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DELVING INTO THE NEUTRALINO SECTOR OF THE MSSM*

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Production of neutralinos in cascade decays of selectrons with subsequent three-body leptonic neutralino decays is discussed. It is shown that a high degree of polarization of such neutralinos and possibility of reconstructing their rest frames could provide new tools to verify the Majorana nature of neutralinos and to measure the CP violation in the neutralino sector of the MSSM.

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1. Introduction

Supersymmetry (SUSY) is one of the most promising extensions of the Standard Model (SM) [1] since, among other things, it solves the hierarchy problem, provides a natural candidate for dark matter, introduces new sources of CP violation that are needed to explain baryon asymmetry of the Universe, *etc.*

All SUSY theories contain neutralinos, the spin-1/2 Majorana superpartners of neutral gauge bosons and Higgs bosons, that are supposed to be one of the lightest supersymmetric particles and can be produced at future colliders — the LHC and the ILC. It is of great importance to confirm that the discovered particles are indeed SUSY partners of the SM particles. Moreover, precise measurements of their quantum numbers, masses, mixing angles, couplings and CP violating phases would allow to reconstruct the fundamental SUSY parameters with great precision and give an insight of physics at very high energy scales.

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In this talk we report on the results obtained in [2], where we provide an alternative method for probing the Majorana nature of neutralinos and the CP properties of the neutralino sector [3] of the Minimal Supersymmetric Standard Model (MSSM). We exploit the charge self-conjugate three body decays of polarized neutralinos into the lightest neutralino and a lepton pair

$$\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+ \ell^- \,, \tag{1}$$

(where $\ell = e$ or μ) reconstructed in the $\tilde{\chi}_2^0$ rest frame. Here we rely on two crucial observations: neutralinos produced in $\tilde{e}_{\rm L}^{\pm}$ decays are 100% polarized, having negative helicity in $\tilde{e}_{\rm L}^- \to e^- \tilde{\chi}_i^0$ and positive helicity in $\tilde{e}_{\rm L}^+ \to e^+ \tilde{\chi}_i^0$ [4]. Furthermore, as it was shown in [5], the rest frame of the neutralino $\tilde{\chi}_2^0$ can be reconstructed in some cascade decay processes, *e.g.* $e^+e^- \to \tilde{e}_{\rm L}^+ \tilde{e}_{\rm L}^- \to e^+ \tilde{\chi}_1^0 e^- \tilde{\chi}_2^0$, followed by the three-body decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \mu^+ \mu^-$. Exploiting these two possibilities we show a method of probing the Majorana nature of neutralinos and measuring CP violation in the neutralino sector of the MSSM. Throughout this work we assume that SUSY is a well established theory and that the masses of particles are known very precisely. We assume also that R-parity is conserved and hence that the lightest neutralino $\tilde{\chi}_1^0$ is the lightest supersymmetric particle (LSP) and escapes detection.

2. Neutralino mixing

In the MSSM, the four neutralino mass eigenstates $\tilde{\chi}_i^0$ $(i = 1, 2, 3, 4)$ are the mixtures of the neutral $\mathrm{U_Y}(1)$ and $\mathrm{SU_L}(2)$ gauginos, $\tilde{B}$ and $\tilde{W}^3$, and two Higgsinos $\tilde{H}_1^0$ and $\tilde{H}_2^0$. In the gauge eigenstate basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ the neutralino mass matrix has the form

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix} , \tag{2}$$

where M_1 and M_2 are the $\mathrm{U_Y}(1)$ and $\mathrm{SU_L}(2)$ gaugino mass parameters, respectively, μ is the Higgsino mass parameter, and $\tan\beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two neutral Higgs fields which break the electroweak symmetry.

By redefinition of the fields, without loss of generality M_2 can be taken real and positive, so that the two remaining physical phases may be attributed to M_1 and μ:

$$M_1 = |M_1|\, e^{i\Phi_1}, \quad \mu = |\mu|\, e^{i\Phi_\mu} \quad (0 \le \Phi_1, \Phi_\mu < 2\pi) \,. \tag{3}$$

For the symmetric matrix $\mathcal{M}$, one unitary matrix N is sufficient to diagonalize it and to rotate the gauge eigenstate basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ to the mass eigenstate basis of the Majorana fields $\tilde{\chi}_i^0$

$$\mathcal{M}_{\text{diag}} = N^* \mathcal{M} N^\dagger \,. \tag{4}$$

The mass eigenvalues m_i $(i = 1, 2, 3, 4)$ in $\mathcal{M}_{\text{diag}}$ can be chosen real and positive by a suitable definition of the unitary matrix N and the corresponding mass eigenstates have the form

$$\tilde{\chi}_i^0 = N_{i1}\tilde{B} + N_{i2}\tilde{W}^3 + N_{i3}\tilde{H}_1^0 + N_{i4}\tilde{H}_2^0 \,. \tag{5}$$

In the CP conserving case, *i.e.* when all elements of the mass matrix (2) are real, for a given neutralino $\tilde{\chi}_i^0$ the mixing matrix elements $N_{i\alpha}$ are all either real or purely imaginary.

3. Production and decays of neutralinos

Neutralinos can be produced in high energy colliders either directly or in cascade decays of other particles. We focus here on neutralinos produced in decays of selectrons $\tilde{e}_{\text{L}}^{\pm}$.

In the e^+e^- collider production of selectrons occurs via an s-channel exchange of γ and Z bosons or a t-channel exchange of sneutrino. Contribution from the Higgs bosons exchange is strongly suppressed by the tiny electron mass, hence it can be neglected. We focus on the production process followed by selectrons decays to the lightest neutralino and the second lightest neutralino, *i.e.*

$$\begin{aligned} e^+e^- &\to \tilde{e}_{\text{R}}^+\tilde{e}_{\text{L}}^- \to e^+\tilde{\chi}_1^0 e^-\tilde{\chi}_2^0 \,, \\ e^+e^- &\to \tilde{e}_{\text{L}}^+\tilde{e}_{\text{L}}^- \to e^+\tilde{\chi}_1^0 e^-\tilde{\chi}_2^0 \,, \end{aligned} \tag{6}$$

with a subsequent decay of neutralino $\tilde{\chi}_2^0$ as in Eq. (1).

Another possibility for studying decays of 100% polarized neutralinos would be at a photon linear collider running in the $e\gamma$ mode, a possible extension of the e^+e^- linear collider programme. The possibility of having

$$e^-\gamma \to \tilde{\chi}_1^0\tilde{e}_{\text{L}}^- \to \tilde{\chi}_1^0 e^-\tilde{\chi}_2^0 \tag{7}$$

would be particularly useful if the selectron pair production at e^+e^- collisions turns out to be kinematically shut. Nevertheless, this process needs further studies to assess the possibility of having the $\tilde{\chi}_2^0$ neutralino rest frame fully reconstructed in a realistic experimental setup.

Three diagrams contributing to the three-body leptonic decay of neutralinos $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-$ are shown in Fig. 1. We neglect the exchange of neutral Higgs bosons since their coupling to e and μ are suppressed by small lepton masses. After a simple Fierz transformation of slepton exchange contributions the decay matrix element has a vector-current product form:

$$\mathcal{D}\left(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+ \ell^-\right) = \frac{e^2}{m_2^2} D_{\alpha\beta} \left[\bar{u}(\tilde{\chi}_1^0)\gamma^\mu P_\alpha u(\tilde{\chi}_2^0)\right] \left[\bar{u}(\ell^-)\gamma_\mu P_\beta v(\ell^+)\right] , \quad (8)$$

with the bilinear charges $D_{\alpha\beta}$ ($\alpha, \beta =$L,R) containing internal propagators and couplings [2].

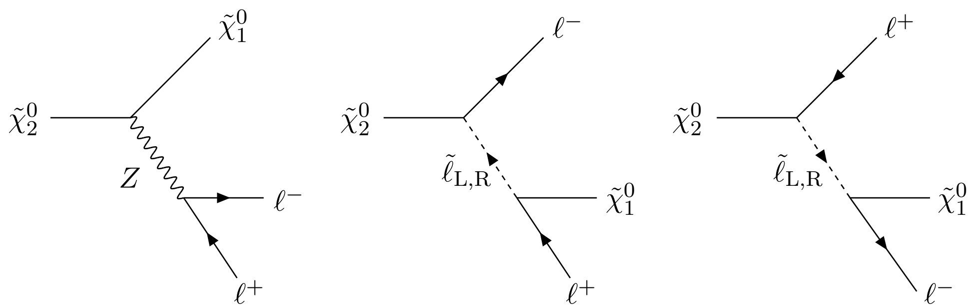

Fig. 1. Diagrams contributing to the leptonic three-body neutralino decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+ \ell^-$.

In the rest frame of the decaying neutralino the neutralino $\tilde{\chi}_2^0$ spin vector $\hat{n} = (0,0,1)$ defines the direction of the z-axis. The x–z plane and the angle θ are fixed by the momentum vector of the negative lepton. The angle α determines the neutralino decay plane, so that by rotating this plane by $-\alpha$ around ℓ^- momentum direction it is brought to the x–z plane, as shown in Fig. 2. In this reference frame, after neglecting lepton masses, we can write the differential decay distribution in terms of two dimensionless energy variables, $x_- = 2E_{e^-}/m_2$, $x_+ = 2E_{e^+}/m_2$, and two angles, θ and α, as

$$\begin{aligned} \frac{d^4\Gamma}{dx_- dx_+ dz d\alpha} &= \frac{\alpha^2\, m_2}{16\pi^2} \Big[F_0(x_-, x_+) + (\hat{q}_- \cdot \hat{n})\, F_1(x_-, x_+) \\ &\quad + (\hat{q}_+ \cdot \hat{n})\, F_2(x_-, x_+) + \hat{n} \cdot (\hat{q}_- \times \hat{q}_+)\; F_3(x_-, x_+) \Big] , \quad (9) \end{aligned}$$

where $z = \hat{q}_- \cdot \hat{n} = \cos\theta$, $\hat{q}_\pm = \vec{q}_\pm / |\vec{q}_\pm|$, and $\vec{q}_\pm$ are the leptons momenta in the $\tilde{\chi}_2^0$ rest frame. The four kinematic functions $F_i(x_-, x_+)$ ($i =$ 0-3) depend on the dimensionless energy variables, x_- and x_+, and the bilinear charges, but not on the orientation angles θ and α.

By applying the CP transformation to the decay matrix element (8) we can derive two relations between bilinear charges, which are consequences of the Majorana nature of neutralinos:

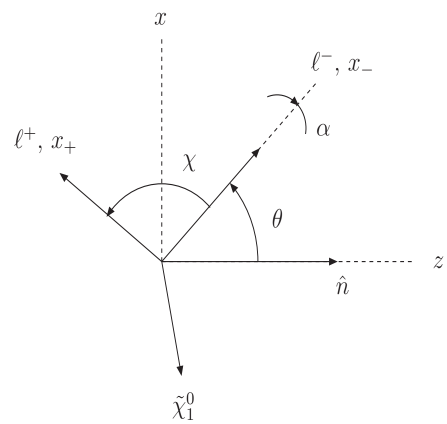

Fig. 2. Kinematic configuration of momenta and the spin vector in the neutralino $\tilde{\chi}^0_2$ rest frame.

$$
\begin{aligned}
D_{\rm LR} &= \eta_1\eta_2 D_{\rm RR}(t \leftrightarrow u)\,,\\
D_{\rm RL} &= \eta_1\eta_2 D_{\rm LL}(t \leftrightarrow u)\,,
\end{aligned}
\tag{10}
$$

where $\eta_{1,2} = \pm i$ are the intrinsic CP parities of $\tilde{\chi}^0_{1,2}$, respectively [7]. These lead to the following CP relations for the kinematic functions (9):

$$
\begin{aligned}
F_0(x_-, x_+) &= +F_0(x_+, x_-)\,,\\
F_1(x_-, x_+) &= -F_2(x_+, x_-)\,,\\
F_3(x_-, x_+) &= -F_3(x_+, x_-)\,.
\end{aligned}
\tag{11}
$$

On the other hand, applying the CP$\tilde{\rm T}$[1] transformation to the decay matrix element results in the following relations among the bilinear charges:

$$
\begin{aligned}
D_{\rm LR} &= -D^*_{\rm RR}(t \leftrightarrow u)\,,\\
D_{\rm RL} &= -D^*_{\rm LL}(t \leftrightarrow u)\,,
\end{aligned}
\tag{12}
$$

in the approximation of neglecting particle widths. Consequently for the kinematic functions we get:

$$
\begin{aligned}
F_0(x_-, x_+) &= +F_0(x_+, x_-)\,,\\
F_1(x_-, x_+) &= -F_2(x_+, x_-)\,,\\
F_3(x_-, x_+) &= +F_3(x_+, x_-)\,.
\end{aligned}
\tag{13}
$$

[1] The naive time reversal transformation $\tilde{\rm T}$ reverses directions of all 3-momenta and spins, but does not exchange the initial and final states.

4. Numerical analyses

For the numerical analyses of the results derived in Sec. 3 we adopt an MSSM scenario defined at the electroweak scale by the following set of parameters:

$$|M_1| = 80\,\mathrm{GeV}, \quad M_2 = 158\,\mathrm{GeV}, \quad \mu = 415\,\mathrm{GeV}, \quad \tan\beta = 10\,. \tag{14}$$

In this analysis we take μ to be real and vary only the phase Φ_1 of M_1. For the neutralino masses we find:

$$m_{\tilde{\chi}^0_1} = 78.1\,\mathrm{GeV}, \qquad m_{\tilde{\chi}^0_2} = 148.5\,\mathrm{GeV}, \tag{15}$$

and for the selectron and sneutrino masses we take:

$$m_{\tilde{e}_\mathrm{L}} = 207.7\,\mathrm{GeV}, \quad m_{\tilde{e}_\mathrm{R}} = 173.1\,\mathrm{GeV}, \quad m_{\tilde{\nu}_e} = 192.1\,\mathrm{GeV}\,. \tag{16}$$

The cross sections for selectron pair production processes with unpolarized e^+e^- beams at $\sqrt{s} = 500$ GeV and the branching ratios relevant for our analysis are as follows [8]:

$$\begin{aligned} \sigma\{\tilde{e}^\pm_\mathrm{R}\tilde{e}^\mp_\mathrm{L}\} &= 113.5\,\mathrm{fb}\,, & \sigma\{\tilde{e}^+_\mathrm{L}\tilde{e}^-_\mathrm{L}\} &= 80.7\,\mathrm{fb}\,, \\ \mathrm{BR}(\tilde{e}_\mathrm{L} \to \tilde{\chi}^0_2 e) &= 28.4\%\,, & \mathrm{BR}(\tilde{e}_\mathrm{L} \to \tilde{\chi}^0_1 e) &= 21.4\%\,, \\ \mathrm{BR}(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 e^+e^-) &= 4.5\%\,, & \mathrm{BR}(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \mu^+\mu^-) &= 4.6\%\,. \end{aligned} \tag{17}$$

Approximately 2×10^5 events of $\tilde{e}^\pm_\mathrm{R}\tilde{e}^\mp_\mathrm{L}$ and $\tilde{e}^+_\mathrm{L}\tilde{e}^-_\mathrm{L}$ production at an integrated luminosity of 1000 fb^{-1} are expected. After combining with the branching ratios in Eqs. (17) a sufficient number of events for the decays $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 e^+e^-$ and $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \mu^+\mu^-$ can be selected. In our Monte Carlo analysis we conservatively assume that at least 1000 neutralino decay events can be reconstructed.

4.1. Lepton energy distribution

The spin averaged differential decay distribution reads:

$$\frac{d^2\Gamma}{dx_- dx_+} \propto F_0(x_-, x_+)\,. \tag{18}$$

The first relation of Eqs. (11) implies that the function F_0 has to be symmetric with respect to the energy variables x_+ and x_- in the CP invariant case (and to a good approximation in the CP non-invariant case). Hence one of the clear signatures of the Majorana nature of neutralinos is the symmetric distribution of events in the (x_-, x_+) Dalitz plane [6].

The left panel of Fig. 3 shows the Dalitz plot of the decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-$ for the parameter set (14) with $\Phi_1 = 0$. We find that the asymmetry in the number of events with $\mathrm{sign}(x_- - x_+) = -$ and $\mathrm{sign}(x_- - x_+) = +$ is small, $\Delta N_\mathrm{ev} = 24$ which is within the statistical error of $\Delta N_\mathrm{stat} = \sqrt{N_\mathrm{ev}} \simeq 32$ for $N_\mathrm{ev} = 1000$.

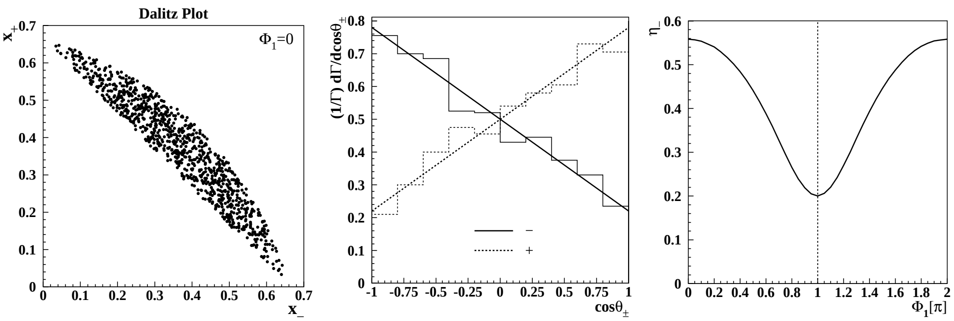

Fig. 3. Left: the Dalitz plot for the neutralino $\tilde{\chi}^0_2$ decay in the (x_-, x_+) Dalitz plane. Middle: the normalized lepton angle distribution (19); the solid line is for the negative charge lepton and the dotted line for the positive charge lepton. Right: the Φ_1 dependence of the slope parameter η_- for the parameter set (14).

4.2. Lepton angular distribution

Another interesting distribution is the lepton angle distribution with respect to the neutralino polarization vector; the angle distribution with respect to the beam direction in the e^+e^- center of mass frame is discussed in [3]. The normalized lepton angle distribution in terms of $\theta_\pm$ (defined as the polar angle between the $\ell^\pm$ momenta and $\hat{n}$) can be written as

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz_\pm} = \frac{1}{2}\left(1 \,\pm\, \eta_\pm\, z_\pm\right) , \tag{19}$$

with $z_\pm = \hat{q}_\pm \cdot \hat{n} = \cos\theta_\pm$. As a result of the CP$\tilde{\mathrm{T}}$ invariance and the Majorana nature of neutralinos we get $\eta_- = \eta_+$, irrespective of whether the theory is CP invariant or not.

The middle panel of Fig. 3 shows the lepton angle distribution for the parameter set (14) with the phase $\Phi_1 = 0$. A simple numerical analysis based on the number of events $N_{\mathrm{ev}} = 1000$ shows that the CP$\tilde{\mathrm{T}}$ relation and the Majorana nature of neutralinos can be confirmed within 1-σ statistical uncertainty of about 10% for the range[2] of $|\cos\theta_\pm| < 0.8$. The dependence of the slope parameter η_- on the Φ_1 phase for the parameter set (14) is shown in the right panel of Fig. 3.

4.3. Lepton invariant mass and opening angle distribution

The next interesting observables are the lepton invariant mass and the lepton opening angle distributions. They give us the possibility to check the relative CP parities of two neutralinos involved in the decay (1). Near

[2] The cut may be necessary to avoid distortions of the $\ell^\pm$ distributions by experimental selection criteria [9].

the end point of the lepton invariant mass distribution the neutralino $\tilde{\chi}^0_1$ is produced nearly at rest. In this case we can expand the squared matrix element in powers of the neutralino velocity $\beta \sim \sqrt{1 - m_{\ell\ell}/(m_2 - m_1)}$:

$$|\mathcal{D}|^2 \sim r_{21}\,(1 - r_{21})^2 \left[\Re(D_{\rm LL})^2 + \Re(D_{\rm LR})^2\right] + O(\beta^2)\,, \tag{20}$$

where $r_{21} = m_1/m_2$. If neutralinos $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$ are of the same (opposite) parity then all bilinear charges are purely real (purely imaginary) and the invariant mass distribution exhibits a characteristic steep S-wave (slow P-wave) decrease proportional to β (β^3) near the maximum of $m_{\ell\ell}$, as can be seen in the left panel of Fig. 4.

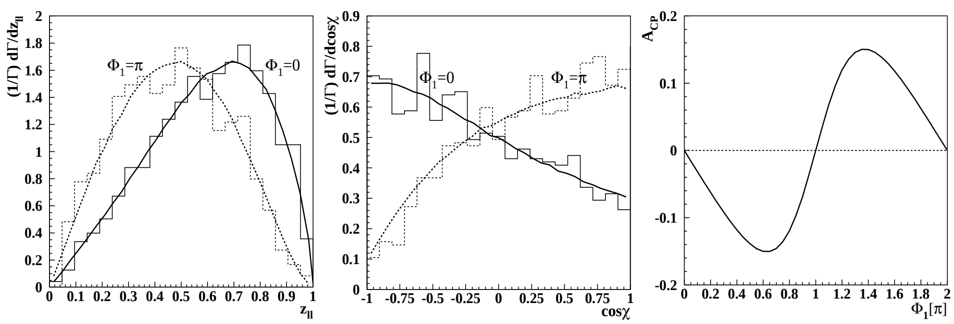

Fig. 4. The lepton invariant mass distribution (left panel) and the opening angle distribution (middle panel) in the three-body leptonic neutralino decay. The solid lines are for $\Phi_1 = 0$, (when neutralinos have the same CP parities) and the dotted lines for $\Phi_1 = \pi$, (when neutralinos have the opposite CP parities). The right panel: the Φ_1 dependence of the CP-odd and CP$\tilde{\rm T}$-even asymmetry $A_{\rm CP}$ of the triple scalar product for the parameter set (14).

Another way of probing the relative CP parities of two neutralinos is the opening angle distribution. The invariant mass of two leptons with respect to the opening angle χ of the lepton pair is given by

$$m^2_{\ell\ell} = \frac{m^2_2}{2}\,x_+ x_-\,\left(1 - \cos\chi\right)\,. \tag{21}$$

At its maximum, for $\cos\chi = -1$, directions of lepton momenta are opposite. Because the helicities of leptons coupled to a vector current are opposite, the angular momentum conservation forces the orbital angular momentum to be zero. On the other hand, since the selection rule of the orbital angular momentum L by the CP symmetry reads:

$$1 = -\eta_1\eta_2(-1)^L\,, \tag{22}$$

for neutralinos of the same (opposite) parity the opening angle distribution is enhanced (suppressed) near $\cos\chi = -1$, as can be seen in the middle panel of Fig. 4.

4.4. CP-odd triple spin-momentum product

If CP is violated, using the CP and CP$\tilde{\mathrm{T}}$ relations (11) and (13) we can construct a CP-odd but CP$\tilde{\mathrm{T}}$-even distribution:

$$F_{\mathrm{CP}}(x_-, x_+) = \frac{1}{2}\left[\, F_3(x_-, x_+) + F_3(x_+, x_-)\,\right] . \tag{23}$$

We observe in (9) that this distribution is connected with a triple neutralino spin and leptons momenta product:

$$O_{\mathrm{CP}} = \hat{n}\cdot(\hat{q}_+ \times \hat{q}_-)\,. \tag{24}$$

With this observable we can construct a CP-odd asymmetry:

$$\begin{aligned} A_{\mathrm{CP}} \;&\equiv\; \frac{N(O_{\mathrm{CP}}>0) - N(O_{\mathrm{CP}}<0)}{N(O_{\mathrm{CP}}>0) + N(O_{\mathrm{CP}}<0)} \\ &= \frac{\int\limits_{\mathcal{D}} \frac{1}{2}\sin\chi\, F_{\mathrm{CP}}(x_-, x_+)\, dx_- dx_+}{\int\limits_{\mathcal{D}} F_0(x_-, x_+)\, dx_- dx_+}\,, \end{aligned} \tag{25}$$

where $\mathcal{D}$ denotes the kinematically allowed (x_-, x_+) region in the Dalitz plot, see Fig. 3.

The right panel of Fig. 4 shows the Φ_1 dependence of the asymmetry A_{CP}. This asymmetry would enable us to measure the CP violating phases in the neutralino system with a 1-σ statistical uncertainty of $\sqrt{(1-A^2_{\mathrm{CP}})/N_{\mathrm{ev}}} \simeq$ 3.1% for 1000 events.

5. Summary

The leptonic decay of a polarized neutralino in its rest frame can provide us with a powerful tool for probing the Majorana nature of neutralinos and their CP properties.
The Majorana nature of neutralinos can be checked through:

- the lepton energy distribution,
- the lepton angle distribution with respect to the neutralino polarization vector.

The relative CP parity of two neutralinos can be identified using:

- the threshold behavior of the lepton invariant mass distribution,
- the opening angle distribution of the lepton pair.

Finally, if CP is violated, one can measure CP violating phases in the neutralino system using the CP-odd quantity built from the neutralino spin vector and two lepton momenta.

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