}(x) \;=\; \bar{\Psi}_f(x)\,\gamma^\mu\,\tau_j\,\Psi_f(x), \qquad j = 1, 2, 3. \tag{10}$$

The charges $Q_{f,j}(t) \equiv \int d^3\boldsymbol{x}\, J^0_{f,j}(x)$ satisfy the su(2) algebra.

We define the *flavor charges* for mixed fields as

$$Q_e(t) \;\equiv\; \int d^3\boldsymbol{x}\,\nu_e^\dagger(x)\nu_e(x) = \frac{1}{2}Q + Q_{f,3}(t)\,, \tag{11}$$

$$Q_\mu(t) \;\equiv\; \int d^3\boldsymbol{x}\,\nu_\mu^\dagger(x)\nu_\mu(x) = \frac{1}{2}Q - Q_{f,3}(t)\,, \tag{12}$$

where $Q_e(t) + Q_\mu(t) = Q$. They are related to the Noether charges[1] Q_i as

$$Q_\sigma(t) \;=\; G^{-1}_\theta(t)\,Q_i\,G_\theta(t) \tag{13}$$

with $(\sigma, i) = (e, 1), (\mu, 2)$.

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[1] The conserved charges Q_i, $i = 1, 2$ are related to the invariance of the Lagrangian (3) under U(1) transformations acting on ν_1 and ν_2 separately.

From Eq. (13), it follows that the flavor charges are diagonal in the flavor ladder operators:

$$Q_\sigma(t) = \sum_r \int d^3\boldsymbol{k} \left(\alpha^{r\dagger}_{\boldsymbol{k},\sigma}(t)\alpha^r_{\boldsymbol{k},\sigma}(t) \, - \, \beta^{r\dagger}_{-\boldsymbol{k},\sigma}(t)\beta^r_{-\boldsymbol{k},\sigma}(t)\right) , \qquad (14)$$

with $\sigma = e, \mu$. We work in the Heisenberg picture and define the state for a particle with definite (electron) flavor, spin and momentum as:

$$|\alpha^r_{\boldsymbol{k},e}\rangle \equiv \alpha^{r\dagger}_{\boldsymbol{k},e}(0)|0\rangle_{e,\mu} = G^{-1}_\theta(0)\alpha^{r\dagger}_{\boldsymbol{k},1}|0\rangle_{1,2} \, , \qquad (15)$$

where $|0\rangle_{e,\mu} \equiv |0(0)\rangle_{e,\mu}$. Note that the $|\alpha^r_{\boldsymbol{k},e}\rangle$ is an eigenstate of $Q_e(t)$, at $t = 0$: $Q_e(0)|\alpha^r_{\boldsymbol{k},e}\rangle = |\alpha^r_{\boldsymbol{k},e}\rangle$. We thus have ${}_{e,\mu}\langle 0|Q_\sigma(t)|0\rangle_{e,\mu} \; = \; 0$ and

$$\begin{aligned} \mathcal{Q}_{\boldsymbol{k},\sigma}(t) \; &\equiv \; \langle\alpha^r_{\boldsymbol{k},e}|Q_\sigma(t)|\alpha^r_{\boldsymbol{k},e}\rangle \\ &= \; \left|\left\{\alpha^r_{\boldsymbol{k},\sigma}(t), \alpha^{r\dagger}_{\boldsymbol{k},\rho}(0)\right\}\right|^2 \; + \; \left|\left\{\beta^{r\dagger}_{-\boldsymbol{k},\sigma}(t), \alpha^{r\dagger}_{\boldsymbol{k},\rho}(0)\right\}\right|^2 . \end{aligned} \qquad (16)$$

Charge conservation is ensured at any time: $\mathcal{Q}_{\boldsymbol{k},e}(t) + \mathcal{Q}_{\boldsymbol{k},\mu}(t) \; = \; 1$. The oscillation formulas for the flavor charges are then [5]

$$\begin{aligned} \mathcal{Q}_{\boldsymbol{k},e}(t) \; &= \; 1 - \sin^2(2\theta)\,|U_{\boldsymbol{k}}|^2\, \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}t\right) \\ &\quad + \sin^2(2\theta)\,|V_{\boldsymbol{k}}|^2\, \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}t\right) , \end{aligned} \qquad (17)$$

$$\begin{aligned} \mathcal{Q}_{\boldsymbol{k},\mu}(t) \; &= \; \sin^2(2\theta)\,|U_{\boldsymbol{k}}|^2\, \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}t\right) \\ &\quad + \sin^2(2\theta)\,|V_{\boldsymbol{k}}|^2\, \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}t\right) . \end{aligned} \qquad (18)$$

This result is exact. There are two differences with respect to the usual formula for neutrino oscillations: the amplitudes are energy dependent, and there is an additional oscillating term.

3. Neutrino oscillations and Lorentz invariance

We now consider the situation in which there are two inertial observers, O and O', related by a Lorentz transformation and relate the flavor charges defined by both of them.

We observe that the flavor currents have non-zero divergence:

$$\begin{aligned} \partial^\rho J^\sigma_\rho(x) \; &= \; \partial^\rho\left[G^{-1}(t)J^i_\rho(x)G(t)\right] = \dot{G}^{-1}(t)J^i_0(x)G(t) \, + \, G^{-1}(t)J^i_0(x)\dot{G}(t) \\ &= \; \left[J^\sigma_0(x), G^{-1}(t)\dot{G}(t)\right] , \end{aligned} \qquad (19)$$

where we denoted by a dot the time derivative and have used the fact that $G^{-1}\dot{G} = -\dot{G}^{-1}G$.

The above term can be calculated explicitly:

$$\begin{aligned}\partial^\rho J^e_\rho(x) &= i(m_2 - m_1)\sin\theta\cos\theta\,[\bar{\nu}_2(x)\nu_1(x) - \bar{\nu}_1(x)\nu_2(x)] \\ &= -\partial^\rho J^\mu_\rho(x)\,.\end{aligned} \tag{20}$$

Let us denote with $Q_e(t)$ the (electron neutrino) charge defined in a reference frame at a given time and by $Q'_e(t')$ the corresponding quantity defined in the transformed frame. We obtain

$$\begin{aligned}Q_e(t) &= \int_\Sigma J^e_\rho(x) d\Sigma^\rho = \int_{\Sigma'} J'^e_\rho(x') d\Sigma'^\rho + \int_\Omega d^4x\,\partial^\rho J^e_\rho(x) \\ &= Q'_e(t') + \int_\Omega d^4x\,\partial^\rho J^e_\rho(x)\,,\end{aligned} \tag{21}$$

where Σ and Σ' are two space-like hypersurfaces defined by $t =$ const. and $t' =$ const., respectively. Ω is the 4-volume delimited by them.

A similar relation is valid for the muon charge:

$$Q_\mu(t) = Q'_\mu(t') + \int_\Omega d^4x\,\partial^\rho J^\mu_\rho(x)\,. \tag{22}$$

The above expressions are to be compared with the corresponding relations for the Noether charges Q_1 and Q_2, which represent the flavor charges in absence of mixing: to such quantities correspond divergenceless currents and therefore they are time-independent and Lorentz scalars.

Adding Eqs. (21) and (22), and using Eq. (20), we get the (time-independent) total charge Q, which is a Lorentz scalar. Thus the total charge, measured on a flavor neutrino state, is a Lorentz invariant quantity and all inertial observers agree on its value. On the other hand, the flavor charges do not need to have the same value in every inertial frame, as shown by Eqs. (21) and (22).

Let us then calculate the correction to the oscillation formula for a neutrino coming from a moving (boosted) source:

$$\begin{aligned}&\langle\alpha^r_{\boldsymbol{k},e}|\,Q'_e(t')\,|\alpha^r_{\boldsymbol{k},e}\rangle = \langle\alpha^r_{\boldsymbol{k},e}|\,Q_e(t)\,|\alpha^r_{\boldsymbol{k},e}\rangle \;-\; \int_\Omega d^4x\,\langle\alpha^r_{\boldsymbol{k},e}|\,\partial^\mu J^e_\mu(x)\,|\alpha^r_{\boldsymbol{k},e}\rangle \\ &= \langle\alpha^r_{\boldsymbol{k},e}|\,Q_e(t)\,|\alpha^r_{\boldsymbol{k},e}\rangle \\ &-i(m_2 - m_1)\sin\theta\cos\theta\int_\Omega d^4x\,\langle\alpha^r_{\boldsymbol{k},e}|\Big[\bar{\nu}_2(x)\nu_1(x) - \bar{\nu}_1(x)\nu_2(x)\Big]|\alpha^r_{\boldsymbol{k},e}\rangle\end{aligned} \tag{23}$$

The expectation value can be calculated giving

$$\langle\alpha^r_{\boldsymbol{k},e}|\left[\bar{\nu}_2(x)\nu_1(x)-\bar{\nu}_1(x)\nu_2(x)\right]|\alpha^r_{\boldsymbol{k},e}\rangle$$
$$= -\frac{2i}{(2\pi)^3}\sin\theta\cos\theta\left\{F_{\boldsymbol{k}}\,\sin\left[(\omega_{k,2}-\omega_{k,1})t\right]+G_{\boldsymbol{k}}\,\sin\left[(\omega_{k,2}+\omega_{k,1})t\right]\right\}, \tag{24}$$

where we have defined $F_{\boldsymbol{k}} \equiv |U_{\boldsymbol{k}}||W_{\boldsymbol{k}}|$ and $G_{\boldsymbol{k}} \equiv -|V_{\boldsymbol{k}}||Y_{\boldsymbol{k}}|$. For a definition of the functions $|W_{\boldsymbol{k}}|$ and $|Y_{\boldsymbol{k}}|$ see Appendix.

Eq. (24) shows that the second term in Eq. (23) is, in general, different from zero, thus implying a correction to the oscillation formula coming from the boosted source (or detector). A quantitative analysis of such a correction term will be given elsewhere.

A plot of the functions $F_{\boldsymbol{k}}$ and $G_{\boldsymbol{k}}$ is given in Fig. (1) for sample values of the masses. From their behavior, we see that the above correction term disappears in the extreme relativistic limit $|\boldsymbol{k}| \gg \sqrt{m_1 m_2}$, thus recovering the result of Ref. [16].

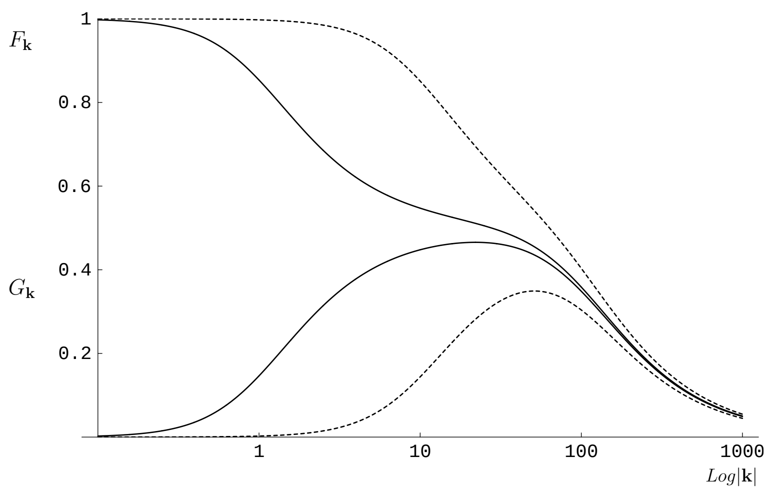

Fig. 1. $F_{\boldsymbol{k}}$ and $G_{\boldsymbol{k}}$ as functions of $|\boldsymbol{k}|$ for $m_1 = 1$, $m_2 = 100$ (solid line) and $m_1 = 10$, $m_2 = 100$ (dashed line).

Finally, we note that:

$${}_{e,\mu}\langle 0|\left[\bar{\nu}_2(x)\nu_1(x)-\bar{\nu}_1(x)\nu_2(x)\right]|0\rangle_{e,\mu} = 0 \tag{25}$$

thus ensuring that ${}_{e,\mu}\langle 0|\,Q_e(t)\,|0\rangle_{e,\mu} = {}_{e,\mu}\langle 0|\,Q'_e(t')\,|0\rangle_{e,\mu} = 0\,.$

4. Conclusions

We have discussed some aspects of the issue of Lorentz invariance for mixed particles (neutrinos). This discussion is motivated by the recent discovery of the inequivalence of mass and flavor representations, in the context of Quantum Field Theory [4]. Thus the basic entities turn out to be the flavor states (eigenstates of the flavor charges) rather than the mass eigenstates, as it is usually assumed. In Ref. [14] it was investigated the meaning of dispersion relations for mixed particles and its consequences on the Lorentz invariance.

In the present paper, we have shown that the concept of flavor charge for mixed particles is not Lorentz invariant, a fact which may have phenomenological consequences. In particular, we have shown that the oscillation formula for mixed neutrinos (two flavors) gets an additional contribution due to the relative (inertial) motion of source and detector. The Lorentz invariance of flavor oscillations is however recovered in the relativistic limit, in agreement with the conclusions of Ref. [16].

Effects of the Lorentz violation on neutrino oscillations have been recently discussed also in different contexts [17–19], where the Lorentz (and CPT) invariance breakdown occurs at the level of effective theory, implying a slight deformations of the standard dispersion relations of the particles propagating in the vacuum [19, 20].

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Appendix: Some useful formulas

$$\begin{aligned}\int d^3x\, \bar{\nu}_i(x)\nu_j(x) \;=\; \sum_{\boldsymbol{k},r,s} \Big(& \bar{u}^r_{\boldsymbol{k},i}(t)u^s_{\boldsymbol{k},j}(t)\alpha^{r\dagger}_{\boldsymbol{k},i}\alpha^s_{\boldsymbol{k},j} \\ & +\bar{v}^r_{-\boldsymbol{k},i}(t)v^s_{-\boldsymbol{k},j}(t)\beta^r_{-\boldsymbol{k},i}\beta^{s\dagger}_{-\boldsymbol{k},j}+\bar{v}^r_{-\boldsymbol{k},i}(t)u^s_{\boldsymbol{k},j}(t)\beta^r_{-\boldsymbol{k},i}\alpha^s_{\boldsymbol{k},j} \\ & +\bar{u}^r_{\boldsymbol{k},i}(t)v^s_{-\boldsymbol{k},j}(t)\alpha^{r\dagger}_{\boldsymbol{k},i}\beta^{s\dagger}_{-\boldsymbol{k},j}\Big) \qquad (26)\end{aligned}$$

and, for $\boldsymbol{k} = (0, 0, |\boldsymbol{k}|)$, we get:

$$\int d^3x \bar{\nu}_1(x)\nu_2(x) \; = \; \sum_{\boldsymbol{k},r} \Big(W^*_{\boldsymbol{k}}(t) \alpha^{r\dagger}_{\boldsymbol{k},1} \alpha^r_{\boldsymbol{k},2} + \epsilon^r \, Y^*_{\boldsymbol{k}}(t) \, \beta^r_{-\boldsymbol{k},1} \alpha^r_{\boldsymbol{k},2}$$
$$+ \epsilon^r Y_{\boldsymbol{k}}(t) \alpha^{r\dagger}_{\boldsymbol{k},1} \beta^{r\dagger}_{-\boldsymbol{k},2} - W_{\boldsymbol{k}}(t) \beta^r_{-\boldsymbol{k},1} \beta^{r\dagger}_{-\boldsymbol{k},2} \Big) \,, \qquad (27)$$

where we have defined:

$$W_{\boldsymbol{k}}(t) \; \equiv \; (\bar{u}^r_{\boldsymbol{k},2} u^r_{\boldsymbol{k},1}) = -(\bar{v}^r_{-\boldsymbol{k},1} v^r_{-\boldsymbol{k},2}) = |W_{\boldsymbol{k}}| e^{i(\omega_{k,2} - \omega_{k,1})t} \,, \qquad (28)$$
$$Y_{\boldsymbol{k}}(t) \; \equiv \; \epsilon^r \; (\bar{u}^r_{\boldsymbol{k},1} v^r_{-\boldsymbol{k},2}) = +\epsilon^r \; (\bar{u}^r_{\boldsymbol{k},2} v^r_{-\boldsymbol{k},1}) = |Y_{\boldsymbol{k}}| \; e^{i(\omega_{k,2} + \omega_{k,1})t} \qquad (29)$$

with

$$|W_{\boldsymbol{k}}| \; = \; \left(\frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{\frac{1}{2}} \left(\frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{\frac{1}{2}} \left(1 - \frac{|\boldsymbol{k}|^2}{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)} \right), \quad (30)$$
$$|Y_{\boldsymbol{k}}| \; = \; \left(\frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{\frac{1}{2}} \left(\frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{\frac{1}{2}} \left(\frac{|\boldsymbol{k}|}{(\omega_{k,2} + m_2)} + \frac{|\boldsymbol{k}|}{(\omega_{k,1} + m_1)} \right), \quad (31)$$

$$|W_{\boldsymbol{k}}|^2 \; + \; |Y_{\boldsymbol{k}}|^2 = 1 \,. \qquad (32)$$

Note also that $|W_{\boldsymbol{k}}||Y_{\boldsymbol{k}}| = |\boldsymbol{k}|(m_1 + m_2)/(2\omega_{k,2}\omega_{k,1})$ and $|W_{\boldsymbol{k}}|^2 - |Y_{\boldsymbol{k}}|^2 = (m_1 m_2 - |\boldsymbol{k}|^2)/(\omega_{k,2}\omega_{k,1})$.

For $m_2 = m_1 = m$, one has $|W_{\boldsymbol{k}}| = \frac{m}{\omega_k}$ and $|Y_{\boldsymbol{k}}| = \frac{|\boldsymbol{k}|}{\omega_k}$.

Finally, we have the following relations:

$$|W_{\boldsymbol{k}}||U_{\boldsymbol{k}}| = \frac{1}{2} \left(\frac{m_1}{\omega_{k,1}} + \frac{m_2}{\omega_{k,2}} \right) \; ; \quad |Y_{\boldsymbol{k}}||V_{\boldsymbol{k}}| = \frac{1}{2} \left(\frac{m_1}{\omega_{k,1}} - \frac{m_2}{\omega_{k,2}} \right) \,. \qquad (33)$$

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