

THE INFLUENCE OF EXTERNAL UV RADIATION
FIELD ON PRIMORDIAL GAS CLOUDS:
FORMULATION OF THE PROBLEM* **

J. STASIELAK^a, S. STACHNIEWICZ^b, M. KUTSCHERA^{a,b}

^aM. Smoluchowski Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Kraków, Poland

^bH. Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland

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Our goal is to study the effects of the UV radiation from the first stars, quasars and decays of the hypothetical Super Heavy Dark Matter (SHDM) particles on the formation of primordial bound objects in the Universe. We trace the evolution of a spherically symmetric density perturbation in the Lambda Cold Dark Matter (LCDM) and MOND models, solving the frequency-dependent radiative transfer equation, non-equilibrium chemistry, and one-dimensional gas hydrodynamics. We concentrate on the destruction and formation processes of the H₂ molecule, which is the main coolant in the primordial objects.

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1. Introduction

Formation of primordial objects such as young galaxies and globular clusters in the early Universe is a fundamental problem in the modern cosmology. There was a rapid progress of observations in the last years. It gave theoreticians invaluable possibility to compare their theories with the observations. One can say that a consistent scenario of the structure formation is established. However, there are still many unanswered questions, especially regarding the influence of the background radiation field from the first objects on the star formation rate. In our work we address this problem.

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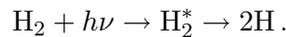
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The formation of the first objects is a direct consequence of the growth of the primordial density fluctuations. At the beginning, there are linear density perturbations which expand with the overall Hubble flow. Subsequently, these perturbations can grow and form primordial clouds. Clouds with enough density contrast decouple from this flow and start to collapse. The kinetic energy of the infalling gas is dissipated through shocks and the cloud becomes pressure supported. The further evolution of the cloud is determined by its ability to cool sufficiently fast. Clouds which could not cool fast enough will stay in a pressure-supported stage and will not form any stars. The existence of the efficient cooling mechanism is necessary to ensure the collapse of the cloud, its subsequent fragmentation and star formation.

In our work we are interested in the first generation of stars, called population III stars, which are still forming in the low mass clouds when first luminous objects already exist. These objects are made from the primordial gas so they are metal-free. It is simply because the first stars did not have much time to produce any metals. These objects could be irradiated by the UV and X-rays radiation produced by the first stars, quasars and hypothetical SHDM particles decays [1,2].

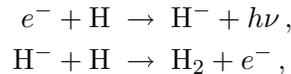
In the absence of metals, the most important cooling mechanism for low-mass primordial clouds is so called ‘H₂ cooling’, *i.e.* cooling by radiation of excited rotational and vibrational states of H₂ molecule. We can look at this mechanism as the collisional excitation of the H₂ molecule, subsequent spontaneous de-excitation and photon emission. Emitted photons can further escape from the cloud and take away kinetic energy of the collapsing cloud. The presence of initial mass fraction of the molecular hydrogen H₂ as small as 10⁻⁶ is enough to trigger the final collapse of low mass clouds.

Molecular hydrogen is fragile and can easily be photodissociated by photons with energies of 11.26–13.6 eV (Lyman and Werner bands) [3] through the following reaction:



Destruction of the H₂ molecules would stop the collapse of the low mass clouds and would decrease star formation rate.

In primordial gas cloud, H₂ molecules can form mainly through the coupled reactions



in which the electrons act only as a catalyst. X-rays radiation can increase production of H₂ by enhancement of free electron fraction.

From the above we see that the UV and X-ray radiation background alters the growth rate of cosmic structures. It regulates star formation rate so it has important implications for the re-ionization history of the Universe [4]. In addition, photoionization by UV photons can be regarded as the process which leads to the inhibition of the low mass galaxies formation, so it could explain the so called ‘low-mass galaxies overproduction problem’ in the hierarchical bottom-up theory of galaxy formation. It is therefore crucial to determine quantitatively the consequences of radiation feedback on the formation of early generation objects.

Feedback of the UV background to the collapse of the spherically symmetric primordial gas cloud in CDM model was studied by many authors [5–9]. However, their calculations were simplified. They have been using self-shielding function [10] instead of solving the full transfer equation. Our approach includes solving the frequency-dependent radiative transfer equation in the ‘exact’ way. We are trying to follow the evolution of the collapsing cloud in the LCDM and MOND models.

2. Basic equations

In Cold Dark Matter models and in case of spherical symmetry, the collapse may be described by the following equations:

$$\frac{dM}{dr} = 4\pi r^2 \varrho, \quad (1)$$

$$\frac{dr}{dt} = v, \quad (2)$$

$$\frac{dv}{dt} = -4\pi r^2 \frac{dp}{dM} - \frac{GM(r)}{r^2}, \quad (3)$$

$$\frac{du}{dt} = \frac{p}{\varrho^2} \frac{d\varrho}{dt} + \frac{\Lambda}{\varrho}, \quad (4)$$

where r is the radius of a sphere of mass M , u is the internal energy per unit mass, p is pressure and ϱ is mass density. Here, Eq. (1) is the continuity equation, (2) and (3) give acceleration and (4) accounts for energy conservation. The last term in Eq. (4) describes gas cooling/heating, where Λ is energy absorption (emission) rate per unit volume.

Λ consists of two parts, that is, the chemical cooling rate Λ_{chem} , and the radiative cooling rate Λ_{rad} :

$$\Lambda(r) = \Lambda_{\text{chem}}(r) + \Lambda_{\text{rad}}(r). \quad (5)$$

The chemical cooling rate, Λ_{chem} , can be written as

$$\Lambda_{\text{chem}} = \varrho \frac{\partial \epsilon_{\text{chem}}}{\partial t} \quad (6)$$

where $\bar{\epsilon}_{\text{chem}}$ is the chemical binding energy per unit mass. The radiative cooling rate $\Lambda_{\text{rad}}(r)$ is obtained by solving the radiative transfer equation.

We use the equation of state of perfect gas

$$p = (\gamma - 1)\varrho u, \quad (7)$$

where $\gamma = 5/3$, as the primordial baryonic matter after recombination is assumed to be composed mainly of monoatomic hydrogen and helium, with the fraction of molecular hydrogen H_2 always less than 10^{-3} .

In the case of modified gravity [11], equation (3) changes:

$$\frac{dv}{dt} = -4\pi r^2 \frac{dp}{dM} - g_H - a_0 f \left(\frac{GM(r)}{a_0 r^2} - \frac{g_H}{a_0} \right), \quad (8)$$

where $f(x)$ is some function, asymptotically equal to x for $x \gg a_0$ and to $\sqrt{a_0 x}$ for $x \ll a_0$, while g_H may be expressed as

$$g_H = \frac{1}{2} H_0^2 [(z+1)^3 \Omega_b + 2((z+1)^4 \Omega_r - \Omega_\Lambda)] r. \quad (9)$$

Here, H_0 is the current value of the Hubble parameter, Ω_b , Ω_r and Ω_Λ are the current fractions of baryons, radiation and dark energy in terms of the critical density of the Universe, and z is the redshift. We have made similar assumption like in [12] and we apply MOND to net acceleration over Hubble flow only.

3. Radiative transfer equation in spherical symmetry

The time-independent, non-relativistic equation for radiation transport in spherical geometry can be written in the form:

$$\mu \frac{\partial I_\nu}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial I_\nu}{\partial \mu} = \varrho \{ \eta_\nu(r, \mu) - \chi_\nu(r) I_\nu \}, \quad (10)$$

where $I_\nu = I_\nu(r, \mu)$ is the intensity of radiation of frequency ν , at radius r and in the direction $\mu = \cos \theta$, where θ is the angle between the outward normal and photon direction. The functions $\eta_\nu(r, \mu)$ and $\chi_\nu(r)$ are the total emissivity and opacity at frequency ν , which can be expressed as:

$$\eta_\nu(r, p) = \eta_\nu^t(r) + \eta_\nu^s(r, p), \quad (11)$$

$$\chi_\nu(r) = \kappa_\nu(r) + \sigma_\nu(r). \quad (12)$$

Here η_ν^t , η_ν^s , κ_ν and σ_ν are thermal emissivity, scattering emissivity, true absorption coefficient and scattering coefficient at frequency ν , respectively.

Following Hummer and Rybicki [13], we introduce a more convenient system of coordinates (r, p) rather than (r, μ) defined by the transformation formula

$$(r, \mu) \rightarrow (r, p = r\sqrt{1 - \mu^2}) \quad \text{for} \quad -1 \leq \mu \leq 1. \quad (13)$$

For a given radius r the ‘impact’ parameter p can vary between 0 and r . Because the parameter p cannot distinguish between $\mu > 0$ and $\mu < 0$, the radiation intensity I_ν have to be separated into outward I_ν^+ and inward I_ν^- directed intensity:

$$I_\nu^+ = I_\nu(r, p) \quad \mu \geq 0, \quad (14)$$

$$I_\nu^- = I_\nu(r, p) \quad \mu < 0. \quad (15)$$

In the new system of coordinates, Eq. (10) can be written as two separate equations,

$$\mu \frac{\partial I_\nu^+}{\partial r} = -\varrho \chi_\nu \{I_\nu^+ - S_\nu(r, p)\}, \quad (16)$$

$$\mu \frac{\partial I_\nu^-}{\partial r} = \varrho \chi_\nu \{I_\nu^- - S_\nu(r, p)\}, \quad (17)$$

where $\mu = \sqrt{1 - \frac{p^2}{r^2}} \geq 0$ and $S_\nu(r, p) = \eta_\nu(r, p) / \chi_\nu(r)$ is the source function.

Let us introduce new functions [14]:

$$j_\nu(r, p) = \frac{1}{2} (I_\nu^+(r, p) + I_\nu^-(r, p)), \quad (18)$$

$$h_\nu(r, p) = \frac{1}{2} (I_\nu^+(r, p) - I_\nu^-(r, p)). \quad (19)$$

Eqs. (16) and (17) obtain then the form

$$\mu \frac{\partial j_\nu(r, p)}{\partial r} = -\varrho \chi_\nu h_\nu(r, p), \quad (20)$$

$$\mu \frac{\partial h_\nu(r, p)}{\partial r} = -\varrho \chi_\nu \{j_\nu(r, p) - S_\nu(r, p)\}. \quad (21)$$

To solve these equations we need also a boundary condition. We consider a spherical envelope with an inner boundary at radius $r = r_{\min}$ and an outer boundary $r = r_{\max}$. The cloud is immersed in an external time dependent, isotropic radiation field. We obtain the following boundary condition

$$\begin{aligned} h_\nu(r_{\min}, p) &= \alpha_\nu(p, t) & 0 \leq p \leq r_{\min}, \\ h_\nu(p, p) &= 0 & r_{\min} \leq p \leq r_{\max}, \\ j_\nu(r_{\max}, p) - h_\nu(r_{\max}, p) &= \beta_\nu(p, t) & 0 < p < r_{\max}, \end{aligned} \quad (22)$$

where α_ν and β_ν describe the changes of the radiation at frequency ν impinging upon the inner and the outer boundary with the time t .

By the standard procedure one obtains from Eq. (10) the zeroth and the first moment equations [15]

$$\frac{\partial (f_\nu(r) J_\nu(r))}{\partial r} + \frac{3f_\nu(r) - 1}{r} J_\nu(r) + \varrho \chi_\nu H_\nu(r) = 0, \quad (23)$$

$$\frac{\partial H_\nu(r)}{\partial r} + \frac{2H_\nu(r)}{r} + \varrho \kappa_\nu J_\nu(r) - \varrho \eta_\nu^t = 0, \quad (24)$$

where $f_\nu(r) = K_\nu(r)/J_\nu(r)$ is the Eddington factor. J_ν , H_ν , K_ν are the zeroth, first and second moment of the radiation field at frequency ν :

$$J_\nu(r) = \int_0^1 j_\nu d\mu, \quad (25)$$

$$H_\nu(r) = \int_0^1 h_\nu \mu d\mu, \quad (26)$$

$$K_\nu(r) = \int_0^1 j_\nu \mu^2 d\mu. \quad (27)$$

We can cast Eq. (23) into a more convenient form, by introducing a sphericity factor q_ν , defined in the following way:

$$q_\nu(r) = \exp \left[\int_{r_c}^r \left(3 - \frac{1}{f_\nu(r')} \right) \frac{dr'}{r'} \right], \quad (28)$$

where r_c is the core radius, that is, the inner boundary of the medium. Using this factor and making some simplification we can rewrite Eqs. (23) and (24) as

$$\frac{\partial (f_\nu(r) q_\nu(r) J_\nu(r))}{\partial r} = -\rho \chi_\nu q_\nu(r) H_\nu(r), \quad (29)$$

$$\frac{\partial (H_\nu(r) r^2)}{\partial r} = r^2 \rho (\eta_\nu^t - \kappa_\nu J_\nu(r)). \quad (30)$$

The corresponding boundary conditions for the moment equations (29) and (30) can be written as follows [16]:

$$H_\nu(r_{\min}) = \int_0^1 \alpha_\nu \mu d\mu, \quad (31)$$

$$J_\nu(r_{\max}) = \int_0^1 \beta_\nu d\mu + \int_0^1 h_\nu(r_{\max}, p) d\mu. \quad (32)$$

We see that these boundary conditions can be determined only after Eqs. (20) and (21) have been solved for j_ν and h_ν .

In the following sections we shall refer to Eqs. (20) and (21) with the boundary conditions (22) as system I equations. The moment equations (29) and (30) with the boundary condition (31), (32) will henceforth be referred to as system II equations.

Solution of the radiative energy transfer equations gives us information about the luminosity function $L(r)$:

$$L(r) = 16\pi^2 r^2 H(r), \quad (33)$$

where

$$H(r) = \int_0^\infty H_\nu(r) d\nu. \quad (34)$$

With the given luminosity we can calculate radiative part of cooling function Λ_{rad} :

$$\Lambda_{\text{rad}}(r) = -\frac{1}{4\pi r^2} \frac{\partial L(r)}{\partial r}. \quad (35)$$

It can be used further in the hydrodynamical equations.

4. Chemical reactions and thermal effects

In our calculations, we include all the relevant thermal and chemical processes in the primordial gas. We take into account nine species: H, H⁻, H⁺, He, He⁺, He⁺⁺, H₂, H₂⁺ and e⁻. Their abundance varies with time due to chemical reactions, ionization and dissociation photoprocesses. The chemical reactions include such processes as ionization of hydrogen and helium by electrons, recombination of ions with electrons, formation of negative hydrogen ions, formation of H₂ molecules, *etc.* A full list of the relevant chemical reactions and appropriate formulas are given in [17].

The time evolution of the number density of component n_i is described by the kinetic equation:

$$\frac{dn_i}{dt} = \sum_{l=1}^9 \sum_{m=1}^9 a_{lm} k_{lm} n_l n_m + \sum_{j=1}^9 b_{ji} \kappa_j n_j. \quad (36)$$

The first component on the right-hand side describes the chemical reactions, and the other one accounts for photoionization and photodissociation processes. Coefficients k_{lm} are reaction rates, quantities κ_n are photoionization

or photodissociation rates, and a_{lmi} and b_{ji} are numbers equal to 0, ± 1 or ± 2 depending on the reaction. All reaction rates, as well as photoionization and photodissociation rates are given in [17].

The cooling (or heating) function Λ includes effects of collisional ionization of H, He and He^+ , recombination to H, He and He^+ , collisional excitation of H and He^+ , Bremsstrahlung, Compton cooling and cooling by de-excitation of H_2 molecules. The heating/cooling rates are given in [17].

5. Numerical strategy and initial conditions

In the simulations we used the code described in [17], based on those presented in [18] and [19]. This is a standard, one-dimensional, second-order accurate, Lagrangian finite-difference scheme, slightly modified to our purposes. Our code supports flat and non-flat CDM and LCDM models as well as Milgrom's Modified Newtonian Dynamics. In the latter case it was necessary to do significant changes in the initial conditions. We start our calculations at the end of the radiation-dominated era instead of $z = 500$. For $\Omega_b = \Omega_M = 0.02/h^2$, $z_{\text{eq}} = 485$ as given by the formula provided by [20], $z_{\text{eq}} = 2.50 \times 10^4 \Omega_0 h^2 \Theta_{2.7}^{-4}$, where $\Theta_{2.7} = T_\gamma/2.7$ K, assuming $h = 0.72$ and $T_\gamma = 2.7277$ K. We assume that in MOND, like in the standard cosmology, initial overdensities may grow only in the matter-dominated era. In both cases we use our own code to calculate the initial chemical composition and initial gas temperature. Initial overdensities may be calculated from the matter power spectrum which may be obtained *e.g.* using the CMBFAST program [21].

We apply the initial density profiles in the form of a single spherical Fourier mode, also used in [19]

$$\varrho_b(r) = \Omega_b \varrho_c \left(1 + \delta \frac{\sin kr}{kr} \right), \quad (37)$$

where ϱ_c is the critical density of the Universe, $\varrho_c = 3H^2/8\pi G$, with H being the value of the Hubble parameter at the initial time.

For this profile, we can distinguish two radius values, R_0 and R_z , which correspond to the first zero and the first minimum of the function $\sin(kr)/kr$, respectively. Inside the sphere of radius $R_0 = \pi/k$ which contains mass M_0 , the local density contrast is positive. The mass M_0 and the radius R_0 will be referred to as the cloud mass and the cloud radius, respectively. The local density contrast is negative for $R_z > r > R_0$, with average density contrast vanishing for the sphere of radius $R_z = 4.49341/k$ and mass M_z . The shell of radius R_z will expand together with the Hubble flow, not undergoing any additional deceleration. This is why this profile is very convenient in numerical simulations. It eliminates the numerical edge effects and the mentioned

shell simply follows the Hubble expansion of the Universe. Thus, it can be regarded as the perturbation boundary, with mass M_z referred to as the bound mass.

It is worth to note that for radii not greater than $3/4R_0$ this profile is very similar to the Gaussian profile

$$\varrho_i(r) = \Omega_i \varrho_c \left[1 + \delta_i \exp\left(\frac{-r^2}{2R_f^2}\right) \right] \quad (38)$$

with $R_f = 1/2R_0$.

As the initial velocity, we use the Hubble velocity:

$$v(r) = Hr. \quad (39)$$

Our numerical procedure to trace the dynamical evolution of the primordial cloud under the UV background is the following:

- We solve the hydrodynamic equations of motion along with equations for energy conservation, ionization, and dissociation of molecular and atomic species. From the solution of these equations we can calculate the properties and distribution of the absorbing components $\kappa_\nu(r)$, $\sigma_\nu(r)$ and the thermal emissivity $\eta_\nu^t(r)$. It will give us the initial value of the source function S_ν .
- With the initial value of S_ν , we solve the system I equations for the geometrical factors f_ν and q_ν .
- We solve the system II equations for J_ν and H_ν .
- The source function is independent of the radiation field (J_ν) only when there is no light scattering. So, in general we have to update S_ν and solve the system I once more.
- Iterative procedure between system I and II is continued until convergence.
- We calculate luminosity $L(r)$ from Eq. (33) and then the cooling function.
- We update abundance of different species and number densities of each atomic and molecular state.
- We repeat all of the above steps.

Detailed results of our simulations will be presented in a separate paper, which is under preparation.

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