Vol. 36 (2005) ACTA PHYSICA POLONICA B No 2

FIDELITY AND WILSON LOOP FOR QUARKS IN CONFINEMENT REGION*

V.I. Kuvshinov

JIPNR, National Academy of Science
Acad. Krasin str. 99, 220109 Minsk, Belarus
e-mail: v.kuvshinov@sosny.bas-net.by

and P.V. Buividovich

Belarusian State University
F. Skoriny av. 4, 220050 Minsk, Belarus
e-mail: buividovich@tut.by

(Received September 27, 2004)

Connection between the stability of quantum motion in random fields and quark confinement in QCD is investigated. The analogy between the fidelity and the Wilson loop is conjectured, and the fidelity decay rates for different types of quark motion are expressed in terms of the parameters which are commonly used in phenomenological and lattice QCD.

PACS numbers: 12.38.Aw, 05.45.Mt

The property of quark confinement in QCD is believed to be determined by the presence of chaotic solutions in the spectrum of Yang–Mills equations [1, 2]. In this case it is important to investigate connection between the stability of quark motion in random fields and the property of quark confinement. The stability of quantum motion is usually described in terms of the fidelity, and the confining properties are analyzed using the Wilson loop. The aim of this paper is to demonstrate the similarity between this two quantities in QCD and therefore to reveal the analogy between the stability of quark motion and the quark confinement.

* Presented at the XXXIV International Symposium on Multiparticle Dynamics, Sonoma County, California, USA, July 26–August 1, 2004.

One usually uses Wilson's area law for the Wilson loop as a litmus test for quark confinement. Wilson loop is usually defined as the trace of an averaged multiplicative integral over a closed contour C:

$$\hat{U}(C) = \overline{\hat{P}\exp\left(ig\int\limits_C \left(\hat{A}^0_\mu + \hat{A}^s_\mu\right) dx^\mu\right)}, \quad W(C) = \mathrm{Tr}\left(\hat{U}(C)\right), \qquad (1)$$

where $\hat{A}^0_\mu$ is a determined field, $\hat{A}^s_\mu$ is a random field and averaging is performed over the ensemble of random fields $\hat{A}^s_\mu$. The hat symbol denotes operators in the colour space. If the "area law" holds for the Wilson loop $W(C)$, that is $W(C) \sim \exp(-\sigma S_c)$, S_c being the minimal area of the surface spanned over the contour C, quarks are said to be tied by a string with the constant "tension" σ [1, 2]. In the case of stochastic vacuum in QCD one usually chooses the curvature tensor of gauge field $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - ig\left[\hat{A}_\mu, \hat{A}_\nu\right]_-$ as a random variable, because for such a field gauge invariance is explicitly preserved. As the stochastic vacuum should be colour neutral, here the random field of curvature tensor $\hat{F}_{\mu\nu}$ is assumed to be statistically homogeneous with zero mean value and with correlators proportional to identity in the colour space [1]:

$$\begin{aligned} \overline{\hat{F}_{\mu\nu}} &= 0\,, \quad g^2\overline{\hat{F}_{\mu\nu}(x_1)\,\hat{F}_{\alpha\beta}(x_2)} = \hat{C}_{\mu\nu\alpha\beta}\cdot f(x_1 - x_2)\,, \\ f(0) &= 1\,, \quad \hat{C}_{\mu\nu\alpha\beta} = F^2\hat{I}\,, \quad \int d\xi^2 f(\xi) = l^2_{\mathrm{corr}}\,. \end{aligned} \qquad (2)$$

For the stochastic vacuum the Wilson loop can be explicitly calculated under conditions (2) for the contour sizes considerably exceeding the correlation length l_{corr}. For topologically trivial fields the integral over the contour in (1) can be represented as the integral over the surface spanned over the contour by applying the non-Abelian Stokes theorem [1–3]:

$$\hat{U}(C) = \overline{\hat{P}\exp\left(ig\int\limits_{S(C)} \tilde{F}_{\mu\nu} dS^{\mu\nu}\right)}, \qquad (3)$$

where $\tilde{F}_{\mu\nu} = \hat{U}(x, y)\,\hat{F}_{\mu\nu}(y)\,\hat{U}(y, x)$ is the shifted curvature tensor, $\hat{U}(x, y) = \hat{P}\exp\left(ig\int_y^x \hat{A}_\mu dx^\mu\right)$.

By applying the van Kampen expansion [4] to the integral (3) one can express it in terms of the accumulants of the random field $\tilde{F}_{\mu\nu}$:

$$W(C) = \exp\left(\sum_k i^k \Delta^k[S]\right), \qquad (4)$$

where

$$\Delta^k[S] = \frac{g^k}{k!}\int\limits_S d\sigma_{\mu_1\nu_1}\ldots\int\limits_S d\sigma_{\mu_k\nu_k} D_{\mu_1\nu_1\ldots\mu_k\nu_k}\,,$$

$$D_{\mu_1\nu_1\ldots\mu_k\nu_k} = \mathrm{Tr}\,\overline{\left(\hat{F}_{\mu_1\nu_1}\ldots\tilde{F}_{\mu_k\nu_k}\right)} - \mathrm{Tr}\left(\overline{\hat{F}}_{\mu_1\nu_1}\ldots\overline{\tilde{F}}_{\mu_k\nu_k}\right) - \ldots\,.$$

An efficient estimation for accumulants in (4) can be obtained for sufficiently large contours after taking (2) into account:

$$\Delta^k[S] \approx \frac{g^k}{k!}F^k l_{\mathrm{corr}}^{2(k-1)}S\,. \tag{5}$$

The contribution of the second-order accumulant dominates in the Wilson loop if the condition $\Delta^k[S] \ll \Delta^2[S]$, $k > 2$ holds, or, according to the estimation (5):

$$g\sqrt{\overline{\hat{F}^2}}\,l_{\mathrm{corr}}^2 \ll 1\,. \tag{6}$$

This condition is called the condition of the Gaussian-dominated vacuum in QCD [1, 2]. Under the assumption of the Gaussian-dominated vacuum the final expression for the Wilson loop (1) is:

$$W(C) \sim \exp\left(-\Delta^2[S]\right) \sim \exp\left(-\frac{g^2}{2}l_{\mathrm{corr}}^2 F^2 S\right). \tag{7}$$

Thus the Wilson area law for the Wilson loop holds, and the Gaussian-dominated stochastic vacuum possesses confining properties. The string tension σ is also obtained from (7): $\sigma = \frac{g^2}{2}l_{\mathrm{corr}}^2 F^2$.

Similar to confinement and Wilson loop in QCD, the stability of quantum motion is commonly described in terms of fidelity decay. Fidelity is usually defined as the scalar product of the state vectors of perturbed and unperturbed systems [5, 6]. For the purposes of semiclassical analysis of quark motion without taking spin into account it is convenient to define the fidelity as the scalar product of the state vectors in the colour space:

$$f = \overline{\langle f_1|\, f_2\rangle}\,, \quad |f\rangle_1\,, |f\rangle_2 \in C^3\,, \quad |f_1| = |f_2| = 1\,, \tag{8}$$

where averaging is performed over random perturbations of the system. According to the standard treatment of state vectors in quantum theory it is more natural to average the square of the absolute value of the fidelity $|f|^2$. However for the estimation of the fidelity decay rate such averaging is possible. One can naturally expect the absolute value of the fidelity to decay, the decay rate being approximately equal for almost all "typical" implementations of the random field. Exponential decay of the absolute value of the fidelity for the fidelity values close to unity is described by the linear term in the Taylor expansion: $f(t) \approx 1 - \alpha t$, α being the fidelity decay rate and t being some parameter on which the fidelity depends. Other possible mechanism for the decay of the averaged fidelity is due to the randomly changing phase of the fidelity. However for the estimation of the fidelity decay rate this mechanism can be neglected, as it is possible to show that the fidelity decay in this case is described by the factor $1 - (\alpha t)^2$, and αt is small for the fidelity values close to unity. In this case the averaged fidelity should be close enough to the absolute value of the fidelity for the "typical" implementations of random variables [7, 8].

The first interesting case is the motion of coloured quark in the different paths γ_1 and γ_2 in chaotic environment. The paths start from the point x and join in the point y. In the point x the state vector is $|f_0\rangle$. In the limit of very massive quarks [1, 2] the evolution of the state vector in the colour space is described by the multiplicative integral introduced in (1):

$$
\begin{aligned}
|f_1\rangle &= \hat{P}\exp\left(ig\int\limits_{\gamma_1} \hat{A}_\mu dx^\mu\right)|f_0\rangle = \hat{U}\left(\gamma_1\right)|f_0\rangle \ , \\
|f_2\rangle &= \hat{P}\exp\left(ig\int\limits_{\gamma_2} \hat{A}_\mu dx^\mu\right)|f_0\rangle = \hat{U}\left(\gamma_2\right)|f_1\rangle \ .
\end{aligned}
$$

The operators $\hat{U}(\gamma_1)$ and $\hat{U}(\gamma_2)$ are unitary because $\hat{A}_\mu$ is hermitian. Taking this into account, one can rewrite the expression for the fidelity:

$$
f = \langle f_0|\,\overline{\hat{U}\left(\gamma_1\right)\cdot\hat{U}^{+}\left(\gamma_2\right)}\,|f_0\rangle = \langle f_0|\,\overline{\hat{U}\left(\gamma_1\bar{\gamma}_2\right)}\,|f_0\rangle \ , \tag{9}
$$

where $\gamma_1\bar{\gamma}_2$ is the path obtained by travelling from the point x to the point y in the path γ_1 and back to the point x in the path γ_2, $\hat{U}\left(\gamma_1\bar{\gamma}_2\right) = \hat{P}\exp\left(ig\int\limits_{\gamma_1\bar{\gamma}_2} \hat{A}_\mu dx^\mu\right)$. The averaging in (9) is performed over the random field of the curvature tensor of the gauge field. It is evident that the fidelity is directly related to the Wilson operator $\hat{U}(C)$ introduced in (1). The van Kampen decomposition can be again applied for the estimation of the fidelity (9) under the assumption of Gaussian-dominated colour-neutral vacuum. The final result is:

$$
f \sim \exp\left(-\frac{g^2}{2} l_{\mathrm{corr}}^2 F^2 S\right) . \tag{10}
$$

The error of this estimation is approximately equal to those of the estimation of the Wilson loop, which does not exceed few percent [2]. Thus for the gauss-dominated stochastic vacuum the fidelity for the quark moving in different paths decays exponentially with the area of the surface spanned over the paths, the decay rate being equal to the string "tension" σ. This hints at the close connection between the stability of quark motion and quark confinement.

Another possible situation, which is more close to the standard treatment of the fidelity, is realized when γ_1 and γ_2 are two random paths which are very close to each other. The corresponding expression for the fidelity is similar to (9), but now the averaging is performed with respect to all random paths which are close enough. The final result for the averaged path-ordered integral is obtained in the way similar to (3)–(7), but in this case the field variables are regarded as predetermined. The final expression for the fidelity in this case is:

$$f = \langle f_0 | \, \hat{P} \exp \left(-\frac{g^2 l_{\mathrm{corr}}}{2} \int\limits_{\gamma_1} \tilde{F}_{\chi\alpha} \tilde{F}_{\nu\beta} n^{\chi} \overline{\delta x^{\alpha} \delta x^{\beta}} dx^{\nu} \right) | f_0 \rangle \,, \tag{11}$$

where δx^{α} is the deviation of the path γ_2 from the path γ_1, n^{χ} is the four-dimensional velocity. A rough estimation of the fidelity (11) is:

$$f \sim \exp \left(-\tfrac{1}{2} g^2 l_{\mathrm{corr}} \, F^2 \, \overline{\delta x^2} \, L \right) , \tag{12}$$

where F^2 in this estimation is the trace of the square of the curvature tensor $\hat{F}_{\mu\nu}$, l_{corr} is the correlation length of quark path expressed in terms of world line length, and the length L characterizes the length of the average path both in time and space. For example, if the average path is parallel to the time axis in the Minkowski space, the quark moves randomly around some point in three-dimensional space. The fidelity in this case decays exponentially with time, as would be expected.

Thus the fidelity decay is related to the mechanism of quark confinement, at least for the simple model of Gaussian-dominated stochastic vacuum. The stronger the quarks are coupled to each other, the greater is the fidelity decay rate. The exact proofs and the consistent treatment of this phenomena in quantum field theory require further investigations.

REFERENCES

[1] Yu. A. Simonov, *Usp. Fiz. Nauk* 4 (1996).
[2] D.S. Kuz'menko, Yu.A. Simonov, V.I. Shevchenko, *Usp. Fiz. Nauk* 1 (2004).
[3] Ying Chen, Bing He, Ji-Min Wu, *Mod. Phys. Lett.* **A15**, 1127 (2000).
[4] N.G. van Kampen, *Physica* **74**, 215 (1974).
[5] V.I. Kuvshinov, A.V. Kuzmin, *Phys. Lett.* **A316**, 391 (2003).
[6] T. Prosen, M. Znidaric, *J. Phys. A: Math. Gen.* **34**, L681 (2001).
[7] M. Mehta, *Random Matrices*, 2nd edition, Academic Press Inc., Boston 1991.
[8] R. Durett, *Probability: Theory and Examples*, 2nd edition, Duxbury Press, 1996.