

FLUCTUATIONS AND CORRELATIONS: INTRODUCTION AND OVERVIEW*

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Higher-order correlations have been observed as particle-density fluctuations. Approximate scaling with improving resolution provides evidence for a self-similar correlation effect. Quantum-Chromodynamics branching is a good candidate for a dynamical explanation of these correlations in e^+e^- collisions at CERN/LEP and, as expected, also of those in pp collisions at future CERN/LHC energies. However, also other sources such as identical-particle Bose–Einstein interference effects contribute.

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1. The correlation formalism

We start by defining symmetrized inclusive q -particle distributions

$$\rho_q(p_1, \dots, p_q) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_q(p_1, \dots, p_q)}{\prod_{i=1}^q dp_i}, \quad (1)$$

where $\sigma_q(p_1, \dots, p_q)$ is the inclusive cross section for q particles to be at $p_1, \dots, p_q$, irrespective of the presence and location of any further particles, p_i is the (four-) momentum of particle i and σ_{tot} is the total hadronic cross section of the collision under study. For the case of identical particles, integration over an interval Ω in p -space yields

$$\begin{aligned} \int_{\Omega} \rho_1(p) dp &= \langle n \rangle, & \int_{\Omega} \int_{\Omega} \rho_2(p_1, p_2) dp_1 dp_2 &= \langle n(n-1) \rangle, \\ \int_{\Omega} dp_1 \dots \int_{\Omega} dp_q \rho_q(p_1, \dots, p_q) &= \langle n(n-1) \dots (n-q+1) \rangle, \end{aligned} \quad (2)$$

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where n is the multiplicity of identical particles within Ω in a given event and the angular brackets imply the average over the event ensemble.

Besides the interparticle *correlations* we are looking for, the inclusive q -particle number densities $\rho_q(p_1, \dots, p_q)$ in general contain “trivial” contributions from lower-order densities. It is, therefore, advantageous to consider a new sequence of functions $C_q(p_1, \dots, p_q)$ as those statistical quantities which vanish whenever one of their arguments becomes statistically independent of the others [1–3]:

$$C_2(1, 2) = \rho_2(1, 2) - \rho_1(1)\rho_1(2), \quad (3)$$

$$C_3(1, 2, 3) = \rho_3(1, 2, 3) - \sum_{(3)} \rho_1(1)\rho_2(2, 3) + 2\rho_1(1)\rho_1(2)\rho_1(3), \quad (4)$$

etc. In the above relations, we have abbreviated $C_q(p_1, \dots, p_q)$ to $C_q(1, 2, \dots, q)$; the summations indicate that all possible permutations must be taken. Expressions for higher orders can be derived from the related formulae given in [4]. Deviations of these functions from zero shall be addressed as *genuine* correlations.

It is often convenient to divide the functions ρ_q and C_q by the product of one-particle densities, which leads to the definition of the normalized inclusive densities and correlations:

$$R_q(p_1, \dots, p_q) = \rho_q(p_1, \dots, p_q) / \rho_1(p_1) \dots \rho_1(p_q), \quad (5)$$

$$K_q(p_1, \dots, p_q) = C_q(p_1, \dots, p_q) / \rho_1(p_1) \dots \rho_1(p_q). \quad (6)$$

In terms of these functions, correlations have been studied extensively for $q = 2$. Results also exist for $q = 3$, but usually the statistics (*i.e.* number of events available for analysis) are too small to isolate genuine correlations. To be able to do that for $q \geq 3$, one must apply factorial moments F_q defined via the integrals in Eq. (2), but in limited phase-space cells [5, 6].

2. Density spikes

To see whether it is worth the effort, we first look for density fluctuations in single events, signaling high-order correlations. A notorious JACEE event [7] at a pseudo-rapidity resolution (binning) of $\delta\eta = 0.1$ has local fluctuations up to $dn/d\eta \approx 300$ with a signal-to-background ratio of about 1:1. An NA22 event [8] contains a “spike” at a rapidity resolution $\delta y = 0.1$ of $dn/dy = 100$, as much as 60 times the average density in this experiment.

Bialas and Peschanski [5] suggested that this type of spikes could be a manifestation of “intermittency”, a phenomenon well known in fluid dynamics [9]. The authors argued that if intermittency indeed occurs in particle production, large density fluctuations are not only expected, but should also

exhibit self-similarity with respect to the size of the phase-space volume. Ideas on self-similarity and fractals in jet physics had already been formulated in [10, 11]. For soft hadronic processes, fractals and self-similarity were first considered in [12] and their quantitative measures in [13].

In multiparticle experiments, the number of hadrons produced in a single collision is small and subject to considerable noise. To exploit the techniques employed in complex-system theory, a method had to be devised to separate fluctuations of purely statistical (Poisson) origin, due to finite particle numbers, from the possibly self-similar dynamical fluctuations of the underlying particle densities. A solution, already used in quantum optics [14] and suggested for multiparticle production in [5], consists in measuring $F_q(\delta y)$ in given phase-space volumes (resolution) δy of ever decreasing size.

Note that this approach of explicitly eliminating “trivial” effects is recently being complemented by a more “holistic” approach presented here by Liu Qin [15].

3. Power-law scaling

Besides the property of noise-suppression, high-order factorial moments act as a filter and resolve the large-multiplicity tail of the multiplicity distribution. They are thus particularly sensitive to large density fluctuations at the various scales δy used in the analysis. As shown in [5], a smooth density distribution, which does not show any fluctuations except for the statistical ones, has the property of normalized factorial moments $F_q(\delta y)$ being independent of the resolution δy in the limit $\delta y \rightarrow 0$. On the other hand, if self-similar dynamical fluctuations exist, the F_q obey the power law

$$F_q(\delta y) \propto (\delta y)^{-\phi_q}, \quad (\delta y \rightarrow 0). \quad (7)$$

The powers ϕ_q (slopes in a double-log plot) are related [16] to the anomalous (or co-) dimensions $d_q = \phi_q/(q-1)$, a measure for the deviation from an integer dimension. Equation (7) is a scaling law since the ratio of the factorial moments at resolutions L and ℓ

$$R = \frac{F_q(\ell)}{F_q(L)} = \left(\frac{L}{\ell}\right)^{\phi_q} \quad (8)$$

only depends on the ratio L/ℓ , but not on L and ℓ , themselves.

One further has to stress the advantages of normalized factorial cumulants K_q compared to factorial moments, since the former measure *genuine* correlation patterns.

As an example, high statistics data of the OPAL experiment [17] are given in Fig. 1 in terms of K_q , as a function of the number $M \propto 1/\delta y$ of

phase space partitions for $q = 3$ to 5. In the leftmost column, the one-dimensional rapidity variable y is used for the analysis. The data (black dots) show an increase of K_q with increasing M for small M , but a saturation at larger M . Even though weaker, some saturation still persists when the analysis is done in the two-dimensional plane of rapidity y and azimuthal angle Φ (middle column), but approximate power-law scaling is indeed observed for the analysis in three-dimensional momentum space (right column). Thus, in high-energy collisions, fractal behavior is fully developed in three dimensions, while projection effects lead to saturation in lower dimension.

In Fig. 1, the data are also compared to a number of parametrizations of the multiplicity distributions, as well as to the Monte Carlo models

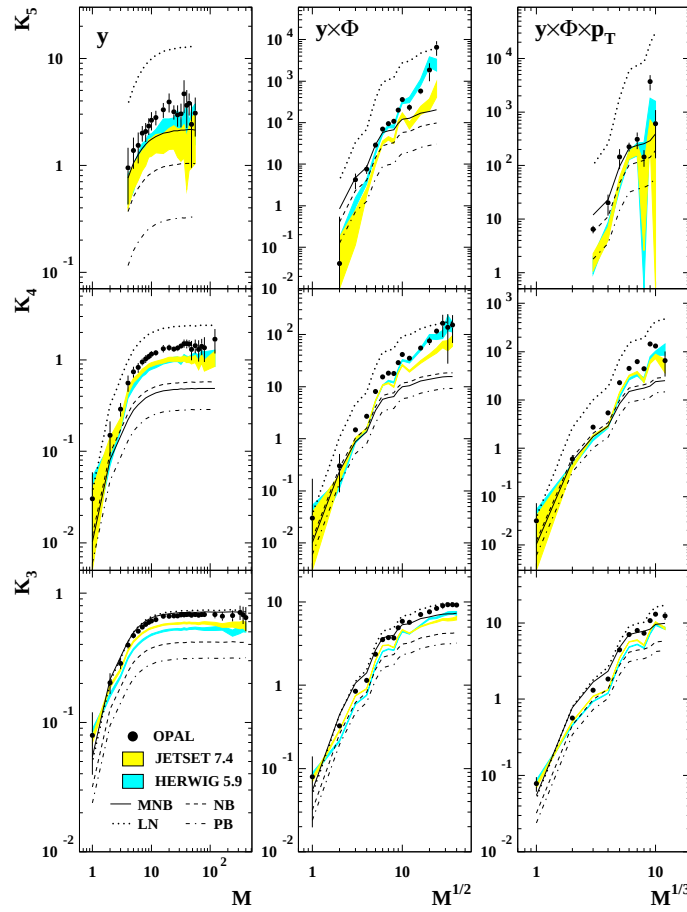


Fig.1. Cumulants of order $q = 3$ to 5 as a function of $M^{1/D}$ in comparison with the predictions of various multiplicity parametrizations and two Monte Carlo models [17].

JETSET and HERWIG. One can see that the fluctuations given by the negative binomial (NB) (dashed line) are weaker than observed in the data. Contrary to the NB, the log-normal (LN) distribution (dotted line) overestimates the cumulants, while these expected for a pure birth (PB) process (dash-dotted) underestimate the data even more significantly than the NB. Among the distributions shown, a modified NB (MNB) gives the best results, even though significant underestimation is observed also there. The Monte Carlo models do surprisingly well.

4. Density and correlation integrals

A fruitful development in the study of density fluctuations is the density and correlation strip-integral method [18]. By means of integrals of the inclusive density over a strip domain in y_1, y_2 space, rather than a sum of box domains, one not only avoids unwanted side-effects such as splitting

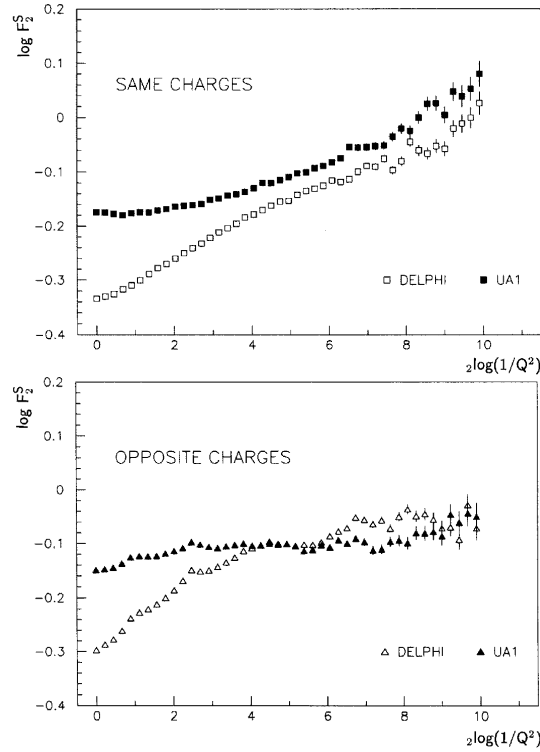


Fig. 2. Comparison of density integrals for $q = 2$ in their differential form ΔF_2 (in intervals $Q^2, Q^2 + dQ^2$) as a function of ${}_2\log(1/Q^2)$ for e^+e^- (DELPHI) and hadron-hadron collisions (UA1) [22].

of density spikes, but also drastically increases the integration volume (and therefore the statistical significance) at given resolution. In terms of the strips (or hyper-tubes for $q > 2$), the density integrals can be evaluated directly from the data after selection of a proper distance measure, as *e.g.* the four-momentum difference $Q_{ij}^2 = -(p_i - p_j)^2$, and after definition of a proper multiparticle topology (GHP integral, [18] snake integral, [19] star integral [20]). Similarly, *correlation* integrals can be defined by replacing the density ρ_q in the integral by the correlation function C_q .

Of particular interest is a comparison of hadron-hadron to e^+e^- results in terms of same and opposite charges of the particles involved. Such a comparison is shown in Fig. 2 for $q = 2$. An important difference between UA1 and DELPHI can be observed in a comparison of the two sub-figures: For relatively large $Q^2 (> 0.03 \text{ GeV}^2)$, where Bose-Einstein effects do not play a major role, the e^+e^- data increase much faster with increasing $-\log Q^2$ than the hadron-hadron results. For e^+e^- , the increase in this Q^2 region is very similar for same and for opposite-sign charges. At small Q^2 , however, the e^+e^- results approach the hh results. For e^+e^- annihilation at LEP at least two processes are responsible for the power-law behavior: Bose-Einstein correlation at small Q^2 following the evolution of jets at larger Q^2 .

5. Multifractal versus monofractal behavior

Anomalous dimensions d_q fitted over the (one-dimensional) range $0.1 < \delta y < 1.0$ are compiled in Fig. 3 [26]. They typically range from $d_q = 0.01$ to 0.1 , which means that the fractal (Rényi) dimensions $D_q = 1 - d_q$ are close to one. The d_q are larger and grow faster with increasing order q in μp and e^+e^- (Fig. 3(a)) than in hh collisions (Fig. 3(b)) and are small and almost independent of q in heavy-ion collisions (Fig. 3(c)). For hh collisions, the q -dependence is considerably stronger for NA22 ($\sqrt{s} = 22 \text{ GeV}$, all p_T) than for UA1 ($\sqrt{s} = 630 \text{ GeV}$, $p_T > 0.15 \text{ GeV}/c$).

In multiplicative cascade models, the one-dimensional moments follow the generalized power law [27]

$$F_q \propto (g(\delta y))^{\phi_q}, \quad (9)$$

where $g(\delta y)$ is a general function of δy . Expressing g in terms of F_2 , one finds the linear relation

$$\ln F_q = c_q + \frac{\phi_q}{\phi_2} \ln F_2, \quad (10)$$

from which the ratio of anomalous dimensions is directly obtained. This has been confirmed by experiment, not only in one dimension, but up to

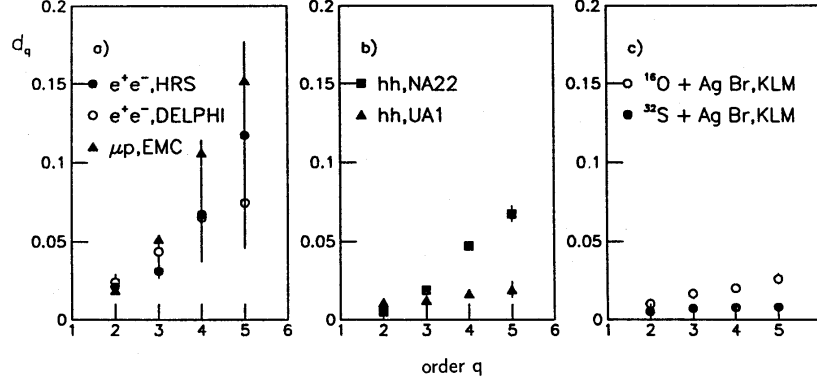


Fig. 3. Anomalous dimension d_q as a function of the order q , for (a) μp and e^+e^- collisions, (b) NA22 and UA1, (c) KLM [26].

3D [28]. Moreover, the ratios ϕ_q/ϕ_2 are found to be largely independent of the dimension of phase space and of the type of collision. The q dependence is indicative of the mechanism causing intermittent behavior. For a (multiplicative) cascade mechanism, in the log-normal approximation (long cascades), the moments satisfy the relation

$$\frac{d_q}{d_2} = \frac{\phi_q}{\phi_2} \frac{1}{q-1} = \frac{q}{2}. \quad (11)$$

However, the use of the Central Limit Theorem for a multiplicative process, such as in the α -model, is a very crude approximation [29] particularly in the tails. As argued in [30], a better description is obtained if the density probability distribution is assumed to be a log-Lévy-stable distribution, characterized by a Lévy index μ . In that case (11) generalizes to

$$\frac{d_q}{d_2} = \frac{1}{2^\mu - 2} \frac{q^\mu - q}{q - 1}. \quad (12)$$

For $\mu = 0$, implying an order-independent anomalous dimension, the multifractal behavior characterized by (11)–(12) reduces to a monofractal behavior [31, 32] with $d_q/d_2 = 1$. This would happen if intermittency were due to a second-order phase transition.

The data are best fitted with a Lévy index of $\mu = 1.6$, but important exceptions exist: While a fit to the combined NA22 data [33] on all variables and dimensions, as well as a weighted average over all individual fits give μ values in rough agreement with those of [28], the 3D-data have $\mu > 2$, not allowed in the sense of Lévy laws. Even larger values of μ , ranging from 3.2 to 3.5, have been found for μp deep-inelastic scattering in [30].

6. Self-affinity versus self-similarity

Comparing log–log plots for one phase-space dimension, one notices that the $\ln F_q$ saturate, but at different F_q values for different variables y , Φ or $\ln p_T$. However, also in three-dimensional analysis the power law is not exact. The 3D hh data even bend upward. It has been shown in [37] that this can be understood by taking the anisotropy of occupied phase space into account. In view of this phase-space anisotropy, also its partition should be anisotropic. If the power law holds when space is partitioned by the same factor in different directions, the fractal is called *self-similar*. If, on the other hand, it holds and only holds when space is partitioned by different factors in different directions, the corresponding fractal is called *self-affine* [38].

If the phase-space structure is indeed self-affine, it can be characterized by a parameter called roughness or Hurst exponent [38], defined as

$$H_{ij} = \frac{\ln M_i}{\ln M_j} \quad (0 \leq H_{ij} \leq 1) \quad (13)$$

with M_i ($i = 1, 2, 3$; $M_1 \leq M_2 \leq M_3$) being the partition numbers in the self-affine transformations $\delta y_i \rightarrow \delta y_i / M_i$, of the phase-space variables y_i . The Hurst exponents can be obtained [37] from the experimentally observed saturation curves of the one-dimensional $F_2(\delta y_i)$ distributions,

$$F_2^i(M_i) = A_i - B_i M_i^{-\gamma_i} \quad (14)$$

as $H_{ij} = (1 + \gamma_j)/(1 + \gamma_i)$. For hh collisions, H_{ij} was indeed determined to be of order 0.5 [39] for the longitudinal-transverse combinations, while it was found consistent with unity within the transverse plane (Φ, p_T) .

The anisotropy is consistent with the fact that the longitudinal direction is privileged over the transverse directions in hadron–hadron collisions. On the contrary, no upward bending is observed in the three-dimensional self-similar analysis of e^+e^- data [40], so the H_{ij} are expected to be compatible with unity. This observation is confirmed with the help of a full self-affine analysis performed with a JETSET 7.4 Monte Carlo sample at 91.2 GeV [41] and a full analysis of L3 data is underway [42] indicating an approximately self-similar behavior for full e^+e^- events, but a self-affine one for single jets.

7. Local fluctuations and QCD

Substantial progress has been made to derive analytical QCD predictions for fluctuations [23–25] in small angular phase-space intervals. Assuming LPHD [43], these predictions for the parton level can be compared to experimental data [44–46]. QCD is inherently intermittent and QCD

predictions [23–25] grant the scaling behavior

$$F_q(\Theta) \propto \left(\frac{\Theta_0}{\Theta} \right)^{(D-D_q)(q-1)}, \quad (15)$$

where Θ_0 is the half opening angle of a cone around the jet-axis, Θ is the angular half-width of a ring around the jet-axis centered at Θ_0 , D is the underlying topological dimension ($D = 1$ for single angle Θ), and D_q are the Rényi dimensions.

A new scaling variable [25], $z = \ln(\Theta_0/\Theta)/\ln(E\Theta_0/\Lambda)$, where the maximum possible region ($\Theta = \Theta_0$) corresponds to $z = 0$, is used in Fig. 4(a). In a fixed coupling regime, for moderately small angular bins,

$$D_q = \gamma_0(Q) \frac{q+1}{q}, \quad (16)$$

where $\gamma_0(Q) = \sqrt{2 C_A \alpha_s(Q)/\pi}$ is the anomalous QCD dimension calculated at $Q \simeq E\Theta_0$, $E = \sqrt{s}/2$, and gluon color factor $C_A = N_c = 3$. This corresponds to the thin solid lines in Fig. 4(a). In the running-coupling regime, for small bins, the Rényi dimensions become a function of the size of the angular ring ($\alpha_s(Q)$ increases with decreasing Θ). Three approximations derived in DLLA are compared in Fig. 4(a), according to (a) [24], (b) [25], (c) [23]. In [24], an estimate for D_q has, furthermore, been obtained in MLLA.

The fixed coupling approximates the running coupling for small z , but does not exhibit the saturation effect seen in the data. For second order, the running- α_s predictions lead to the saturation effects observed in the data, but significantly underestimate the observed signal. Predictions for the higher moments are too low for low values of z , but tend to overestimate the data at larger z . The DLLA approximation differs significantly at large z . The MLLA predictions do not differ significantly from the DLLA result.

Using transverse momentum p_T rather than Θ , within DLLA, the normalized factorial moments of gluons which are restricted as $p_T < p_T^{\text{cut}}$ are expected [47] to follow,

$$F_q(p_T^{\text{cut}}) \simeq 1 + \frac{q(q-1)}{6} \frac{\ln(p_T^{\text{cut}}/Q_0)}{\ln(P/Q_0)}, \quad (17)$$

where P is again the initial energy of the outgoing quark and p_T is defined relative to the direction of this quark.

Again, the DLLA predictions are on the parton level and should be regarded asymptotic, *i.e.* valid at small p_T^{cut} . Therefore, they should be considered only as qualitative predictions when compared to the data in

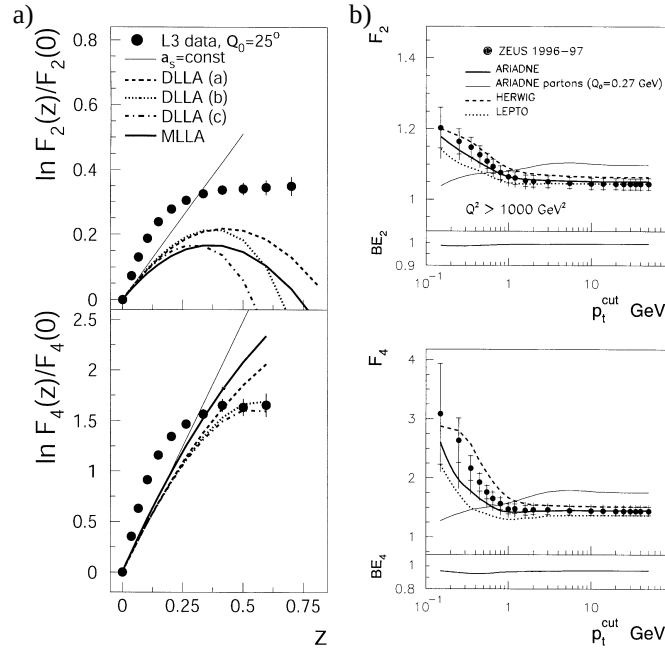


Fig. 4. The L3 data [45] compared to the analytical QCD predictions for $\Lambda = 0.16$ GeV and $\Theta_0 = 25^\circ$: $\alpha_s = \text{const}$ (thin solid line); DLLA (a) [24]; DLLA (b) [25]; DLLA (c) [23]; MLLA [24]. (b) Factorial moments for charged particles in the current region of the Breit frame of e^+p collisions at HERA, as a function of p_T^{cut} , compared to Monte Carlo models at the hadron level (thick lines) and ARIADNE with $Q_0 = 0.27$ GeV at the parton level (thin solid line). The data are corrected for Bose–Einstein correlations by the BE factor indicated [48].

conjugation with the LPHD hypothesis. Such a comparison has been made by ZEUS [48] (see Fig. 4(b)). While DLLA (Eq. (17)) predicts the moments to approach unity from above as p_T^{cut} decreases, the data show the opposite. The Monte Carlo models follow the trend of the data, with ARIADNE giving the best overall description.

To check the effect of energy-momentum conservation, the moments were also determined at the parton level of ARIADNE, the physics implementation of which strongly resembles the analytic calculations [47]. To satisfy LPHD, the cut-off parameter Q_0 was reduced to 0.27 GeV, also ensuring the parton multiplicity to equal that of the hadrons. The results are given as the thin solid line in Fig. 4(b). They indeed show the behavior expected from Eq. (17), *i.e.*, they disagree with the hadronic data. Analogous differences between the hadron and parton levels of ARIADNE have been observed

in e^+e^- annihilation [47]. So, one has to conclude with the authors that here the limits of LPHD are crossed, *i.e.* the F_q are particularly sensitive to dynamical details of non-perturbative QCD.

8. Bose–Einstein correlations

Whether derived as Fourier transform of a (static and chaotic) pion source distribution, a covariant Wigner-transform of the (momentum dependent) source density matrix, or from the string model, identical-pion correlation leads to a positive, non-zero two-particle correlator $K_2(Q)$ (see Eqs. (9) and (10)), *i.e.* to

$$R_2(Q) = 1 + K_2(Q) > 1 \quad (18)$$

at small four-momentum difference Q . These Bose–Einstein Correlations, by now, are a well-established effect in all types of collisions, even in hadronic Z^0 decay (for recent reviews see [49, 50]) originally expected to be too coherent to show an effect. If existent also as inter- W BEC in fully hadronic WW decay at LEP2, this could serve as an important laboratory for research on the behavior of two (partially) overlapping strings. The status of this is reviewed here by Todorova [51].

Other important recent observations are given in abstract form below.

1. When evaluated in two (or better three) dimensions in the Bertsch–Pratt system, an *elongation* of the emission region (better region of homogeneity [52]) is observed along the event axis in all types of collisions (hadron–hadron [53], all four LEP experiments [54], ZEUS [55], RHIC [56]). However, it is important to note that the longitudinal radius of homogeneity is much shorter than the length of the string (of order 1%).

The recent observation that the out-radius does not grow beyond the side-radius at RHIC [56] points to a short duration of emission and causes a problem for some hydrodynamical models [57], but not for *e.g.* the Buda–Lund hydro model. The latter, in fact gives a beautifully consistent description of single-particle spectra and BEC in hadron–hadron and heavy-ion collisions at SPS and RHIC [58]. The emission function resembles a Gaussian shaped fire-ball for AA collisions, but a fire-tube for hh collisions.

2. The form of the correlator at small Q is steeper than Gaussian, in fact consistent with a power law as would be expected from the intermittency phenomenon described above. Recent unifying progress is reported here by Csörgő [59].

3. The approximate transverse mass $m_T^{-1/2}$ scaling first observed in heavy-ion collisions at the SPS [60] and usually blamed on collective flow, is now observed at RHIC [56], but also in e^+e^- collisions [61]. Quite generally, it follows from a strong position momentum correlation [62], be it due to collective flow or to string fragmentation.

4. *Genuine three-pion correlations* exist in all types of collisions and, in principle, allow a phase to be extracted from

$$\cos \phi \equiv \omega(Q_3) = K_3(Q_3)/2\sqrt{K_2(Q_3)}. \quad (19)$$

At small Q , this ω is near unity (as expected from incoherence) for hh [63] and e^+e^- [64] collisions, as well as for PbPb [65,66] and AuAu [67] collisions at SPS and RHIC, while it is near zero (compatible with full coherence) in collisions of light nuclei [65]. This contradiction can be solved [49,68] if ω is interpreted as a ratio of normalized cumulants (Eq. (10)). Since $K_q^{(N)}$ of N independent overlapping sources gets diluted like $1/N^{q-1}$, ω would be reduced if strings produced by light ions (or in WW decay!) do not interact. If, in *heavy* ion collisions, the string density gets high enough for them to coalesce, some kind of percolation sets in (see also the talks of Dias de Deus [69] and Ferreiro [70]) and full inter-string BEC gets restored.

5. Azimuthal anisotropy is now also observed in configuration space of non-central heavy-ion collisions at AGS energies [71], but also at RHIC [72]. Contrary to elliptic flow, it is directed out of the event plane, but consistent with the elliptic nuclear overlap in a non-central collision. Due to larger pressure in the event plane, the anisotropy gets reduced but not destroyed at RHIC. Also this is evidence for a short duration of pion emission.

Since a reaction plane also exists in hA , hh , and three-jet e^+e^- collisions, application to those would be interesting. Of course, a three-dimensional (*e.g.* Bertsch-Pratt) analysis in bins of azimuthal angle requires a very high statistics. For hA collisions, this will hopefully soon become available from HERA-B (see Mureşan [73]).

9. Summary

Multiparticle production in high-energy collisions is an ideal field to study genuine higher-order correlations. They are directly accessible in their full multi-dimensional characteristics, under well controlled experimental conditions. Methods also used in other fields are being tested and extended here for general application. Indications for genuine, approximately self-similar higher-order correlations are indeed found in high-energy particle collisions. At large four-momentum distance Q^2 , they are not only expected to be an inherent property of perturbative QCD, but are directly related

to the anomalous multiplicity dimension and, therefore, to the running coupling constant α_s . At small Q^2 , the QCD effects are complemented by Bose–Einstein interference of identical mesons carrying information on the unknown space-time development of particle production during the collision. The interplay between these two mechanisms, important for an understanding of the process of hadronization, is a particular challenge at the moment.

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