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PROBING HADRON-PRODUCTION PROCESSES BY USING NEW STATISTICAL METHODS TO ANALYZE DATA*

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It is pointed out that the powerful statistical methods introduced by Bachelier and Mandelbrot in Economics, and those introduced by Hurst and Feder in Marine Sciences, can be readily used to examine fluctuation phenomena in hadron-production processes. Evidences for the existence of non-Gaussian stable, stationary, scale invariant distributions, fractal dimensions, and the validity of Hurst's empirical law are presented. Since none of the observed features is directly related to the basis of the conventional physical picture, it is not clear whether (and if yes, how and why) these striking empirical regularities can be understood in the framework of the conventional picture including QCD.

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1. Introduction and motivation

The talk is a brief summary of two recent papers [1, 2] written in collaboration with Prof. Meng Ta-chung. What you see in this talk are the results obtained in a series of *preconception-free* data-analyses. The *purpose* of these analyses is to extract useful information on the reaction mechanism(s) of hadron-production processes, directly from experimental data. The *methods* we used to perform such analyses are borrowed from other sciences, namely, Mandelbrot's approach in Economics [3], and Hurst's R/S analysis in Marine Sciences [4].

First of all, we see a great need for preconception-free data-analyses! This is because the currently popular ways of describing hadron-production processes are based either on a "Three-Step Scenario" in terms of parton-momentum distributions, pQCD, and parton-fragmentation functions, or on a "Two-Component Picture" in which the fluctuations in every experimental distribution are separated by hand into a "pure statistical part" and a part

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which is considered to be physical. Under this assumption, the method of Bialas and Peschanski [5] is used to calculate factorial moments. The common goal of these conventional approaches (for a recent review in which various experimental and theoretical aspects of hadron-production processes are discussed, see Ref. [6]) seems to be the following: Explain all the details about the reaction mechanism(s). The price one has to pay for such details is, however, the large number of inputs (assumptions, adjustable parameters, *etc.*) which one must make in order to do calculations. Hence, a rather natural question is: Do we really need all these inputs, if we only wish to know the key features of such hadron-production processes?

2. Fluctuation

It is well-known that fluctuations are of considerable importance in non-equilibrium as well as in equilibrium systems. There are several reasons for this. One of them is that fluctuation studies provide a natural framework for understanding a large class of phenomena, among which the best known one is "Brownian motion" or "random walk".

Bachelier, a then young French student, was the first [7] who used the idea of random walk to study fluctuations. It was in year 1900, which is five years earlier than Einstein's [8]. Bachelier's Gaussian hypothesis says: (1) price changes, $z(t+T)-z(t)$, are independent random variables; (2) these changes are approximately Gaussian. The mathematical basis of Bachelier's theory is the classical central limit theorem [9], but a fatal defect of his theory is that the empirical data are *not* Gaussian.

In Mandelbrot's well-known 1963-paper [3], he pointed out an important observation namely that the variances of the empirical distribution of price changes,

$$L_{\mathrm{M}}(t,T)=\ln z(t+T)-\ln z(t), \tag{1}$$

can behave as if they were infinite. An immediate result of this observation is that the Gaussian distribution in Bachelier's approach should be replaced by a family of limiting distributions called *stable distributions* which contain Gaussian as the only member with *finite* population variance. The mathematical basis of this result is the generalized central limit theorem [9], and the main advantage of Mandelbrot's approach is that the empirical data conform best to the non-Gaussian members of stable distributions.

To study fluctuations in subnuclear reactions, we consider the two well-known JACEE events [10] as examples. They exhibit significant fluctuations of multiplicities in rapidity distributions. In analogy with Mandelbrot's $L_{\mathrm{M}}(t,T)$, we introduce the quantity:

$$L(\eta,\Delta\eta)=\ln\frac{dN}{d\eta}(\eta+\Delta\eta)-\ln\frac{dN}{d\eta}(\eta)\,, \tag{2}$$

where $dN/d\eta$ is a measurable quantity, and $\Delta\eta$ can be integer times of the resolution power which is 0.1 in JACEE events. Under the assumption that these $L(\eta, \Delta\eta)$'s are identically distributed random variables, we examine their resulting distributions by using the data for the two JACEE events (usually known as JACEE1 and JACEE2) and show that the obtained distributions are *stable, stationary, and scale invariant*.

To study the space-time properties of such fluctuations, we introduce, in analogy with rapidity, a quantity l which we call "locality"; and its corresponding "pesudolocality" is

$$\lambda = \frac{1}{2} \ln \frac{r - x_{||}}{r + x_{||}} \,. \tag{3}$$

The uncertainty principles lead, in particular, to

$$\Delta\lambda\Delta\eta \sim \text{constant}\,, \tag{4}$$

which is useful for the further discussion of scale invariance in space-time.

For stability test, we make use of the fact that a non-degenerate random variable X is stable, if and only if for all integers $m > 1$, there exist constants $c_m = m^{1/\alpha}$ with $\alpha \in (0, 2]$ and $d_m \in R$ such that

$$S_m \equiv X_1 + X_2 + \cdots + X_m \stackrel{d}{=} c_m X + d_m\,, \tag{5}$$

where X_i's are independent, identical copies of X. Let $X \equiv L(\eta, \Delta\eta)$, and for the convenience of data-analyses we use $c_m^{-1}(S_m - d_m) \stackrel{d}{=} L$ to check whether the variable is stable. Here, $\{L_1, L_2, \cdots, L_m\}$ stands for a m-dimensional random variable the components of which can be considered as independent. It is seen (see Figs. 5 and 6 of Ref. [1]) that the above-mentioned c_m's can be readily found, and thus the two sets of $L(\eta, \Delta\eta)$ obtained from the two JACEE events are indeed *stable random variables*.

Stationarity expresses the invariance principle with respect to time. Hence in hadron-production processes, the property of stationarity manifests itself in the sense that the $L(\eta, \Delta\eta)$'s obtained from the η-distribution measured at different times (or time-intervals) have the same statistical properties. It is seen (*cf.* Fig. 7 of Ref. [1]) that the tail distributions of JACEE1 and JACEE2 are very much the same. The fact that these two events occurred at different times; in reactions at different energies; by using different projectiles and targets, makes the observed similarity particularly striking.

For scale invariance test, we propose to evaluate the running sample variance

$$S_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} [L(\eta_i, \Delta\eta) - \bar{L}_n(\eta, \Delta\eta)]^2 \tag{6}$$

of JACEE-data and then plot their frequency distributions. (See Figs. 12 and 13 of Ref. [1].) From the straight-line structure the scale invariance property is evident. In contrast, we also consider two sets of sample values of a standard Gaussian random variable as many as those in the two JACEE events, and wish to see in particular also how their running sample variances behave. The results are plotted in the same figures (see Figs. 12 and 13 of Ref. [1]) as those of JACEE cases for comparison. Here we see clearly that for such variables with finite variance, there exists a scale, which is in sharp contrast to the power-law structure of JACEE events. Combined with the result that $L(\eta, \Delta\eta)$ is stable, we are led to the conclusion that *it is not only stable but also non-Gaussian*.

3. Correlations

In order to find out, whether/how the $L(\eta, \Delta\eta)$'s and/or the $dN/d\eta(\eta)$'s are (statistically) co-related with one another, we propose to follow Feder [11], Mandelbrot and his collaborators [12], and apply Hurst's *rescaled range analysis* (also known as R/S analysis) to the rapidity distributions of the two above-mentioned JACEE events. The obtained results (see Fig. 1 of Ref. [2]) can be summarized as follows: First, the Hurst's empirical law and the scaling behavior are valid for $dN/d\eta(\eta)$'s with universal features of $H = 0.9$ for both JACEE events, where H is the Hurst exponent. Second, the Hurst exponent $H(y_i)$ is independent of the selected starting point y_i. Third, the fact that $H = 0.9 > 0.5$ shows the existence of global statistical dependence and thus global structure in the two data sets. Furthermore, we see that not only the divider (or trail) dimension $D_{\mathrm{T}} = 1/H$, but also the self-affine properties of the system of produced hadrons together with its associated fractal dimension $D_{\mathrm{G}} = 2 - H$ can be readily determined, once the corresponding Hurst exponents are found. Further studies along this line are underway.

4. Concluding remarks

The results obtained from the preconception-free data-analyses performed in Refs. [1] and [2] have led us to the following conclusions: First, the fact that non-Gaussian stable distributions which are stationary and scale-invariant describe the existing data remarkably well calls for further attention, and it would be very helpful to have a comparison with data taken at other energies and/or for other collision processes. Second, the validity of Hurst's empirical law with the same exponent for the two JACEE events is not only another example for the existence of universal features in the complex system of produced hadrons, but also implies the existence of global statistical dependence and thus the existence of global structure between the different parts of the system. Third, the fact that the extremely robust quantities such as the frequency distribution of running sample variance and the rescaled range R/S obey universal power-laws which are independent of the colliding energy, independent of the colliding objects, and independent of the size of the rapidity intervals, strongly suggests that the system under consideration has *no intrinsic scale* in space-time. Finally, since none of the above-mentioned features can be directly related to the basis of the conventional picture, it is not clear whether, and if yes how and why, these striking empirical regularities can be understood in terms of the conventional approaches, including QCD.

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