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LOOKING FOR CP VIOLATION IN $B \to \tau^+\tau^-$ DECAYS*

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In supersymmetry with large $\tan\beta$ the decays $B^0(\bar{B}^0) \to l^+l^-$ are dominated by the scalar and pseudoscalar Higgs penguin diagrams leading to strong enhancement of leptonic decay rates with potentially large CP asymmetries in the $\tau^+\tau^-$ decay modes measurable in BELLE or BABAR experiments. The TAUOLA τ-lepton decay library supplemented by its universal interface can efficiently be used to search for $B^0(\bar{B}^0) \to \tau^+\tau^-$ decays, and to investigate how the CP asymmetry is reflected in realistic experimental observables.

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1. Introduction

Understanding the origin of CP violation is one of the most important tasks of particle physics. Until now, CP violation has been firmly established in K- and B-physics in a series of high statistics experiments. One way of testing the conventional CKM description of CP violation is by more precise measurements and joint analysis of CP asymmetries measured in different channels with a hope of finding some inconsistency signalling a contribution from physics beyond the Standard Model (SM) to CP violation. Alternatively, observation of a non-zero effect in channels in which no (or negligibly small) effects violating CP are predicted by the SM would be an unambiguous signal of new physics contribution to CP violation.

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In this talk presented are results of recent analyses of flavour changing decays of the neutral B mesons into lepton pairs, $B^0_{d,s} \to l^+l^-$ [1], for which the SM predicts no CP violating effects. This decay mode is very sensitive to new physics which affects the b-quark Yukawa couplings [2–4]. Approximate and full one-loop calculations in the supersymmetric extension of the SM with large $\tan\beta$ (the ratio of the vacuum expectation values of the two Higgs doublets) [3–5] showed that truly spectacular enhancement of the rates of the decays $B^0_{d,s} \to l^+l^-$ can be expected. Moreover, new physics can also lead to observable CP violation in these decays [1]. CP violation could manifest itself through non-equal leptonic decay rates of the $B^0(t)$ and $\bar{B}^0(t)$ states (tagged at $t=0$ as B^0 and $\bar{B}^0$, respectively). If polarisation of final state leptons can be determined, additional information on CP violation could be provided by non-equal $\Gamma(B^0(t) \to l^+_{\rm L} l^-_{\rm L})$ and $\Gamma(\bar{B}^0(t) \to l^+_{\rm R} l^-_{\rm R})$ [or $\Gamma(B^0(t) \to l^+_{\rm R} l^-_{\rm R})$ and $\Gamma(\bar{B}^0(t) \to l^+_{\rm L} l^-_{\rm L})$] decay rates [6, 7].

Leptonic decays are theoretically clean as the only non-perturbative quantities they depend on, are the B^0 meson decay constants $F_{B_{d,s}}$, which cancel out in suitably defined CP asymmetries. However, none of these decays have been seen so far: the best upper limits on the $B^0_{d,s} \to \mu^+\mu^-$ and $\tau^+\tau^-$ branching fractions at present are listed in Table I.

TABLE I

Upper limits for the branching fractions of leptonic B-meson decays.

Mode	SM expectation	Exp. limit	Ref.
$B_d \to \mu^+\mu^-$	1×10^{-10}	8.3×10^{-8}	[8]
		3.8×10^{-8}	[9]
$B_d \to \tau^+\tau^-$	2.8×10^{-8}	3.2×10^{-3}	[10]
$B_s \to \mu^+\mu^-$	3.7×10^{-9}	1.5×10^{-7}	[9]
$B_s \to \tau^+\tau^-$	1×10^{-6}	$\sim 5\times10^{-2}$	[11]

With the rates as predicted by the SM, the detection of the $\mu^+\mu^-$ decay channel will become possible only at the LHC; in the hadronic collider the $\tau^+\tau^-$ channel is extremely challenging. New physics (like supersymmetry) can increase significantly their rates to a level that they can be observed at BaBar, BELLE or Tevatron in near future. We find, however, that the ratio of time integrated leptonic decay rates of $B^0(t) \to \mu^+\mu^-$ and $\bar{B}^0(t) \to \mu^+\mu^-$ is unlikely to deviate appreciably from unity. Polarisation measurement also seems very difficult in the case of the $\mu^+\mu^-$ channel. In the $\tau^+\tau^-$ channel the situation can be quite different: large CP violating effects can be expected, and τ polarisation measurement is possible. Results of first experimental analyses of this channel have recently been made public [10]. Increasing the experimental efficiency of identifying τ leptons and suppressing the background might result in a much stronger limit or, hopefully, in observing the signal in B meson factories. We show that the existing `TAUOLA` package [12] and its `universal interface` [13] may prove useful to search for these decays *and* for the CP violation since in realistic scenarios of large $\tan\beta$ MSSM the decay rates can significantly be increased to a level measurable at the running *BaBar* and BELLE experiments and the CP asymmetry in the $B_d^0(\bar{B}_d^0) \to \tau^+\tau^-$ channel can be quite large and potentially measurable. We also identify, in addition to the ratio of time integrated leptonic $B_d^0(t)$ and $\bar{B}_d^0(t)$, two realistic CP-sensitive experimental observables which reflect τ-lepton polarisations: the $\pi^\pm$ energies from $\tau \to \pi\nu$ decays and the acoplanarity angle between the decay planes of the ρ mesons which originate from $\tau \to \rho\nu$. The former is sensitive to the longitudinal, while the latter to the transverse polarisations of τ's coming from B_d^0 and $\bar{B}_d^0$ decays.

2. Preliminaries

In leptonic B-meson decays if, for example,

$$\begin{aligned} \Gamma(B^0 &\to l_{\rm L}^+ l_{\rm L}^-) \neq \Gamma(\bar{B}^0 \to l_{\rm R}^+ l_{\rm R}^-)\,, \\ \Gamma(B^0 &\to l_{\rm R}^+ l_{\rm R}^-) \neq \Gamma(\bar{B}^0 \to l_{\rm L}^+ l_{\rm L}^-) \end{aligned} \tag{1}$$

CP is violated because the initial and final states on both sides transform into each other under CP [6]. The amplitudes of B^0 decays into two helicity eigenstates read

$$\begin{aligned} \mathcal{A}_{\rm L} &\equiv \langle l_{\rm L}^+ l_{\rm L}^- | B^0 \rangle = M_B\,(a + b\,\beta)\,, \\ \mathcal{A}_{\rm R} &\equiv \langle l_{\rm R}^+ l_{\rm R}^- | B^0 \rangle = M_B\,(a - b\,\beta)\,. \end{aligned} \tag{2}$$

Similar formulae with a and b replaced by $\bar{a}$ and $\bar{b}$, respectively, give the amplitudes $\bar{\mathcal{A}}_{\rm L}$ and $\bar{\mathcal{A}}_{\rm R}$ for the corresponding $\bar{B}^0$ decays. Here $\beta = (1 - 4m_l^2/M_B^2)^{1/2}$ and a, b, $\bar{a}$ and $\bar{b}$ are the coefficients in the effective Lagrangian describing $B^0(\bar{B}^0) \to l^+l^-$ decays

$$\mathcal{L}_{\rm eff} = B_{s,d}^0 \bar{\psi}_l (b_{s,d} + a_{s,d}\gamma^5)\psi_l + \bar{B}_{s,d}^0 \bar{\psi}_l (\bar{b}_{s,d} + \bar{a}_{s,d}\gamma^5)\psi_l \tag{3}$$

(the subscripts d and s referring to non-strange and strange B^0 mesons, unless explicitly written, will be omitted). Hermiticity (CPT invariance) implies $\bar{b} = b^*$ and $\bar{a} = -a^*$.

Since in leptonic decays no strong phases are involved, inequality (1) can occur only through the mixing of the B^0 and $\bar{B}^0$ mesons. In the standard formalism [14] the state which at $t = 0$ is a pure B^0 ($\bar{B}^0$) evolves in time as

$$|B_{\rm phys}^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p} g_-(t)|\bar{B}^0\rangle \tag{4}$$

(for $\bar{B}^0_{\rm phys}(t)$ replace $B^0 \leftrightarrow \bar{B}^0$ and $p \leftrightarrow q$). The coefficients $g_\pm(t)$, neglecting the difference of the decay widths of the two B^0 mass eigenstates and denoting $\Delta M \equiv M_{B^0_H} - M_{B^0_{\rm L}} \ll M_B \equiv (M_{B^0_H} + M_{B^0_{\rm L}})/2$, read

$$g_+(t) = e^{-iM_Bt-\frac{\Gamma}{2}t}\ \cos\frac{\Delta M}{2}t\ , \quad g_-(t) = e^{-iM_Bt-\frac{\Gamma}{2}t}\ i\ \sin\frac{\Delta M}{2}t\ , \tag{5}$$

and the ratio p/q is calculated from the effective Hamiltonian

$$p/q = (H^*_{12}/H_{12})^{1/2} \tag{6}$$

with $H_{12} \equiv M_{12} + \frac{i}{2}\Gamma_{12} = \langle B^0|\mathcal{H}_{\rm eff}|\bar{B}^0\rangle$, *etc.* [14].

Since $|\mathcal{A}_{\rm L}| = |\bar{\mathcal{A}}_{\rm R}|$, $|\mathcal{A}_{\rm R}| = |\bar{\mathcal{A}}_{\rm L}|$ and $\bar{\mathcal{A}}_{\rm L}/\mathcal{A}_{\rm L} = \left(\mathcal{A}_{\rm R}/\bar{\mathcal{A}}_{\rm R}\right)^*$, CP is violated if either

$$|q/p| \neq 1 \qquad \text{or} \qquad {\rm Im}(\lambda_{\rm L}) \neq {\rm Im}(\lambda_{\rm R}^{-1})\ ,$$

where $\lambda_{\rm L} \equiv q\,\bar{\mathcal{A}}_{\rm L}/p\,\mathcal{A}_{\rm L}$ and $\lambda_{\rm R} \equiv q\,\bar{\mathcal{A}}_{\rm R}/p\,\mathcal{A}_{\rm R}$.

The simplest quantitative measures of CP violation are provided by the asymmetries constructed out of time integrated polarised decay rates

$$A^1_{\rm CP}(t_1,t_2) \equiv \frac{\int_{t_1}^{t_2} dt\ [\Gamma(B^0_{\rm phys}(t) \to l^+_{\rm L}l^-_{\rm L}) - \Gamma(\bar{B}^0_{\rm phys}(t) \to l^+_{\rm R}l^-_{\rm R})]}{\int_{t_1}^{t_2} dt\ [\Gamma(B^0_{\rm phys}(t) \to l^+_{\rm L}l^-_{\rm L}) + \Gamma(\bar{B}^0_{\rm phys}(t) \to l^+_{\rm R}l^-_{\rm R})]}\ , \tag{7}$$

$$A^2_{\rm CP}(t_1,t_2) \equiv \frac{\int_{t_1}^{t_2} dt\ [\Gamma(B^0_{\rm phys}(t) \to l^+_{\rm R}l^-_{\rm R}) - \Gamma(\bar{B}^0_{\rm phys}(t) \to l^+_{\rm L}l^-_{\rm L})]}{\int_{t_1}^{t_2} dt\ [\Gamma(B^0_{\rm phys}(t) \to l^+_{\rm R}l^-_{\rm R}) + \Gamma(\bar{B}^0_{\rm phys}(t) \to l^+_{\rm L}l^-_{\rm L})]}\ , \tag{8}$$

and the ratio of integrated unpolarised decay rates

$$R_l(t_1,t_2) \equiv \frac{\int_{t_1}^{t_2} dt\ \Gamma(B^0_{\rm phys}(t) \to l^+l^-)}{\int_{t_1}^{t_2} dt\ \Gamma(\bar{B}^0_{\rm phys}(t) \to l^+l^-)}\ . \tag{9}$$

If the statistics of tagged events is low, or experimental determination of the decay time t is difficult, the fully integrated asymmetries $A^1_{\rm CP} \equiv A^1_{\rm CP}(0,\infty)$, $A^2_{\rm CP} \equiv A^2_{\rm CP}(0,\infty)$ and the ratio $R_l \equiv R_l(0,\infty)$ can be exploited. If $|q/p| = 1$ (as in the SM and many SUSY extensions), the expressions for the asymmetries $A^1_{\rm CP}$, $A^2_{\rm CP}$ and the ratio R_l simplify to

$$A^1_{\rm CP} = \frac{-2\ x\ {\rm Im}\ \lambda_{\rm L}}{2 + x^2 + x^2\ |\lambda_{\rm L}|^2}\,, \qquad A^2_{\rm CP} = \frac{-2\ x\ {\rm Im}\lambda_{\rm R}}{2 + x^2 + x^2\ |\lambda_{\rm R}|^2}\,, \tag{10}$$

$$R_l = \frac{\left(|\mathcal{A}_{\rm L}|^2 + |\mathcal{A}_{\rm R}|^2\right)(1+x^2) - x\left\{|\mathcal{A}_{\rm L}|^2{\rm Im}(\lambda_{\rm L}) + |\mathcal{A}_{\rm R}|^2{\rm Im}(\lambda_{\rm R})\right\}}{\left(|\mathcal{A}_{\rm L}|^2 + |\mathcal{A}_{\rm R}|^2\right)(1+x^2) + x\left\{|\mathcal{A}_{\rm L}|^2{\rm Im}(\lambda_{\rm L}) + |\mathcal{A}_{\rm R}|^2{\rm Im}(\lambda_{\rm R})\right\}}\,, \tag{11}$$

where $x \equiv \Delta M/\Gamma$. The asymmetries $A^1_{\rm CP}$, $A^2_{\rm CP}$, as functions of $\lambda_{\rm L,R}$, are therefore bounded from above by [6]

$$\left|A^{1,2}_{\rm CP}\right| \leq (2+x^2)^{-1/2} \tag{12}$$

which has important consequences. Since $x_s > 20.6$ for the B^0_s–$\bar{B}^0_s$ system, the CP asymmetries in the leptonic $B^0_s(\bar{B}^0_s)$ decays can reach at best $\sim 4.5\%$ irrespectively of the amount of CP violation. In contrast, for the B^0_d–$\bar{B}^0_d$ system, for which $x_d = 0.771 \pm 0.012$ they can be as large as $\sim 60\%$ and hopefully measurable at BABAR and BELLE in a relatively clean environment. Therefore, below we will consider only the CP asymmetries in the $B^0_d(\bar{B}^0_d) \to l^+l^-$ decays.

The time dependent B^0-meson mixing can easily be dealt with by introducing time dependent effective factors $a_{\rm eff}$, $b_{\rm eff}$, $\bar{a}_{\rm eff}$ and $\bar{b}_{\rm eff}$. The factor $a_{\rm eff}$ is defined as

$$a_{\rm eff}(t) = a\ g_+(t) + \bar{a}\ \frac{q}{p}\ g_-(t) \tag{13}$$

and the other factors are defined in a similar manner [1]. Then the instantaneous B^0 widths into left- or right-handed l's read

$$\Gamma\left(B^0_{\rm phys}(t) \to l^+_{\rm L} l^-_{\rm L}\ [l^+_{\rm R} l^-_{\rm R}]\right) = \frac{M_B}{16\pi}\beta\left|a_{\rm eff}(t) + [-]\beta\ b_{\rm eff}(t)\right|^2 \tag{14}$$

and those for $\bar{B}^0$ are given by similar formulae with $a_{\rm eff}(t)$, $b_{\rm eff}(t)$ replaced by $\bar{a}_{\rm eff}(t)$, $\bar{b}_{\rm eff}(t)$. CP is violated because in general $\bar{a}_{\rm eff}(t) \neq -a^*_{\rm eff}(t)$, and $\bar{b}_{\rm eff}(t) \neq b^*_{\rm eff}(t)$.

In the case of the $\tau^+\tau^-$ decay mode the τ polarisations are best identified by measuring the $\pi^\pm$ energy spectra from $\tau \to \pi\nu$ decays [15]. The spin density matrix formalism of Ref. [16] allows to construct also observables sensitive to transverse polarisation of the final state τ's. The spin weight for the complete event ($B \to \tau\tau \to$ decay products) can be written in the form

$$\mathrm{WT} = \frac{1}{4}\left(1 + \sum_{i,j=x,y,z} R_{ij}\ h^i_1\ h^j_2 + \sum_{i=x,y,z} R_{i0} h^i_1 + \sum_{j=x,y,z} R_{0j}\ h^j_2\right), \tag{15}$$

where the polarimetric vectors $\vec{h}_1$ and $\vec{h}_2$ are determined solely by the dynamic of the τ decay processes, and

$$\begin{aligned} R_{00} &= +1\,, & R_{x0} = R_{y0} = R_{0x} = R_{0y} = 0\,,\\ R_{zz} &= -1\,, & R_{xz} = R_{yz} = R_{zx} = R_{zy} = 0\,, \end{aligned}$$

$$R_{0z} = -R_{z0} = \frac{2\mathrm{Re}(a_{\mathrm{eff}} b^*_{\mathrm{eff}})\ \beta}{|b_{\mathrm{eff}}|^2\beta^2 + |a_{\mathrm{eff}}|^2}\,,$$
$$R_{xy} = -R_{yx} = -\frac{2\mathrm{Im}(a_{\mathrm{eff}} b^*_{\mathrm{eff}})\ \beta}{|b_{\mathrm{eff}}|^2\beta^2 + |a_{\mathrm{eff}}|^2}\,,$$
$$R_{xx} = R_{yy} = \frac{|b_{\mathrm{eff}}|^2\beta^2 - |a_{\mathrm{eff}}|^2}{|b_{\mathrm{eff}}|^2\beta^2 + |a_{\mathrm{eff}}|^2}\,. \tag{16}$$

Eq. (15) with $R_{\mu\nu}$ replaced by $\bar{R}_{\mu\nu}$ computed as above but with $\bar{a}_{\mathrm{eff}}(t)$ and $\bar{b}_{\mathrm{eff}}(t)$ replacing $a_{\mathrm{eff}}(t)$ and $b_{\mathrm{eff}}(t)$, respectively, gives the spin weight for the events from $\bar{B}^0$ decays. CP violating effects in the $B^0 \to \tau^+\tau^-$ and $\bar{B}^0 \to \tau^+\tau^-$ decays are absent if $R_{0z} = \bar{R}_{z0}$, $R_{xy} = \bar{R}_{yx}$ and $R_{xx} = \bar{R}_{xx}$.

To simulate how the CP asymmetry in $B^0_d(\bar{B}^0_d) \to \tau^+\tau^-$ decays are reflected in realistic observables the `TAUOLA` τ-lepton decay library has been used (the method and numerical algorithm is given in [12]). The input to the `TAUOLA Universal Interface` [13] is the spin density matrix of the $\tau^+\tau^-$ system resulting from the decay of a neutral particle.

For simulations of the time integrated measurements, the time averaged matrix $\langle R_{\mu\nu}\rangle$ has to be used

$$\langle R_{\mu\nu}\rangle \equiv \frac{\int dt\ \Gamma(B^0_{\mathrm{phys}}(t) \to \tau^+\tau^-)\ R_{\mu\nu}(t)}{\int dt\ \Gamma(B^0_{\mathrm{phys}}(t) \to \tau^+\tau^-)}$$

and $\langle \bar{R}_{\mu\nu}\rangle$ given by a similar formula. The asymmetry (7) is then given by

$$A^1_{\mathrm{CP}} = \frac{(1-\langle R_{z0}\rangle)\Gamma_{\mathrm{int}} - (1+\langle \bar{R}_{z0}\rangle)\bar{\Gamma}_{\mathrm{int}}}{(1-\langle R_{z0}\rangle)\Gamma_{\mathrm{int}} + (1+\langle \bar{R}_{z0}\rangle)\bar{\Gamma}_{\mathrm{int}}}\,, \tag{17}$$

where

$$\Gamma_{\mathrm{int}} = \int dt\Gamma(B^0_{\mathrm{phys}}(t) \to \tau^+\tau^-)\,, \qquad \bar{\Gamma}_{\mathrm{int}} = \int dt\Gamma\left(\bar{B}^0_{\mathrm{phys}}(t) \to \tau^+\tau^-\right)\,.$$

A^2_{CP} defined in (8) is given by (17) reversing the signs in the brackets.

In the limit $a = \pm b\beta$ the two integrated rates Γ_{int} and $\bar{\Gamma}_{\mathrm{int}}$ are equal resulting in $\langle R_{z0}\rangle = -\langle \bar{R}_{z0}\rangle$. It implies that even for $\delta_{\mathrm{CP}} \neq 0$ there can be no CP violating effects in the observables sensitive to the longitudinal polarisation of τ's, like $A^{1,2}_{\mathrm{CP}}$, nor in R_τ. However, in this limit the elements xx and xy of these matrices *need not* satisfy $\langle R_{xy}\rangle = -\langle \bar{R}_{xy}\rangle$ and $\langle R_{xx}\rangle = \langle \bar{R}_{xx}\rangle$. Hence, the observables sensitive to transverse τ polarisation can reveal CP violation even if the observables sensitive to longitudinal τ polarisation cannot.

3. Supersymmetry scenarios

In models of new physics $\mathrm{Im}(\lambda_\mathrm{L}) \neq \mathrm{Im}(\lambda_\mathrm{R}^{-1})$ and/or $|q/p| \neq 1$ can be expected. However, if the decay rates are not at the same time strongly enhanced, the detection of the $\mu^+\mu^-$ decay mode will become possible only at the LHC, while for the $B_d^0 \to \tau^+\tau^-$ mode at BELLE and BaBar the number of reconstructed events might be too small to detect any CP violation.

Much more promising situation can occur in the supersymmetric scenario with a large ratio of the vacuum expectation values of the two Higgs doublets, $v_u/v_d \equiv \tan\beta \sim 40 \div 50$. Contributions from the Higgs penguin diagrams with s-channel H^0 and A^0 Higgs boson exchanges [3, 4, 7] can become dominant easily saturating the experimental limits in Table I if the Higgs particles H^0 and A^0 in the range of order $\lesssim 500$ GeV even for other supersymmetric particle masses quite large, say in the TeV range [4, 17].

In Ref. [1] two different supersymmetric scenarios have been considered: minimal (MFV) and non-minimal (NMFV) flavour violating, both with large ratio of VEVs. In both scenarios in which the $B_{d,s}^0 \to l^+l^-$ amplitudes have been dominated by the exchange of H^0 and A^0 Higgs bosons, it was found that

$$a \approx b \qquad \text{or} \qquad a \approx -b\,,$$

(up to $\lesssim 15\%$). For $a = b$ the factors λ_L and λ_R simplify to:

$$\lambda_\mathrm{L} = -\frac{q}{p}\,\frac{a^*}{a}\frac{1-\beta}{1+\beta}\,, \qquad\qquad \lambda_\mathrm{R} = -\frac{q}{p}\,\frac{a^*}{a}\frac{1+\beta}{1-\beta} \tag{18}$$

and all CP-sensitive quantities could be expressed in terms of one effective phase which can be taken as

$$\delta_\mathrm{CP} = -\frac{1}{2}\arg(\lambda_\mathrm{L})\,. \tag{19}$$

The effective CP phase is a function of the CKM phase and complex soft SUSY breaking Lagrangian parameters.

The immediate consequence of $a = b$ with $|q/p| \sim 1$ is that for the $\mu^+\mu^-$ final state, for which $\beta = (1 - 4m_\mu^2/M_B^2)^{1/2}$ is almost 1, the parameters $|\lambda_\mathrm{L}|$ and $|\lambda_\mathrm{R}|$ assume values $\sim 4 \times 10^{-4}$ and $\sim 2.5 \times 10^3$, respectively. As a result, the expected asymmetries are very small, at most $|A_\mathrm{CP}^1| \lesssim 2 \times 10^{-4}$, $|A_\mathrm{CP}^2| \lesssim 10^{-3}$. On the other hand, for the $\tau^+\tau^-$ final states $\beta = (1 - 4m_\tau^2/M_B^2)^{1/2}$ differs substantially from 1 giving $|\lambda_\mathrm{L}| \sim 0.15$, $|\lambda_\mathrm{R}| \sim 6.7$ and the maximal possible values of the asymmetries are

$$\left|(A_\mathrm{CP}^1)^\mathrm{max}\right| = 9\% \qquad \text{and} \qquad \left|(A_\mathrm{CP}^2)^\mathrm{max}\right| = 35\%\ . \tag{20}$$

This is reflected in Fig. 1 in which possible CP violating effects in the ratio (11) for $\mu^+\mu^-$ and $\tau^+\tau^-$ decay modes are shown as functions of b/a for four different values of the phase $\delta_{\rm CP}$ (keeping arg(a) =arg(b) and $|p/q| = 1$). The plots demonstrate that the ratios R_l approach unity for $a \approx \pm b\beta$, the feature which can be read also from the formula (11) if one takes into account that $\lambda_{\rm R} \sim 1/(a \mp b\beta)$ for $a \to \pm b\beta$ whereas $|\mathcal{A}_{\rm R}|^2 \sim |a \mp b\beta|^2$. Therefore, for $a \approx \pm b$ the deviation from unity of R_μ is tiny while for R_τ it can be quite substantial.

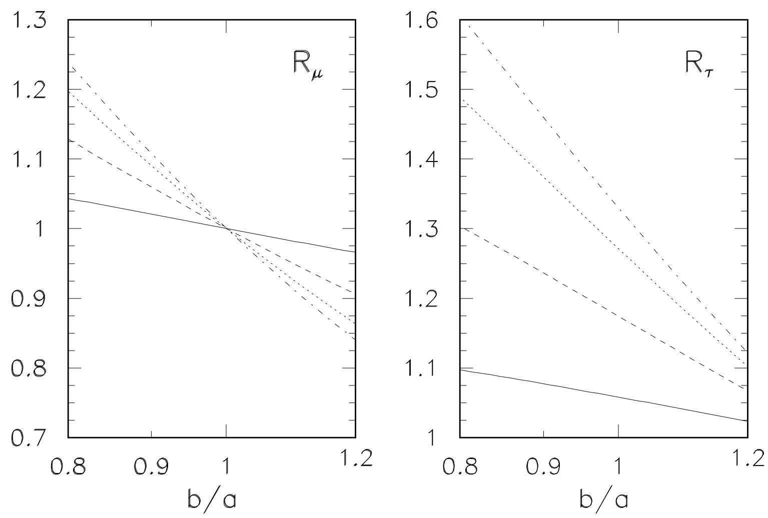

Fig. 1. The ratios R_μ and R_τ as functions of b/a for the phase $\delta_{\rm CP} = -\frac{1}{2}\arg(\lambda_{\rm L}) =$ 0.1 (solid line), 0.3 (dashed), 0.5 (dotted) and 0.75 (dash-dotted). $R_l(-\delta_{\rm CP}) = R_l^{-1}(\delta_{\rm CP})$.

The coefficients a and b ($\bar{a}$ and $\bar{b}$) are constrained by the experimental limit in Table I, which in the case $a \approx \pm b$ gives

$$|a| \approx |b| \lesssim 4.9 \times 10^{-9} \ . \tag{21}$$

In our simulations in the next section we conservatively set $|a| = |b| \lesssim 10^{-9}$ and treat both scenarios simultaneously, as all what matters are the values of $|a| = |b|$ and the single CP violating phase $\delta_{\rm CP}$ which can be of order 1. As supersymmetric box and Z^0 penguin contributions as well as a finite difference of A^0 and H^0 masses spoil the exact equality $a = \pm b$, we investigate the effect of a and b different from each other by some $15 \div 20\%$.

4. Numerical results

Two observables, known to provide valuable and complementary information on the spin state of decaying τ lepton pairs, have been simulated:
(a) $\pi^\pm$ *energy spectra* in the decay channels $\tau^+ \to \pi^+\bar{\nu}_\tau$ (or $\tau^- \to \pi^-\nu_\tau$) reflect the longitudinal polarisation of the individual $\tau^\pm$ leptons. Therefore, they are sensitive to R_{z0} and $\bar{R}_{0z}$, as can be inferred from the expression (15), *i.e.* to $\mathrm{Re}(a_{\rm eff}b^*_{\rm eff})$ as follows from (16). The CP violation is signalled if the energy spectrum of $\pi^-(\pi^+)$ originating from $B^0(\bar{B}^0)$ is different from the energy spectrum of $\pi^+(\pi^-)$ originating from $\bar{B}^0(B^0)$. In the following plots, the energy spectra are measured in the rest frame of the $B^0(\bar{B}^0)$ meson assuming that the reconstruction of the event kinematics in the BELLE and BaBar experiments is sufficiently good for that purpose.
(b) *acoplanarity angle* φ^* between two planes spanned by the momenta of decay products of $\rho^\pm \to \pi^\pm\pi^0$ coming from decays of both τ leptons into $\rho\nu_\tau$ [16]. It is sensitive to correlations between transverse components of τ-lepton spins (*i.e.* to the elements R_{xx} and R_{xy} which in turn probe $\mathrm{Im}(a_{\rm eff}b^*_{\rm eff})$, as can be seen from (16)). The acoplanarity angle is defined as

$$\varphi^* = \begin{cases} \xi & \text{for} \quad \mathrm{sgn}(\boldsymbol{p}_{\pi^-}\cdot\boldsymbol{n}_+) < 0 \\ 2\pi - \xi & \text{for} \quad \mathrm{sgn}(\boldsymbol{p}_{\pi^-}\cdot\boldsymbol{n}_+) > 0 \end{cases} , \tag{22}$$

where $\cos\xi = \frac{\boldsymbol{n}_+\cdot\boldsymbol{n}_-}{|\boldsymbol{n}_+||\boldsymbol{n}_-|}$ and two vectors $\boldsymbol{n}_\pm = \boldsymbol{p}_{\pi^\pm}\times\boldsymbol{p}_{\pi^0}$ are normal to the planes determined by the momenta of pions which originate from $\rho^\pm$ decays. Note that the full range of the variable $0 < \varphi^* < 2\pi$ is of physical interest, and in addition events have to be sorted depending whether $y_1y_2 > 0$ or $y_1y_2 < 0$, where

$$y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}} , \qquad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}} , \tag{23}$$

since otherwise the spin correlations are washed out, as explained in Ref. [18]. The acoplanarity distribution is evaluated in the rest frame of the $\rho^+\rho^-$ pair, but with the energies of $\pi^\pm$ and π^0's in (23) taken in the rest frame of the $B^0(\bar{B}^0)$. The difference between the distributions of the acoplanarity angle φ^* measured in B^0 decays and the angle $2\pi - \varphi^*$ measured in $\bar{B}^0$ decays for the same signs of y_1y_2 signals the CP violation.

Fig. 2 shows the pion energy spectra and the acoplanarity distributions assuming $|q/p| = 1$, $a = b = 10^{-9}$ and the CP violating phase $\delta_{\rm CP} = 0.7$. For all plots the same number of 5×10^5 $\tau^+\tau^-$ events from B^0_d and $\bar{B}^0_d$ decays has been generated with `TAUOLA`, although for the parameters chosen the ratio $R_\tau = 1.32$, see Fig. 1. In the upper left panel the energy spectra of π^- from B^0 decays (thick line) and of π^+ from $\bar{B}^0$ (thin line) are shown, while

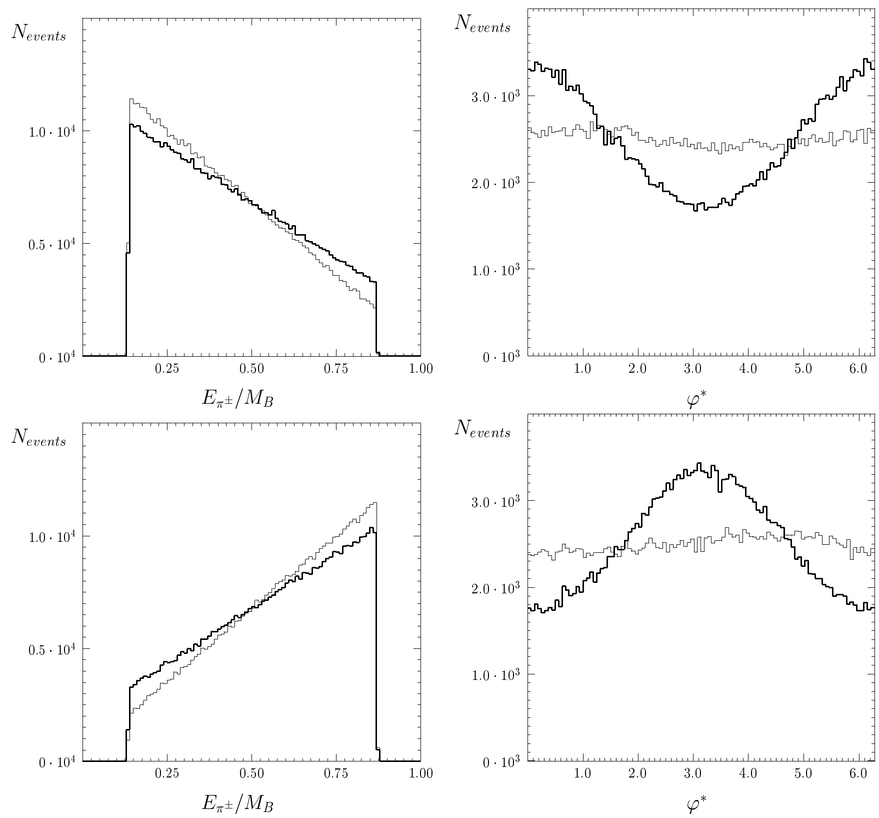

Fig. 2. Results for the CP violating phase $\delta_{\rm CP} = 0.7$ and $a = b$. Left panels: Single $\pi^\pm$ energy spectra in $B^0(\bar{B}^0) \to \tau^+\tau^-$, $\tau^\pm \to \pi^\pm\nu_\tau(\bar{\nu}_\tau)$. In the upper (lower) panel the thick line corresponds to the energy spectrum of π^- (of π^+) from B^0 decays and the thin line to the energy spectrum of π^+ (of π^-) from $\bar{B}^0$. Right panels: acoplanarity distributions of the $\rho^+\rho^-$ decay products in $B^0(\bar{B}^0) \to \tau^+\tau^-$, $\tau^\pm \to \rho^\pm\nu_\tau(\bar{\nu}_\tau)$, $\rho^\pm \to \pi^\pm\pi^0$. The thick lines correspond to the acoplanarity angle φ^* measured in B^0 decays and the thin ones are for the angle $2\pi - \varphi^*$ measured in $\bar{B}^0$ decays. Events in the upper (lower) panel have $y_1y_2 > 0$ ($y_1y_2 < 0$).

in the lower left panel shown are the spectra of π^+ from B^0 decays (thick line) and of π^- from $\bar{B}^0$ (thin line). The harder π^- energy spectrum from B_d^0 decays than π^+ from $\bar{B}_d^0$ in the upper left panel indicates that ${\rm Br}(B_d^0 \to \tau_{\rm R}^+\tau_{\rm R}^-) > {\rm Br}(\bar{B}_d^0 \to \tau_{\rm L}^+\tau_{\rm L}^-)$, which is a clear signal of CP violation. In the acoplanarity plots (right panels) thick lines correspond to the distribution of φ^* measured in B^0 decays, and the thin lines to the distribution of $2\pi - \varphi^*$ measured in $\bar{B}^0$ decays; in the upper right panel $y_1y_2 > 0$, and $y_1y_2 < 0$ in the lower right one. Different shapes of thick and thin lines seen in the right panels of Fig. 2 again indicate CP violation. In both energy and acoplanarity plots the CP violation is clearly seen and should be measurable even for small statistics. Note also that if upper and lower plots are combined (*i.e.* no sorting according to the pion charge or sign of $y_1 y_2$ is made) all CP asymmetries are lost.

With decreasing $|\delta_{\rm CP}|$ the signal of CP violation deteriorates (especially in the pion spectra) and the possibility of distinguishing pion spectra and acoplanarity distributions from B^0 and $\bar{B}^0$, and hence the CP violation, would require increasingly large statistics which may not be attainable at BELLE and *BaBar* without major upgrades.

As we discussed, the relation $a = \pm b$ is only approximate. For $b = 0.8\ a$ with the same value of $\delta_{\rm CP}$ the CP violating effects in $\pi^\pm$ energy spectra get enhanced, while the acoplanarities are only slightly affected. On the other hand, for a approaching $b\beta$ (for $B \to \tau^+\tau^-$ decays $\beta \approx 0.74$) the effects of CP violation in the $\pi^\pm$ energy spectra disappear, as expected and seen in the left panel of Fig. 3. In contrast, the acoplanarities shown in the right panel of Fig. 3 clearly indicate the CP violation even for $a \approx b\beta$ confirming our earlier discussion demonstrating the complementarity of the energy and acoplanarity distributions as a means to detect CP violation.

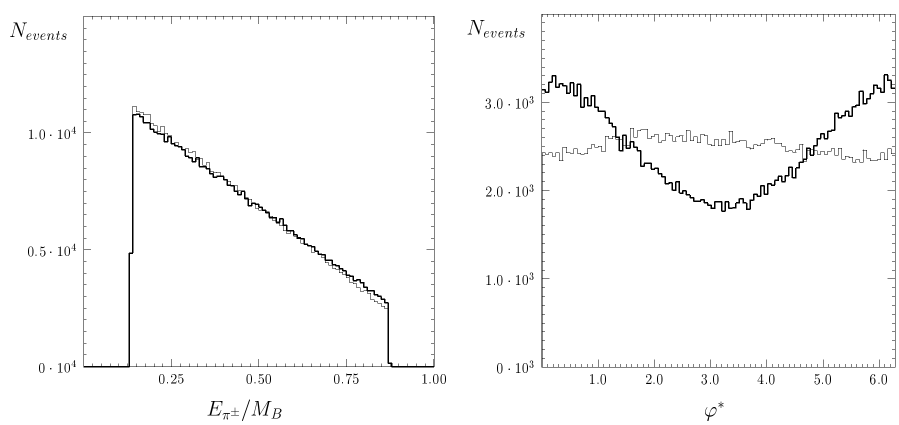

Fig. 3. As in the upper panels of figure 2 but for $\delta_{\rm CP} = 0.7$ and $a = 0.8\,b$.

5. Conclusions

In the supersymmetry scenario with $\tan\beta \sim 40 \div 50$ the rates of $B_d^0(\bar{B}_d^0) \to \tau^+\tau^-$ decays are enhanced and could be detectable in the SLAC and KEK B-factories. Moreover, the effective CP violating phase needs not be small. Therefore, the CP asymmetries can be quite large as opposed to the $B^0(\bar{B}^0) \to \mu^+\mu^-$ decays in which they are kinematically suppressed.

By using Monte-Carlo simulations we have investigated the possible effects of CP violation in two realistic experimental observables and demonstrated that they might be detectable if the CP violating phase is reasonably large, *i.e.* $\mathcal{O}(1)$. We have developed the necessary formalism and numerical tools allowing to apply the `TAUOLA` τ-lepton decay library together with its `universal interface` to simulate fully the effects of the polarisation of τ^+ and τ^- originating from such decays. The tools can also be applied to determine the upper limits on the branching fraction of the $B^0(\bar{B}^0) \to \tau^+\tau^-$ decays by the *BaBar* and BELLE collaborations.

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