Vol. 36 (2005) ACTA PHYSICA POLONICA B No 2

MULTIPLICITY FLUCTUATIONS IN HADRON–HADRON AND NUCLEUS–NUCLEUS COLLISIONS AND PERCOLATION OF STRINGS*

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(Received October 12, 2004)

We argue that recent NA49 results on multiparticle distributions and fluctuations, as a function of the number of participant nucleons, suggest that percolation plays an important role in particle production at high densities.

PACS numbers: 12.38.Mh, 13.85.Ni, 25.75.Nq, 24.85.+p

Recently, the NA49 collaboration has presented results, from the experiment CERN/SPS at 158 A GeV, on multiplicity fluctuations or, to be more precise, on $V(n)/\langle n\rangle$,

$$V(n)/\langle n\rangle \equiv \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle}, \tag{1}$$

as a function of the number $N_{\rm part}$ of participant nucleons, from pp to PbPb collisions [1].

These data are very interesting for several reasons:

(1) They show evidence for universal behaviour: the experimental points in the plot $V(n)/\langle n\rangle$ *versus* $N_{\rm part}$ fall into a unique curve (see Fig. 1).

* Presented at the XXXIV International Symposium on Multiparticle Dynamics, Sonoma County, California, USA, July 26–August 1, 2004.

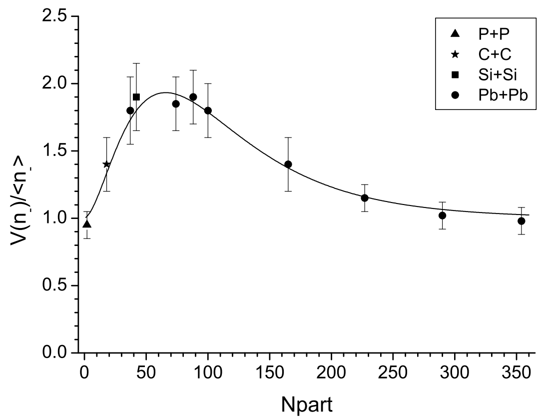

Fig. 1. Variance over average multiplicity, for negative particle production, as a function of the number of participants. The curve is from (5′) with (12), (16) and (22). Data are from NA49 [1].

(2) The physics in the small $N_{\rm part}$ limit ($pp, N_{\rm part} \to 2$) and in the large $N_{\rm part}$ limit (PbPb, $N_{\rm part} \to 2A_{\rm PbPb}$) seems to be quite the same, as in both cases the quantity (1) approaches 1. The fluctuations are larger in the intermediate $N_{\rm part}$ region (see Fig. 1).

(3) The (negative) particle distribution, in the low density and in the high density limits, is in fact a Poisson distribution (see Fig. 2), the distribution being wider than Poisson in the intermediate $N_{\rm part}$ region.

In the framework of the string model with percolation [2], these results are quite natural. On one hand, percolation is a universal geometrical phenomenon, the properties depending essentially on the space dimension (dimension 2, impact parameter plane, in our case), and being controlled by the transverse density variable η,

$$\eta \equiv \left(\frac{r}{R}\right)^2 \bar{N}_{\rm S}\,, \tag{2}$$

where r is the transverse radius of the string ($r \simeq 0.2$ fm), R the radius of the interaction area, and $\bar{N}_{\rm S}$ the average number of strings. The quantity $(R/r)^2$ is nothing but the interaction area in units of the string transverse area. As R and $\bar{N}_{\rm S}$ are functions of the number $N_{\rm part}$ of participants, $N_{\rm part}$, similarly to η, becomes, at a given energy, a universal variable.

On the other hand, in percolation [3], what matters is the fluctuation in the size of the clusters of strings: one starts, at low density (small $N_{\rm part}$), from a situation where strings are isolated, at intermediate density one finds

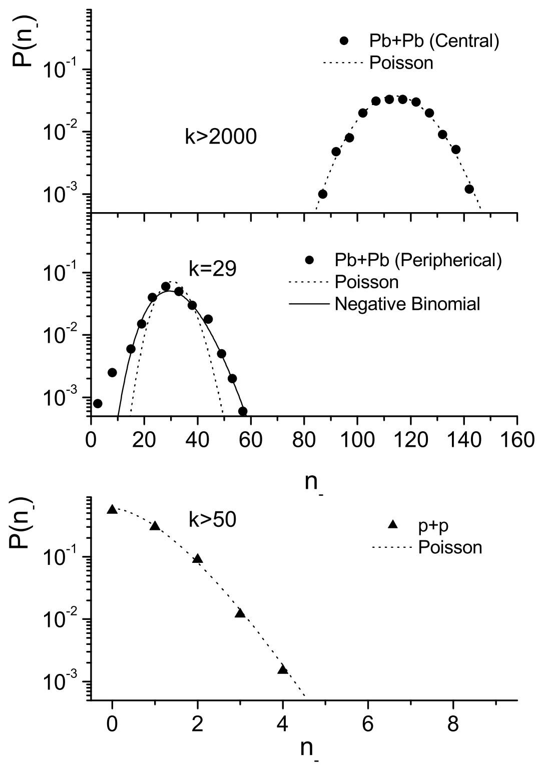

Fig. 2. Multiplicity Distributions, $P(n_-)$, as a function of n_-. The curves are Poisson (dashed lines) and Negative Binomial (full line).

clusters of different sizes, and one ends up, at high density, above the percolation threshold, with a single large cluster. In both, low and high, density limits, fluctuations in cluster size vanish (see Fig. 3). In the simplest string model the particle distribution is Poisson (as observed in e^+e^- and pp at low energy) and $V/\langle n \rangle \to 1$ in both, low and high, density limits (see Figs. 1 and 2).

Let us try to be more specific. In hadron–hadron and nucleus–nucleus collisions, during the collision strings are produced along the collision axis, and these strings may overlap and form clusters of different sizes. In the spirit of percolation theory, we shall assume that fluctuations in the number N of strings per cluster dominate over fluctuations in the number $N_{\rm C}$ of clusters.

From our calculations [4] we obtain:

$$\langle n \rangle = \bar{N}_{\rm C} \langle N \rangle \bar{n} \,, \tag{3}$$

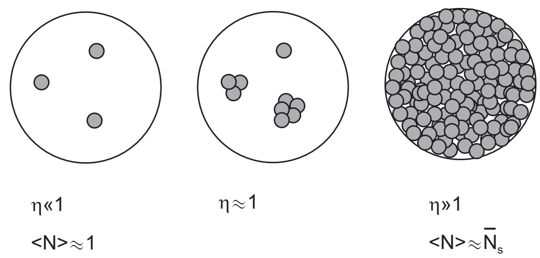

Fig. 3. Impact parameter percolation. For small densities ($\eta \leq \phi$) and for large densities ($\eta \gg 1$) there are no strong fluctuations in the number N of strings per cluster. For intermediate densities ($\eta \simeq 1$) N-fluctuations are large.

and

$$\langle n^2 \rangle - \langle n \rangle^2 = \bar{N}_{\mathrm{C}} \left[(\langle N^2 \rangle - \langle N \rangle^2)\bar{n}^2 + \langle N \rangle (\bar{n}^2 - \bar{n}^2) \right] , \tag{4}$$

where $\bar{n}$ is the single string particle multiplicity, and, finally, for the (variance)/(multiplicity) ratio,

$$\frac{V(n)}{\langle n \rangle} = \bar{n}\frac{V(N)}{\langle N \rangle} + 1 , \tag{5}$$

where

$$\frac{V(N)}{\langle N \rangle} \equiv \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} . \tag{6}$$

In order to have agreement between (5) and Fig. 1, it is required that

$$\frac{V(N)}{\langle N \rangle} \underset{\eta \to 0}{\longrightarrow} 0 \quad \text{and} \quad \frac{V(N)}{\langle N \rangle} \underset{\eta \to \infty}{\longrightarrow} 0 . \tag{7}$$

At low density ($\eta \to 0$, few strings in the interaction area) $\langle N \rangle \simeq 1$ and $\langle N^2 \rangle - \langle N \rangle^2 \simeq 0$, and condition (7), for low η, is satisfied.

At high density, statistical models without percolation, of the kind of "coins-in-boxes" (fixed size clusters or boxes) [5, 6] $\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle$ and (7) is not satisfied for $\eta \to \infty$. In order to satisfy (7), as $\eta \to \infty$, one needs percolation. In fact, in percolation, at high density (above the percolation threshold, η_{c}), one forms a single cluster with all the strings and $\langle N \rangle \simeq \bar{N}_{\mathrm{S}}$, and $\langle N^2 \rangle - \langle N \rangle^2 \simeq 0$.

We shall next develop a simple and general percolation model. If N stands for the number of strings in a cluster, N_{C} the number of clusters and N_{S} the number of strings, two sum rules follow:

$$\bar{N}_{\mathrm{C}} \langle N \rangle = \bar{N}_{\mathrm{S}} , \tag{8}$$

and

$$\bar{N}_{\mathrm{C}}\langle A\rangle = \left(\frac{R}{r}\right)^2 (1-e^{-\eta}) \,, \tag{9}$$

where $\langle A\rangle$ is the average area occupied by a cluster (see, for instance, [3]). In the usual "coin-in-boxes" model $\langle A\rangle$ is fixed — the size of the box — independent of η. In the percolation model $\langle A\rangle$ increases with η, approaching the full area of interaction in the $\eta \to \infty$ limit.

We thus write for $\langle A\rangle$, [7],

$$\langle A\rangle = f(\eta)\left[\left(\frac{R}{r}\right)^2 (1-e^{-\eta}) - 1\right] + 1 \,, \tag{10}$$

where $f(\eta)$ is a percolation function, such that $f(\eta) \to 0$, as $\eta \to 0$, and $\langle A\rangle \to 1$, as expected for isolated strings, and $f(\eta) \to 1$, as $\eta \to \infty$, and $\langle A\rangle \to (\frac{R}{r})^2$ as expected in percolation. For $f(\eta)$ we have chosen

$$f(\eta) = (1 + e^{-(\eta-\eta_{\mathrm{c}})/a})^{-1} \,, \tag{11}$$

with $a = 0.85$ and $\eta_{\mathrm{c}} = 1.15$ [7].

From (8), (9) and (10) we obtain

$$\langle N\rangle = \frac{\eta}{1-e^{-\eta}}\left(f(\eta)\left[\left(\frac{R}{r}\right)^2 (1-e^{-\eta}) - 1\right] + 1\right) , \tag{12}$$

with $\langle N\rangle \to 1$ as $\eta \to 0$, and $\langle N\rangle \to \bar{N}_{\mathrm{S}}$, as $\eta \to \infty$.

Regarding the cluster string variance, $\langle N^2\rangle - \langle N\rangle^2$, we have to satisfy the general constraint,

$$\langle N^2\rangle - \langle N\rangle^2 \geq 0 \,, \tag{13}$$

and the, already mentioned, constraints,

$$\langle N^2\rangle - \langle N\rangle^2 \underset{\eta \to 0}{\longrightarrow} 0 \,, \tag{14}$$

and

$$\langle N^2\rangle - \langle N\rangle^2 \underset{\eta \to \infty}{\longrightarrow} 0 \,. \tag{15}$$

We thus write for the variance,

$$V(N) \equiv \langle N^2\rangle - \langle N\rangle^2 = \left[\frac{1}{b}\frac{1-(1+b\eta)e^{-b\eta}}{e^{b\eta}-1}\right]\langle N\rangle^2 \,, \tag{16}$$

where $b > 0$ is an adjustable parameter (fixed at the value $b = 1.65$). Note that (16) satisfies (13), (14) and (15).

Before making a comparison between our string percolation model and NA49 data, there are two questions to be addressed:

(i) $F(\eta)$ factor due to random colour summation

When strings fuse in a cluster the effective colour charge is not just the sum of the colour charges of the individual strings [8]. In practice, the effective number N of strings is reduced, [9],

$$N \longrightarrow \sqrt{\frac{\langle A\rangle}{\langle N\rangle}}N \longrightarrow F(\eta)N \ , \tag{17}$$

where (see (8) and (9)),

$$F(\eta) = \sqrt{\frac{1-e^{-\eta}}{\eta}} \ , \tag{18}$$

such that, instead of (3) and (5), have

$$\langle n\rangle = F(\eta)\bar{N}_{\mathrm{C}}\langle N\rangle\bar{n} = F(\eta)\bar{N}_{\mathrm{S}}\bar{n} \ , \tag{3'}$$

and

$$\frac{V(n)}{\langle n\rangle} = F(\eta)\bar{n}\frac{V(N)}{\langle N\rangle} + 1 \ . \tag{5'}$$

Note that the $F(\eta)$ correction is more important for $\langle n\rangle$, (3′), then for Eq. (5′). If the single particle distribution is Poisson with average multiplicity $\bar{n}$, the average cluster has also a Poisson distribution with multiplicity $F(\eta)\langle N\rangle\bar{n}$.

(ii) The relation between η and N_{part}

In the definition of η (2), what appears is not N_{part} but rather the average number $\bar{N}_{\mathrm{S}}$ of strings and the radius R of interaction. Making use of simple nuclear physics and multiple scattering arguments, one has [10]

$$R \simeq R_1 N_A^{1/3} \ , \tag{19}$$

and

$$\bar{N}_{\mathrm{S}} \simeq \bar{N}_{\mathrm{S}}^{p} N_A^{4/3} \ , \tag{20}$$

where R_1 is the nucleon radius ($\simeq$ 1fm), $\bar{N}_{\mathrm{S}}^{p}$ is the (energy dependent) number of strings in pp collisions, at the same energy, and N_A is given by

$$N_A = \frac{N_{\mathrm{part}}}{2} \, . \tag{21}$$

We would like to mention that Eqs. (19) and (20) are not rigorous: in (19) geometrical factors are not taken into account, in (20) no distinction is made between valence strings and sea strings, [10].

From (2), (19), (20) and (21) we obtain for the relation between η and $N_{\rm part}$,

$$\eta = \left(\frac{r}{R_1}\right)^2 \bar{N}_{\rm S}^p N_A^{2/3} \tag{22}$$

We shall now present our results:

1. $V(n)/\langle n\rangle$

In Fig. 1 we show our curve, Eq. (5′) with (12), (16) and (22), in comparison with NA49 data. The obtained values for $\bar{n}(\bar{n} = 0.12)$ and $N_{\rm S}^p(N_{\rm S}^p = 4.5)$ are consistent with the additional constraint $\langle n\rangle_p \simeq 0.52$ (as seen in Fig. 2).

In Fig. 2 we show fits of the multiplicity distributions for different values of $N_{\rm part}$, with Negative Binominals. Reasonable fits are obtained with values for the NB parameter: $k = \infty$, Poisson, at low and high density, and $k = 29$, for intermediate density.

2. $\langle n\rangle_{N_A}$ and $\langle n\rangle_p$

We can relate $\langle n\rangle_{N_A}$ to $\langle n\rangle_p$ by making use of (3') and (22):

$$\langle n\rangle_{N_A} = \frac{F(\eta_{N_A})}{F(\eta_p)}\langle n\rangle_p N_A^{4/3} \tag{23}$$

we first note that (25) satisfies saturation as seen at RHIC[10] when $N_A \to \infty$:

$$\frac{1}{N_A}\langle n\rangle_{N_A} \underset{N_A\to\infty}{\longrightarrow} \text{const.} \tag{24}$$

Relation (23) is valid for high energy. At low energy – $\sqrt{s} \simeq 20\,\text{GeV}$ is the energy at SPS — the presence of valence quarks cannot be ignored. We take them into account by writing, instead of (23)

$$\langle n\rangle_{N_A} = \frac{F(\eta_{N_A})}{F(\eta_p)}\langle n\rangle_p N_A^{4/3}\left[1 - c\left(1 - 1/N_A^{1/3}\right)\right] , \tag{23′}$$

where c is a parameter decreasing with energy, $1 \geq c \geq 0$, measuring the relative contribution of valence quarks to multiplicity,

$$c \equiv \frac{\langle n\rangle_p^V}{\langle n\rangle_p} . \tag{25}$$

In Fig. 4 we show (23′) in comparison with NA49 data.

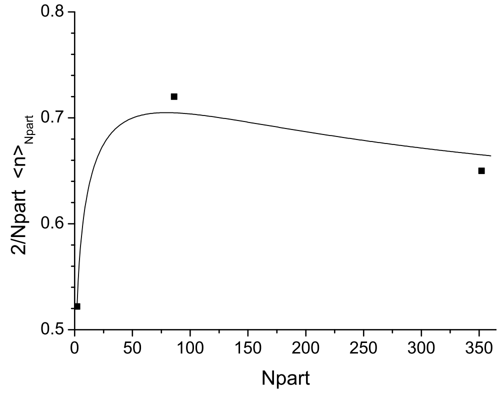

Fig. 4. The average multiplicity divided by $1/2N_{\mathrm{part}}$ as a function of N_{part}. The curve corresponds to Eq. (23'). Multiplicities were calculated from the distributions of Fig. 2. For c was taken the value $c = 0.53$.

In conclusion, we find that the recent NA49 results, regarding the multiplicity distribution dependence on the number of participant nucleons are quite consistent with the impact parameter, percolation description of hadron–hadron and nucleus–nucleus collisions at high energies and high densities.

We would like to thank Elena Ferreiro, Carlos Pajares and Roberto Ugoccioni for many discussions. We would like to thank P. Seyboth and M. Rybiczyński for information on NA49 data. This work has been done under the contract POCTI/36291/FIS/2000, Portugal.

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