

## FRAGMENTATION OF PARTONS\*

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*(Received October 15, 2004)*

The concept of parton fragmentation in QCD hard scattering phenomenology as well as NLO pQCD analysis of fragmentation functions are outlined. Hadroproduction of pions of a few GeV  $p_{\perp}$  is discussed through the example of recent measurements at  $\sqrt{S_{\text{RHIC}}} = 200$  GeV.

PACS numbers: 13.87.Fh, 12.38.Bx

**1. Introduction**

Hard pQCD reactions produce highly virtual partons that will radiate off their virtualities, thus evolving into the non-perturbative states that contain observable hadrons. A quantitative understanding of this process, known broadly as the fragmentation of partons, considerably widens the class of reactions that can be handled within a perturbative QCD approach. Examples are too many to enumerate here (as the reader may convince himself by browsing through the proceedings), two recent cases will be discussed below in Secs. 3.2 and 3.3.

A graph such as in Fig. 1 serves as a popular illustration of QCD factorization [1]. It also displays the role of fragmentation functions  $D_c^{\pi}(z)$  in hard scattering phenomenology when there are detected hadrons in the final state. The early Field & Feynman [2] cascade model for quark fragmentation into mesons is depicted in Fig. 2 side by side with a later field-theoretic definition in terms of a bilocal operator [3]. At leading twist accuracy, these describe fragmentation (or decay) functions (now in a narrower sense of the word) that turn a single parton into a single-hadron inclusive state, with fractional momentum transfer  $z$  from the parton to the hadron. Analogous

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\* Presented at the XXXIV International Symposium on Multiparticle Dynamics, Sonoma County, California, USA, July 26–August 1, 2004.

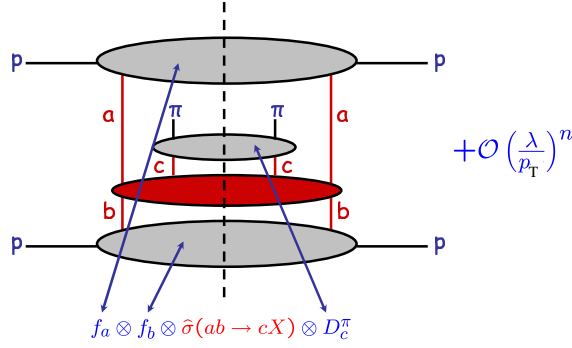


Fig. 1. Factorization in terms of parton densities and fragmentation functions.

to parton densities, renormalization induces evolution of the (transposed) DGLAP type

$$\frac{\partial D_i}{\partial \ln \mu} = \sum_{j=q,\bar{q},g} D_j \otimes P_{ij}. \quad (1)$$

This write-up discusses a few theoretical aspects of fragmentation functions and fragmentation processes. Please confer also Hamacher's contribution to these proceedings [4] for experimental results.

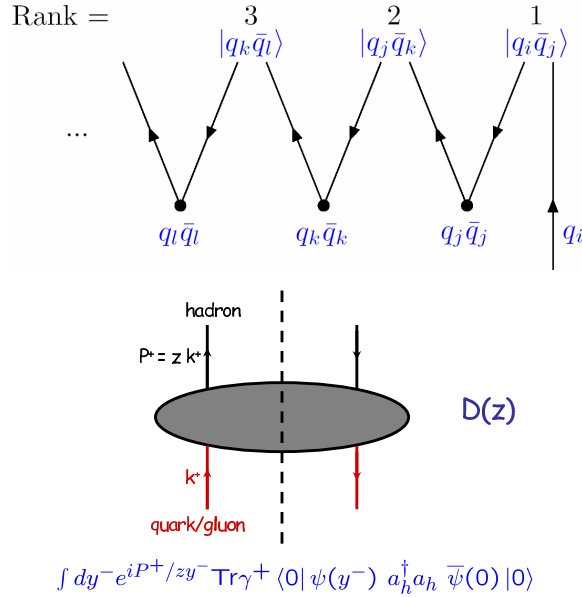


Fig. 2. Field & Feynman model (upper) of cascade fragmentation and bilocal operator (lower) à la Collins & Soper.

## 2. Global analyses of fragmentation functions

Fragmentation functions are determined in NLO QCD fits to data. They are statistically dominated by  $e^+e^-$  annihilations with particularly precise  $e^+e^- \rightarrow hX$  data from  $Z^0$  decays at LEP and SLD. Even though the  $e^+e^-$  data can be reproduced very successfully over a wide range of energy, there are several limitations:

1. The leading order partonic process  $e^+e^- \rightarrow q\bar{q}$  produces quarks only; the subleading terms are too weak to constrain gluon fragmentation sufficiently. Gluon-quark mixing in the evolution Eq. (1) is not enough of a constraint, either.
2. Data are less accurate at large- $z$ . This fact translates into less accurate functions  $D_i(z)|_{\text{large-}z}$ .
3. The individual light flavor channels (corresponding to the “Ranks” in Fig. 2) cannot be disentangled from the flavor inclusive data.

These items limit, at present, the accuracy of the functions  $D_i(z)$ . They require that fragmentation analyses become truly global the way that analyses of parton densities already are, *i.e.* including data from different processes that constrain different kinematic regions and flavor combinations.

Recent global analyses of fragmentation functions, determined at Next-to-Leading-Order accuracy in QCD, were performed in [5]. The individual fits are conceptually very similar, such that a comparison between the fits gives a good first estimate of the uncertainties in the functions  $D_i(z)$ . As explained above, the uncertainties of these functions are data driven and can be large. It can indeed be shown [6] that the electroweak couplings of up and down type quarks at the  $Z^0$  pole suggest that only flavor-insensitive combinations such as the singlet  $\sum_q D_{q+\bar{q}}$  are well determined. Flavor structure can be constrained through the analysis of semi-inclusive DIS [6]. The simple lowest order term

$$d\sigma_{\text{SIDIS}} \propto \sum_q \left[ q(x) D_q(z) + \bar{q}(x) D_{\bar{q}}(z) \right], \quad (2)$$

illustrates that the information is local in  $z$  (no convolution at LO) and that the by now very good knowledge of the parton distributions  $q(x)$  provides reliable flavor weights.

Having discussed item 3, we are still left with items 1, 2 from our shopping list above. They lead over to the analysis of hadroproduction reactions.

### 3. Fragmentation in hadroproduction reactions

#### 3.1. Partonometry at $\sqrt{S_{\text{RHIC}}} = 200 \text{ GeV}$

RHIC ( $pp$  mode) operates at  $\sqrt{S_{\text{RHIC}}} = 200 \text{ GeV}$ , between lower energies (such as ISR or Fermilab), where the pQCD calculations are known to be problematic [7], and higher CMS energies (up to  $\sqrt{S_{\text{Tevatron}}}$ ), where pQCD seems to work [5]. It was quite remarkable to see, how well pion production data in RHIC  $pp$  collisions [8] agree with the NLO QCD calculation [9] at central and forward rapidity and down to  $p_{\perp}$  values as low as  $p_{\perp} \gtrsim 1 \text{ GeV}$ . While it remains a challenge to understand the  $\sqrt{S}$  dependence of hadroproduction spectra in terms of power corrections [10], the good agreement of the pion spectrum with NLO QCD at  $\sqrt{S_{\text{RHIC}}}$  and the good precision of the RHIC data are a gold-mine for fragmentation analyses.

Fig. 3 decomposes RHIC collisions into 2-parton initial and 1-parton-inclusive final states. For Fig. 4 the corresponding scaling variables have been evaluated at their average values, manifesting the symmetric and asymmetric kinematics of central and forward rapidities, respectively. The parton densities are probed in regions of  $x_{1,2}$  where they are well known already whereas new constraints are imposed on the large- $z$  fragmentation functions, and in particular on the gluon function. While the constraint on the gluon fragmentation function is nonlocal (convoluted) in hadroproduction reactions, a QCD analysis of this reaction is more straightforward than an analysis of anti-tagged gluon jets in  $e^+e^- \rightarrow b\bar{b}g$  configurations [4].

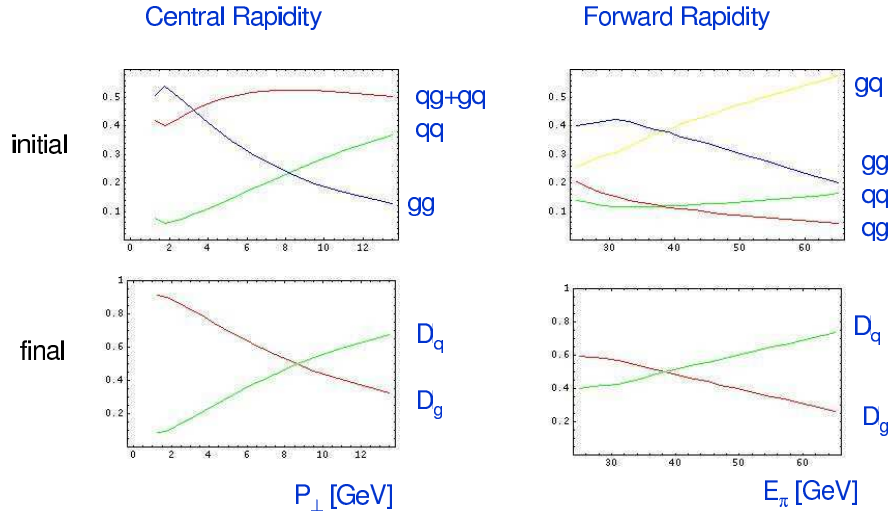


Fig. 3. Partonic decomposition of the initial and final state of  $pp$  collisions at 200 GeV; central and forward rapidity.

The latter are nontrivial due to the fact that jets and fragmentation functions are not the same in QCD.

I will next leave the fragmentation functions and move on to discussing theoretical aspects of two recent pion production measurements at RHIC.

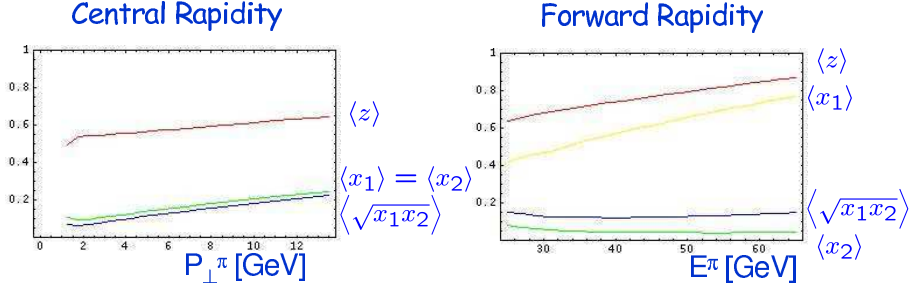


Fig. 4. Scaling variables, averaged over the parton initial and final state of  $pp$  collisions at 200 GeV; central and forward rapidity.

### 3.2. The double-spin asymmetry $A_{LL}^\pi$

The gluon helicity contribution  $\Delta g$  to the proton spin is probed in collisions of longitudinally polarized protons under the condition that the dominant dynamics are perturbative and of leading twist origin. These conditions may apply to the measurement of beam helicity asymmetries

$$A_{LL} = \frac{d\Delta\sigma}{d\sigma} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}} \quad (3)$$

for high transverse momentum reactions where  $d\sigma \equiv d\sigma/dp_\perp|_{\text{large-}p_\perp}$ . For large  $p_\perp$  pion production, a variant of Fig. 1 with polarized beams illustrates that the (glue–glue contribution to the) reaction can schematically be written as

$$A_{LL} \propto (\Delta g_a \otimes \Delta g_b) \otimes \Delta \hat{\sigma}^{ab \rightarrow cX} \otimes D_c^\pi. \quad (4)$$

The Mellin transform

$$\Delta g_a(N) \Delta g_b(N) \Delta \hat{\sigma}^{ab \rightarrow cX}(N) D_c^\pi(N) \quad (5)$$

[with  $\Delta g_a(N) \equiv \int_0^1 x^{N-1} g_a(x)$  etc.] turns the nonlocal (in parton momentum, *i.e.*  $x_a \neq x_b$ ) convolutions

$$\Delta g_a \otimes g_a \dots = \int_{x_T^2}^1 dx_a \int_{x_T^2/x_a}^1 dx_b \Delta g_a(x_a) \Delta g_b(x_b) \dots \quad (6)$$

into products that are local in the moments  $N$ . For small to medium  $p_\perp \lesssim 4 \text{ GeV}$  one can then show [11] by variation  $\delta A_{\text{LL}}/\delta \Delta g = 0$  that  $A_{\text{LL}}^\pi$  has a minimum and is basically positive definite at leading twist, *i.e.*

$$A_{\text{LL}}^\pi|_{p_\perp \lesssim 4 \text{ GeV}} > \mathcal{O}(-10^{-3}) \quad (7)$$

is a necessary condition for leading twist dynamics to be dominant in the production of small to medium  $p_\perp$  pions. To achieve a *slightly* negative  $A_{\text{LL}}^\pi$  of order  $\mathcal{O}(-10^{-3})$  the Mellin inversion of  $\Delta g_{a,b}(N)$  back to  $\Delta g_{a,b}(x_{a,b})$  has to generate a crossing of  $\Delta g(x)$  through zero and explore the fact that the convolution in Eq. (6) can then pick  $x_a \neq x_b$  such that  $\Delta g_a(x_a)\Delta g_b(x_b) < 0$ . For any foreseeable experimental accuracy  $\mathcal{O}(-10^{-3}) \simeq 0$  which is why  $A_{\text{LL}}^\pi$  is “basically” positive definite. At present, data [12] are statistically not conclusive and the positivity of  $A_{\text{LL}}^\pi$  remains yet to be confirmed, or disproved.

### 3.3. Production of $\rho(\rightarrow 2\pi)$ mesons

Recent data [13] indicate that the invariant mass spectrum of pion pairs in  $pp$  collisions peaks at a value around, but significantly below, the  $\rho$  mass as measured in  $e^+e^-$  exclusive production. Now, resonance mass shifts would seem an unlikely manifestation of a perturbative effect if it were not for the fact that the effect extends to large values of transverse momentum  $p_\perp \sim 3 \text{ GeV}$  where the  $\rho$  can be expected to be a parton fragment. The strong decay  $\rho \rightarrow 2\pi$  then contributes a  $p$ -wave resonance to the  $D_i^{2\pi}$  fragmentation function [14]<sup>1</sup>.

In Fig. 1, the fragmenting parton momentum  $k$  runs over the following virtualities and transverse momenta:

$$\int_{M^2/z}^{\hat{s}} dk^2 \int_0^{(1-z)/z(k^2-M^2/z)} dk_\perp^2 \quad . \quad (8)$$

Strictly, Eq. (8) differs from the operator product depicted in Fig. 2 which is defined to be UV-divergent [after integration over  $\int d^4k \delta(k^+ - P^+/z)$ ]. The argument I am about to make, though, is not related to the subtle [15] problems that arise from cutting off momentum integrals.

The partonic center of mass energy  $\hat{s}$  can, for an order of magnitude estimate, be inferred from Fig. 4 through  $\langle \hat{s} \rangle = \langle x_1 x_2 \rangle S_{\text{RHIC}}$ . Under the assumption that the partonic spectrum behaves like  $k^{-2}$ , a Breit-Wigner

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<sup>1</sup> For the semi-quantitative argument we are making here, we will ignore the mixing of  $m$ -hadron fragmentation with  $n$ -hadron ( $n < m$ ) fragmentation, as well as the non-resonant  $s$  wave channel.

peak in the  $2\pi$  mass spectrum — positioned at the  $\rho$  mass for  $p_\perp \rightarrow \infty$  — is then shifted to a lower value by a few 10 MeV around a finite  $p_\perp = 3$  GeV. This suggests a partonic effect that is dual to the pion phase space distortion at low  $p_\perp \rightarrow 0$ . A more detailed evaluation will be given in [16].

#### 4. Conclusions

Parton fragmentation processes explore conceptually and phenomenologically rich aspects of hadronization, as I hope to have demonstrated through the examples of a positivity bound on the spin asymmetry  $A_{LL}^\pi$  and a perturbative ansatz to explain resonance mass shifts at high  $p_\perp$ . I have also outlined the next steps in global analysis of fragmentation functions; a corresponding update will be made available very soon.

Many thanks to M. Grosse-Perdekamp for the invitation and to all organizers of ISMD04, in particular B. Gary, for financial support and a superbly organized workshop. This write-up benefited from discussions and correspondences with A. Bacchetta, K. Barish, S. Brodsky, G. Bunce, P. Fachini, K. Hamacher, E. Laenen, A. Mueller, M. Radici, E. Sichtermann, R. Seto and W. Vogelsang. Work supported by RIKEN-BNL and the U.S. Department of Energy (contract DE-AC02-98CH10886).

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