Vol. 36 (2005) ACTA PHYSICA POLONICA B No 12

ANOMALOUS DIMENSIONS AND REGGEIZED GLUON STATES*

Jan Kotański

M. Smoluchowski Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Kraków, Poland

(Received September 26, 2005)

Solving the BKP equation and comparing with the structure function of hadron for deep inelastic scattering processes we are able to find a relation between reggeized N-gluon states and anomalous dimensions of QCD. To this end we perform analytical continuation of the Reggeon energy and compare exponents of two different twist-series expansions for the hadron structure function. This makes possible to calculate the anomalous dimensions and determine the twist related to them.

PACS numbers: 12.40.Nn, 11.55.Jy, 12.38.–t, 12.38.–t

1. Introduction

During this talk I would like to present the work which was performed about one year ago in collaboration with Korchemsky and Manashov [1]. This work describes the way one can calculate the anomalous dimensions of QCD making use of the reggeized gluon states, *i.e.* Reggeons. This approach was first used in 1980 by Jaroszewicz [2] who calculated anomalous dimensions coming from $N = 2$ Reggeon state corresponding to twist $n = 2$. Later, the cases for higher twists with $N = 2$ were computed by Lipatov [3, 4]. In Ref. [1] we performed calculation for Reggeon states with $N > 2$.

In order to present the calculation, firstly, I will explain briefly what the reggeized gluons are. Next, I will describe the way one can relate them to the anomalous dimensions in QCD. Finally, I will show our results.

---

* Presented at the XLV Cracow School of Theoretical Physics, Zakopane, Poland June 3–12, 2005.

2. Reggeized gluons and Deep Inelastic Scattering

Let us consider the deep inelastic scattering of hadron, $P(p)$, off virtual photon, $\gamma^*(q)$, where the Bjorken $x = Q^2/2(pq)$ with $Q^2 = -q_\mu^2$ and $M^2 = p_\mu^2$. In the Regge limit of small x:

$$M^2 \ll Q^2 \ll s^2 = (p+q)^2 = Q^2 \frac{1-x}{x} \tag{1}$$

resuming appropriate Feynman diagrams one can formulate an effective field theory in which compound states of gluons, *i.e.* reggeized gluons, play a role of a new elementary field.

In the limit (1) the leading contribution to the structure function of hadron $F(x, Q^2)$ comes from the Reggeons with total angular momentum $j \to 1$. Thus, the structure function can be expanded in terms of the moments

$$\widetilde{F}(j, Q^2) \equiv \int_0^1 dx\, x^{j-2} F(x, Q^2) = \sum_{N=2}^{\infty} \bar{\alpha}_{\rm s}^{N-2} \widetilde{F}_N(j, Q^2) \tag{2}$$

with the strong coupling constant $\bar{\alpha}_{\rm s} = \alpha_{\rm s} N_c/\pi$, where

$$\widetilde{F}_N(j, Q^2) = \sum_{\boldsymbol{q}} \frac{1}{j - 1 + \bar{\alpha}_{\rm s} E_N(\boldsymbol{q})} \beta_{\gamma^*}^{\boldsymbol{q}}(Q)\, \beta_p^{\boldsymbol{q}}(M)\,. \tag{3}$$

In the above formula the impact factors

$$\beta_{\gamma^*}^{\boldsymbol{q}}(Q^2) = \int d^2 z_0\, \langle \Psi_{\gamma^*} | \Psi_{\boldsymbol{q}}(\vec{z}_0) \rangle, \qquad \beta_P^{\boldsymbol{q}}(M^2) = \int d^2 z_0\, \langle \Psi_{\boldsymbol{q}}(\vec{z}_0) | \Psi_P \rangle \tag{4}$$

are the overlaps between the Reggeon wave-function $\Psi_{\boldsymbol{q}}(\vec{z}_1, \ldots, \vec{z}_N; \vec{z}_0)$ and the wave-functions of the scattering particles. The parameters $\{\vec{z}_i\}_{i=1,\ldots,N}$ correspond to transverse Reggeon coordinates and are integrated out in the scalar product (4) defined in (24). In order to find the quantized values of $E_N(\boldsymbol{q})$ and $\boldsymbol{q}$ one has to solve the Schrödinger-like equation

$$\mathcal{H}_N\, \Psi_{\boldsymbol{q}}\left(\{\vec{z}_k\}\right) = E_N\, \Psi_{\boldsymbol{q}}\left(\{\vec{z}_k\}\right) \tag{5}$$

which was first formulated for $N = 2$ Reggeons by Balitsky, Fadin, Kuraev and Lipatov [5–7] and later generalized for $N \geq 2$ Reggeon by Bartels, Kwieciński, Praszałowicz and Jaroszewicz [8–10].

It turns out that after performing the multi-color limit [11] Eq. (5) corresponds to the Schrödinger equation of the non-compact Heisenberg SL$(2, \mathbb{C})$ spin chain magnet model [12–14] where the $E_N(\boldsymbol{q})$ plays a role of the total energy. The system becomes integrable with the complete set of the integrals of motion $\boldsymbol{q} = (q_2, \bar{q}_2 \ldots, q_N, \bar{q}_N)$ that are also called conformal charges. Introducing holomorphic and anti-holomorphic coordinates[1] Eq. (5) separates

[1] Variables from the anti-holomorphic sector are denoted by the barred characters.

into two independent equations where

$$\mathcal{H}_N \sim \sum_{k=0}^{N-1} [H(z_k, z_{k+1}) + H(\bar{z}_k, \bar{z}_{k+1})] \tag{6}$$

with $H(z_k, z_{k+1})$ defined in Ref. [13]. The quantity $E_N(\boldsymbol{q})$ is also called the Reggeon energy. The eigenvalue of the lowest conformal charge, q_2, may be parametrized by

$$q_2 = -h(h-1) + Ns(s-1)\,, \tag{7}$$

where in QCD ($s = 0, \bar{s} = 1$) are the complex spins of Reggeons and $(h, \bar{h})$ define a spin of N-Reggeon state

$$h = \frac{1+n_h}{2} + i\nu_h\,, \qquad \bar{h} = \frac{1-n_h}{2} + i\nu_h \tag{8}$$

with $n_h \in \mathbb{Z}$ and $\nu_h \in \mathbb{R}$. The Hamiltonian (6) is invariant under SL$(2, \mathbb{C})$ group transformation

$$z_k \to \frac{az_k + b}{cz_k + d}\,, \qquad \bar{z}_k \to \frac{\bar{a}\bar{z}_k + \bar{b}}{\bar{c}\bar{z}_k + \bar{d}}\,, \qquad ad - bc = 1\,, \quad \bar{a}\bar{d} - \bar{b}\bar{c} = 1 \tag{9}$$

and its eigenstates transform as

$$\Psi_q(\{\vec{z}\}; \vec{z}_0) \to (cz_0 + d)^{2h} (\bar{c}\bar{z}_0 + \bar{d})^{2\bar{h}} \prod_{k=1}^{N} (cz_k + d)^{2s} (\bar{c}\bar{z}_k + \bar{d})^{2\bar{s}}\, \Psi_{\boldsymbol{q}}(\{\vec{z}\}; \vec{z}_0)\,. \tag{10}$$

In order to solve (5) we use an algorithm based on the Q-Baxter method [15]. It is very interesting, however complicated and the reader is referred to Refs. [12–14, 16] for details.

3. Anomalous dimensions and twist series

Due to the scaling symmetry of the Reggeon states (10) one can calculate the dimensions of the impact factors as

$$\beta^{\boldsymbol{q}}_{\gamma^*}(Q^2) = C^{\boldsymbol{q}}_{\gamma^*} Q^{-1-2i\nu_h}\,, \qquad \beta^{\boldsymbol{q}}_{p}(M^2) = C^{\boldsymbol{q}}_{p}\, M^{-1+2i\nu_h}\,, \tag{11}$$

where $C^{\boldsymbol{q}}_{\gamma^*}$ and $C^{\boldsymbol{q}}_{p}$ are dimensionless constants. Substituting (11) into (3) one obtains

$$\widetilde{F}_N(j, Q^2) = \frac{1}{Q^2} \sum_{\boldsymbol{\ell}} \sum_{n_h \geq 0} \int_{-\infty}^{\infty} d\nu_h \frac{C^{\boldsymbol{q}}_{\gamma^*}\, C^{\boldsymbol{q}}_{p}}{j - 1 + \bar{\alpha}_{\rm s} E_N(\boldsymbol{q})} \left(\frac{M}{Q}\right)^{-1+2i\nu_h}\,, \tag{12}$$

where integer n_h and $\boldsymbol{\ell} = (\ell_1, \ldots, \ell_{2N-4})$ enumerate the quantized values of conformal charges $\boldsymbol{q} = \boldsymbol{q}(\nu_h; n_h, \boldsymbol{\ell})$ [14, 17]. The integral over ν_h is calculated by performing analytical continuation of $E_N(\boldsymbol{q}(\nu_h))$ into the complex ν_h-plane, closing the integral contour in infinity and summing the residua inside the contour at $\nu_h(j)$ defined by the condition

$$j - 1 + \bar{\alpha}_{\rm s} E_N(\boldsymbol{q}(\nu_h(j); n_h, \boldsymbol{\ell})) = 0\,. \tag{13}$$

Thus, the moment of the structure function for $j \to 1$ is given by

$$\widetilde{F}_N(j, Q^2) \sim \sum_{\rm res} \frac{1}{Q^2} \left(\frac{M}{Q}\right)^{-1+2i\nu_h(j)} . \tag{14}$$

The above formula will help us to relate the reggeized gluon states to the anomalous dimensions.

On the other hand the moments of $F(x, Q^2)$ can be expanded in inverse powers of the hard scale Q, *i.e.* in the twist series, as

$$\widetilde{F}(j, Q^2) = \sum_{n=2,3,\ldots} \frac{1}{Q^n} \sum_a C_n^a(j, \alpha_{\rm s}(Q^2)) \, \langle p\,|\mathcal{O}_{n,j}^a|p\rangle \tag{15}$$

which is also called operator product expansion (OPE). The Wilson operators, $\mathcal{O}_{n,j}^a$, satisfy

$$Q^2 \frac{d}{dQ^2} \langle p\,|\mathcal{O}_{n,j}^a(0)|p\rangle = \gamma_n^a(j)\, \langle p\,|\mathcal{O}_{n,j}^a(0)|p\rangle\,, \tag{16}$$

where a enumerates operators with the same twist and the anomalous dimensions may be expanded as

$$\gamma_n^a(j) = \sum_{k=1}^{\infty} \gamma_{k,n}^a(j) \left(\frac{\alpha_{\rm s}(Q^2)}{\pi}\right)^k . \tag{17}$$

In the limit $j \to 1$ the moment $\widetilde{F}(j, Q^2)$ takes a form

$$\widetilde{F}(j, Q^2) = \frac{1}{Q^2} \sum_{n=2,3,\ldots} \sum_a \tilde{C}_n^a(j, \alpha_{\rm s}(Q^2)) \, \left(\frac{M}{Q}\right)^{n-2-2\gamma_n^a(j)} . \tag{18}$$

Now we are ready to compare the exponents in (15) and (18) that results in

$$\gamma_n(j) = \frac{n-1}{2} - i\nu_h(j) = \frac{n - (h(j) + \bar{h}(j))}{2}\,. \tag{19}$$

Combining (13) with (19) using the property that $\gamma_n(j) \to 0$ for $\bar{\alpha}_{\rm s} \to 0$ we are able to compute the coefficients in Eq. (17) and to determine the corresponding twist.

Thus making use of the above equations we can extract $\gamma_n(j)$ from the expansion of $E_N(\boldsymbol{q}(\nu_h(j)))$ in the vicinity of its poles:

$$E_N(\boldsymbol{q}) = -\left[\frac{c_{-1}}{\epsilon} + c_0 + c_1\,\epsilon + \ldots\right] \tag{20}$$

at $i\nu_h = i\nu_h^{\rm pole} + \epsilon$. Inverting Eq. (20) and using Eq. (19) one obtains

$$\gamma_n(j) = -c_{-1}\left[\frac{\bar{\alpha}_{\rm s}}{j-1} + c_0\left(\frac{\bar{\alpha}_{\rm s}}{j-1}\right)^2 + \left(c_1 c_{-1} + c_0^2\right)\left(\frac{\bar{\alpha}_{\rm s}}{j-1}\right)^3 + \ldots\right], \tag{21}$$

where the coefficient $c_k = c_k(n, n_h, \boldsymbol{\ell})$ are defined by (20). Moreover, it turns out that the position of the energy poles:

$$E_N(\boldsymbol{q}) \sim \frac{\gamma_n^{(0)}}{i\nu_h - (n-1)/2} \tag{22}$$

determines the twist n:

$$i\nu_h = (n-1)/2 \quad \text{with} \quad n \geq N + n_h\,. \tag{23}$$

4. Results

4.1. Analytical continuation

After performing the analytical continuation of $E_N(\boldsymbol{q}(\nu_h))$ in complex ν_h-space the Reggeon wave functions $\Psi_{\boldsymbol{q}}(\{\vec{z}_i\};\vec{z}_0)$ is no more normalizable with respect to the scalar product

$$\langle \Psi_{\boldsymbol{q}}(\vec{z}_0) | \Psi_{\boldsymbol{q}'}(\vec{z}_0{}') \rangle \equiv \int \prod_{k=1}^{N} d^2 z_k\, \Psi_{\boldsymbol{q}}(\{\vec{z}\};\vec{z}_0)\big(\Psi_{\boldsymbol{q}'}(\{\vec{z}\};\vec{z}_0{}')\big)^* = \delta^{(2)}(z_0 - z_0')\delta_{\boldsymbol{q}\boldsymbol{q}'}\,. \tag{24}$$

Moreover, the quantization conditions for $\boldsymbol{q}$ become relaxed, so that

$$\bar{q}_k \neq q_k^* \quad \text{and} \quad \bar{h} \neq 1 - h^*\,. \tag{25}$$

4.2. Two-Reggeon states

For $N=2$ Reggeon states the energy [5–7] in known analytically

$$E_2(\nu_h, n_h) = \psi\left(\frac{1+|n_h|}{2} + i\nu_h\right) + \psi\left(\frac{1+|n_h|}{2} - i\nu_h\right) - 2\psi(1)\,, \tag{26}$$

with $\Psi(x) = \frac{d}{dx}\Gamma(x)$. It is an analytical function on the complex ν_h-plane without the branching points and with poles at $i\nu_h = \pm(n-1)/2$. The leading twist $n = 2$, which agrees with (23), corresponds to the pole at $i\nu_h = 1/2$, while the anomalous dimension [2]

$$\gamma_2(j) = \frac{\bar{\alpha}_s}{j-1} + 2\zeta(3)\left(\frac{\bar{\alpha}_s}{j-1}\right)^4 + 2\zeta(5)\left(\frac{\bar{\alpha}_s}{j-1}\right)^6 + \mathcal{O}(\bar{\alpha}_s^8)\,. \tag{27}$$

4.3. N-Reggeon states

For more than $N = 2$ Reggeons the energy E_N is a multi-valued function with the branching point in the complex ν_h-plane where the cuts take a form

$$E_N^{\pm} \sim a_k \pm b_k\sqrt{\nu_{\mathrm{br},k} - \nu_h}\,. \tag{28}$$

Poles at $i\nu_h = (n-1)/2$ with twist $n \geq N + n_h$. We do not have a unique analytical formula so we evaluate $E_N(\boldsymbol{q})$ numerically [14].

4.4. $N = 3$ Reggeon states with $n_h = 0$

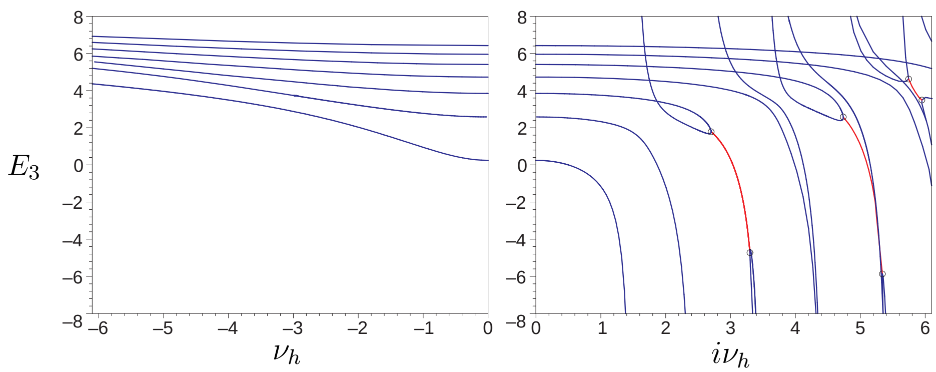

Fig. 1. The energy spectrum of the $N = 3$ Reggeon states $E_3(\nu_h; n_h, \boldsymbol{\ell})$ for $n_h = 0$ and $\boldsymbol{\ell} = (0, \ell_2)$, with $\ell_2 = 2, 4, \ldots, 14$ from the bottom to the top (on the left). Analytical continuation of the energy along the imaginary ν_h-axis (on the right). The branching points are indicated by open circles. The lines connecting the branching points represent $\mathrm{Re}E_3(i\nu_h)$ [1].

Let us consider the case for $N = 3$ and $n_h = 0$ where $\boldsymbol{\ell} = (0, \ell_2)$ with $\ell_2 = 2, 4, \ldots, 14$ what gives a condition $\bar{q}_3 + q_3 = 0$. The spectral surfaces $E_3(\boldsymbol{q}(\nu_h))$ in this case along real and imaginary axes are shown in Fig. 1. On the left panel the energy $E_3(\nu_h)$ is a monotonic function of real ν_h. However, on the right panel, *i.e.* for imaginary values of ν_h, branching points, denoted by open circles, appear. They glue together surfaces with the same quantum numbers. They appear not only for purely imaginary ν_h but also for complex ν_h. However, one can notice that contribution to the structure function from the cuts cancel each other so the (OPE) expansion is not broken. The poles of $E_N(\nu_h)$ are localized at $i\nu^{\rm pole} = (n-1)/2$ which gives possible values of the twists $n = 4, 6, 8, \ldots$.

The leading twist $n = 4$ in this sector comes from the poles at $i\nu_h = 3/2$ where

$$E_3\left(\frac{3}{2}+\epsilon\right) = \epsilon^{-1} + \frac{1}{2} - \frac{1}{2}\,\epsilon + 1.7021\,\epsilon^2 + \ldots \qquad (29)$$

which gives the anomalous dimension

$$\gamma_4^{(N=3)}(j) = \frac{\bar\alpha_{\rm s}}{j-1} - \frac{1}{2}\left(\frac{\bar\alpha_{\rm s}}{j-1}\right)^2 - \frac{1}{4}\left(\frac{\bar\alpha_{\rm s}}{j-1}\right)^3 - 1.0771\left(\frac{\bar\alpha_{\rm s}}{j-1}\right)^4 + \ldots .$$

The energy around the other poles reads as follows

$$\begin{aligned}
E_3\left(\frac{3}{2}+\epsilon\right) &= \frac{1}{\epsilon} + \frac{1}{2} - \frac{1}{2}\,\epsilon + 1.7021\,\epsilon^2 + \ldots\,,\\
E_3\left(\frac{5}{2}+\epsilon\right) &= \frac{2}{\epsilon} + \frac{15}{8} - 1.6172\;\epsilon + 0.719\;\epsilon^2 + \ldots\,,\\
E_3^{(\rm a)}\left(\frac{7}{2}+\epsilon\right) &= \frac{1}{\epsilon} + \frac{11}{12} - 0.6806\;\epsilon - 1.966\;\epsilon^2 + \ldots\,,\\
E_3^{(\rm b)}\left(\frac{7}{2}+\epsilon\right) &= \frac{2}{\epsilon} + \frac{15}{4} - 3.2187\;\epsilon + 3.430\;\epsilon^2 + \ldots\,,\\
E_3^{(\rm a)}\left(\frac{9}{2}+\epsilon\right) &= \frac{2}{\epsilon} + \frac{125}{48} - 2.0687\;\epsilon + 1.047\;\epsilon^2 + \ldots\,,\\
E_3^{(\rm b)}\left(\frac{9}{2}+\epsilon\right) &= \frac{2}{\epsilon} + \frac{53}{12} - 2.4225\;\epsilon + 0.247\;\epsilon^2 + \ldots\,, \qquad (30)
\end{aligned}$$

and can be generally cast in the form:

$$E_3(i\nu_h^{\rm pole}+\epsilon) = \frac{R}{\epsilon} + 2\mathcal{E}(i\nu_h) + \mathcal{O}(\epsilon)\,, \qquad (31)$$

where $R = 2$ (or $R = 1$ for *even* $h = \frac{1}{2} + i\nu_h$) and $\mathcal{E}(h)$ is energy of the Heisenberg model with $SL(2,\mathbb{R})$ spin. Making use of (20) and (21) one can calculate expansion coefficients of anomalous dimensions corresponding to (30).

4.5. $N = 3$ descendent states with $n_h = 1$

Another interesting case for $N = 3$ Reggeon states is when $q_3 = \bar{q}_3 = 0$ and $n_h = 1$. Such states are called descendent of $N = 2$ states because they possess the same quantum numbers as $N = 2$ Reggeon states and the energy

$$E_3^{\rm desc}(q_3 = 0, q_2; n_h = 1) = E_2(q_2; n_h = 1) \tag{32}$$

as a function of ν_h does not have branching points and its poles are situated at $i\nu_h = 1, 2, 3, \ldots$ with twists $n = 3, 5, 7, \ldots$. According to (23) this case is related to $N = 2$ case which gives one more argument to call them descendent.

The leading twist, *i.e.* $n = 3$, corresponds to the energy pole where

$$E_{3,{\rm d}}(1 + \epsilon) = \epsilon^{-1} + 1 - \epsilon - (2\zeta(3) - 1)\,\epsilon^2 + \ldots\,, \tag{33}$$

so that in this case the anomalous dimension

$$\gamma_3^{(N=3)}(j) = \frac{\bar{\alpha}_{\rm s}}{j-1} - \left(\frac{\bar{\alpha}_{\rm s}}{j-1}\right)^2 + (2\zeta(3) + 1)\left(\frac{\bar{\alpha}_{\rm s}}{j-1}\right)^4 + \ldots\,. \tag{34}$$

4.6. Leading twist for higher N

For higher N the leading twist $n = N$. However, considering *even* and *odd* N's separately one can notice that the leading twist comes from completely different sectors. For *even* N it corresponds to the pole at $i\nu_h = (N-1)/2$ localized in the sector where $n_h = 0$. The energy pole residuum equals $(N-2)$ that gives anomalous dimensions

$$\gamma_N^{(N)}(j) = (N-2)\frac{\bar{\alpha}_{\rm s}}{j-1} + \mathcal{O}(\bar{\alpha}_{\rm s}^2) \tag{35}$$

with the twist $n = N$.

For *odd* N's in the sector with $n_h = 0$ the minimal twist $n_{\rm min} = (N+1)$ corresponds to the energy pole at $i\nu_h = N/2$. For example, for $N = 5$ we have

$$E_5(5/2 + \epsilon) = \frac{3}{\epsilon} + \frac{7}{6} + \ldots\,. \tag{36}$$

However, the real leading twist comes from the sector of descendent states with $n_h = 1$ and it corresponds to the pole at $i\nu_h = (N-1)/2$, *e.g.*

$$E_{5,{\rm d}}(2 + \epsilon) = \frac{3 + \sqrt{5}}{2\epsilon} + 1.36180 + \ldots\,. \tag{37}$$

As one can see, the pole residuum in this case has a more complicated form.

In each case the energy pole which gives the leading twist is situated on the same surface as the state with the minimal energy $E_N(\nu_h)$ where $\nu_h \in \mathbb{R}$.

5. Summary

In this work following Ref. [1] we have considered the deep inelastic scattering processes of hadrons described by means of the reggeized gluon states. Expanding the structure function (2) we have used two approaches. Firstly, we have expanded the structure function (2) making use of the reggeized gluon states which was possible due to the analytical continuation of the Reggeon energy $E_N(\nu_h)$ into complex ν_h-plane. Secondly, we have performed (OPE) expansion (15) with evolution equation (16), which defines the anomalous dimensions of QCD. Comparing exponents of these two series we have found relation between $\gamma_n^a(j)$ and N-Reggeon states. Since we are able to solve numerically the BKP equation [13, 14] for N-Reggeon states with $N \geq 2$, we have calculated anomalous dimensions and the twist coming from these states.

Contrary to $N = 2$ Reggeon case for $N \geq 3$ the energy $E_N(\nu_h)$ is a multi-valued function of complex ν_h parameter defined on complex Riemann surface with the infinite number of branching points that glue the surfaces with the same quantum numbers. However, similarly to $N = 2$ case, the energy has poles only for purely imaginary ν_h and their position defines the possible values of the twist (23). Fitting expansion coefficients of the energy in the vicinity of these poles we have calculated the anomalous dimensions of QCD.

It turns out that the leading twist n coming from the N-Reggeon states is equal to a number of reggeized gluons N. However, we have to consider the states with *even* N and the states with *odd* N separately. For *even* N the leading twist comes from the sector where $n_h = 0$, whereas for *odd* N the minimal twist comes from the sector of descendent states with $n_h = 1$.

To sum up I would like to notice that contrary to common approaches [18–20] the present work goes beyond the leading order of the twist expansion of the structure function and describes contributions with non-leading asymptotics that correspond to the higher twists.

I would like to thank warmly G.P. Korchemsky, A.N.Manashov and S.É. Derkachov with whom I have done this work. I am also grateful to M. Praszałowicz and J. Wosiek for fruitful discussions. This work was supported by the Polish State Committee for Scientific Research (KBN) grants PB-2-P03B-43-24 and PB-0349-P03-2004-27.

REFERENCES

[1] G.P. Korchemsky, J. Kotanski, A.N. Manashov, *Phys. Lett.* **B583**, 121 (2004).

[2] T. Jaroszewicz, *Phys. Lett.* **B116**, 291 (1982).

[3] L.N. Lipatov, *Sov. Phys. JETP* **63**, 904 (1986).

[4] H.J. De Vega, L.N. Lipatov, *Phys. Rev.* **D66**, 074013 (2002).

[5] V.S. Fadin, E.A. Kuraev, L.N. Lipatov, *Phys. Lett.* **B60**, 50 (1975).

[6] E.A. Kuraev, L.N. Lipatov, V.S. Fadin, *Sov. Phys. JETP* **45**, (1977).

[7] L.N. Lipatov I.I. Balitsky, *Sov. J. Nucl. Phys.* **28**, 822 (1978).

[8] J. Bartels, *Nucl. Phys.* **B175**, 365 (1980).

[9] J. Kwiecinski, M. Praszalowicz, *Phys. Lett.* **B94**, 413 (1980).

[10] T. Jaroszewicz, Triest preprint IC/80/175.

[11] G. 't Hooft, *Nucl. Phys.* **B72**, 461 (1974).

[12] G.P. Korchemsky, J. Kotanski, A.N. Manashov, *Phys. Rev. Lett.* **88**, 122002 (2002).

[13] S.E. Derkachov, G.P. Korchemsky, A.N. Manashov, *Nucl. Phys.* **B617**, 375 (2001).

[14] S.E. Derkachov, G.P. Korchemsky, J. Kotanski, A.N. Manashov, *Nucl. Phys.* **B645**, 237 (2002).

[15] R.J. Baxter, *Exactly Solved Models in Statistical Mechanics*, Academimc Press, London 1982.

[16] J. Kotanski, M. Praszalowicz, *Acta Phys. Pol. B* **33**, 657 (2002).

[17] S.E. Derkachov, G.P. Korchemsky, A.N. Manashov, *Nucl. Phys.* **B661**, 533 (2003).

[18] E.M. Levin, M.G. Ryskin, A.G. Shuvaev, *Nucl. Phys.* **B387**, 589 (1992).

[19] J. Bartels, *Z. Phys.* **C60**, 471 (1993).

[20] J. Bartels, M.G. Ryskin, *Z. Phys.* **C60**, 751 (1993).