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RESUMMED EVENT SHAPES AT HADRON COLLIDERS*

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We present recently defined jet-observables for hadron–hadron dijet production, which are designed to reconcile the seemingly conflicting theoretical requirement of globalness, which makes it possible to resume them (automatically) at NLL accuracy and the limited experimental reach of detectors, so that they are measurable at the Tevatron and at the LHC.

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1. Introduction

Event shapes and jet-rates are infrared and collinear (IRC) safe observables, which describe the energy and momentum flow of the final state. They constitute an ideal compromise between simplicity and sensitivity to properties of QCD radiation. They provide then a wealth of information, *e.g.* in measurements of the coupling $\alpha_{\rm s}$ and its renormalisation group running, in cross checks/measurements of the values of the colour factors of QCD and, most importantly, in studies of the connection between parton-level (the perturbative (PT) description of quarks and gluons) and hadron-level (the real), for a review see [1].

IRC-safety ensures that event-shape distributions can be computed within perturbation theory, however in the more exclusive phase space region where perturbative radiation is suppressed (conventionally associated to almost vanishing values of the observable, $V \ll 1$) large logarithmic corrections need to be resummed to all orders. More specifically, given an event-shape V, a function of all secondary final state momenta $\{k_1, \dots, k_n\}$ and of the set of recoiling Born momenta $\{\tilde{p}\}$, the probability of "constrained

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events" *i.e.* $V(\{\tilde{p}\}, k_1 \ldots k_n) < v$ has a *divergent* PT expansion for $v \to 0$

$$\Sigma(v) \equiv \text{Prob}\,(V < v) = 1 + \sum_{m\leq 2n} R_{n,m} {\alpha_\text{s}}^n \ln^m v + \ldots\,,$$

i.e. there is a soft and collinear divergence $[\rightsquigarrow \ln v]$ for each emitted gluon. Today's state-of-the-art accuracy accounts for all Leading (LL) and Next-to-Leading Logarithms (NLL) as follows:

$$\Sigma(v) = \exp\{\underbrace{Lg_1(\alpha_\text{s}L)}_{\text{LL}} + \underbrace{g_2(\alpha_\text{s}L)}_{\text{NLL}} + \ldots\}\,, \qquad L \equiv \ln\frac{1}{v}\,. \tag{1}$$

Furthermore, resummations are matched to fixed order results at NLO.

2. Basics of resummation

The main ingredient in resummations is factorisation. Usually two steps are needed to obtain a resummed prediction. The first task is to exploit angular ordering to reduce n-parton matrix elements to a QED-like factorised form, *i.e.* to the product of independent emissions from the hard scattering partons only. Schematically, *e.g.* for $e^+e^- \to 2\,$jets this can be written as

$$w_{p\bar{p}}(k_1,\ldots,k_n) = \frac{1}{n!}\prod_{i=1}^{n} w_{p\bar{p}}(k_i) \sim \frac{1}{n!}\prod_{i=1}^{n}\frac{\alpha_\text{s}C_\text{F}}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}\,, \tag{2}$$

with corrections which contribute beyond NLL to $\Sigma(v)$. The second step requires some analytical understanding of the observable's behaviour in the presence of soft-collinear emissions in order to factorise its definition via Mellin transforms, *e.g.* for the thrust T in $e^+e^- \to 2\,$jets one has $(Q = \sqrt{s}/2)$

$$1 - T \simeq \frac{1}{Q}\sum_{i=1}^{n}\frac{E_i\theta_i^2}{2} \quad\longrightarrow\quad \Theta(1 - T < \tau) = \int \frac{d\nu}{2\pi i\nu}\, e^{\nu\tau}\prod_{i=1}^{n} e^{-\nu\frac{E_i\theta_i^2}{2Q}}\,. \tag{3}$$

By combining Eqs. (2) and (3) one obtains the resummed answer

$$\Sigma(\tau)\int\frac{d\nu}{2\pi i\nu}\, e^{\nu\tau}\exp\left[\int\frac{d\theta}{\theta}\frac{dE}{E}\frac{\alpha_\text{s}(E\theta)C_\text{F}}{\pi}\left(e^{-\nu\frac{E_i\theta_i^2}{2Q}} - 1\right)\right]\,, \tag{4}$$

where the effects due to the running of the coupling have been included.

Despite the simplicity of the ideas underlying resummations, some technicalities cannot be avoided. Indeed, at NLL accuracy, care is needed to treat properly the emission of hard collinear or soft large-angle gluons and the inclusive gluon splittings. Furthermore, it is known that for some observables, when seeking NLL accuracy, the multi-parton matrix-element cannot be factorised as in Eq. (2), notably this is the case for non-global observables [2]. In a similar way some observables, such as jet rates in Jade-like algorithms [3], do not exponentiate, so that it is not possible to write the distribution in the form in Eq. (1). Also, the observable's factorisation itself can be non trivial especially for multi-jet observables like the 3-jet limit of thrust minor, which involves five integral transforms [4].

3. Automated resummed predictions

The observation that the origin of logarithmic enhancement in jet-shape distributions is always the same (*i.e.* it is due to the radiation of soft-collinear gluons) and that the approximations which lead to resummed expressions such as Eq. (4) are very similar for different observables, made it natural to investigate the possibility of developing a general and automated approach to resummation, which does not require analytical treatment of observable specific multiple emission effects via Mellin/Fourier transforms.

Such an approach is feasible, and has been implemented in the computer code CAESAR (Computer Automated Expert Semi-Analytical Resummer) [5]. This program differs from usual Monte Carlos whose task is generally the numerical evaluation of integrals via generation of random events.

CAESAR instead is based on a generic master resummation formula, presented and derived in detail in [5]. This formula applies to a well-defined class of observables, whose requirements can be summarised as follows.

- Given a Born event, when just one soft emission k is radiated collinear to the Born parton p_ℓ, the observable should behave as

$$V(\tilde{p}, k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)\,, \tag{5}$$

  where, for each hard leg ℓ, a_ℓ, b_ℓ, d_ℓ are some numbers and $g_\ell(\phi)$ is a regular function parameterising the azimuthal dependence.

- The observable should be *continuously global*, meaning that it should be *sensitive to emissions everywhere* and the *transverse momentum dependence should be uniform* ($a_1 = \ldots = a_n = a$).

- The observable should be *recursive infrared and collinear (r-IRC) safe*, meaning that *the addition of emissions which are much softer or more collinear should not drastically change the value of the observable.* This new concept was introduced and illustrated in detail [5].

Given the general master formula, together with it's applicability conditions, Caesar works as an expert system, which in a first step establishes whether the observable is within its scope. It's next task is to determine the inputs needed for the evaluation of the master formula, basically the coefficients and functions $a_\ell, b_\ell, d_\ell, g_\ell(\phi)$ together with a NLL correction function $\mathcal{F}$ which accounts for effects due to multiple emissions [5,6]. Finally as a last trivial step, Caesar evaluates the master formula (integrating over different Born configurations when necessary).

Notice that the first two steps are critical: they require high precision arithmetic [7] to take asymptotic (soft & collinear) limits and they follow methods of "Experimental Mathematics" [8] to validate or falsify hypothesis.

Currently implemented processes are 2 & 3 jets in e^+e^-, [1+1] & [1+2] jets in DIS, Drell–Yan + 1 jet, hadron–hadron dijet events. Results as obtained with Caesar have been tested against all known NLL global event shape resummations in e^+e^-, DIS and Drell–Yan. In hadronic dijet events no event-shape resummations had ever been carried out, so there was no standard definitions of event shapes, as in e^+e^- and DIS. In the following we will therefore show how to define observables at hadronic colliders which reconcile the seemingly conflicting theoretical requirement of globalness and the limited experimental reach of detectors. Indeed in hadron–hadron collisions the presence of radiation from the initial state, together with the limited coverage close to the beam region (usually expressed in terms of a maximum rapidity $\eta_{\max}$) makes this a critical issue.

4. Observables in hadron–hadron dijet events

4.1. Directly global observables

Measurements for this class of observables include ideally all particles in the event, *i.e.* the maximum rapidity $\eta_{\max}$ is taken as large as experimentally possible. One then defines variants of the usual e^+e^- observables, *e.g.*:

- The transverse thrust
$$T_{\rm T} \equiv \max_{\vec{n}_{\rm T}} \frac{\sum_i |\vec{q}_{\perp i} \cdot \vec{n}_{\rm T}|}{\sum_i q_{\perp i}}\,,$$
where $q_{\perp i}$ are the transverse momenta with respect to the beam axis and $\vec{n}_{\rm T}$ defines the transverse thrust axis.

- The thrust minor
$$T_m = \frac{\sum_i |q_i^{\rm out}|}{\sum_i q_{\perp i}}\,,$$
here $q_i^{\rm out}$ are the momentum-components out of the event-plane (*i.e.* that containing the beam axis and the $\vec{n}_{\rm T}$-axis).

NLL resummations are then valid as long as $\ln(1/v) < (a+b_{\min})\eta_{\max}$ [9]. Explicit results for the differential distributions show that most of the data lie in this region [10].

4.2. Observables with recoil term

The drawback of observables presented in Sec. 4.1 is that they require measurements quite far in the forward region. While this is possible at the Tevatron (where $\eta_{\max} \sim 3.5$), only calorimeter information is available in forward regions, whose resolution is quite low. A way to circumvent this is to define measurements which explicitly select a restricted, central region and to modify the observable's definition so as to make it indirectly sensitive to emissions in the unobserved phase space region. This sensitivity is typically achieved exploiting recoil effects. The idea is essentially that momentum conservation ensures that the *vectorial* sum of transverse momenta in the central region, exactly balances the vectorial sum of transverse momenta in the unobserved forward regions. The definition of this type of observables proceeds then as follows. One first selects a central region $\mathcal{C}$ (*e.g.* taking all particles with $\eta < \eta_0 \sim 1$), one then defines a central, non-global observable, *e.g.* a central thrust minor

$$T_{m,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}} \sum_{i\in\mathcal{C}} |q_i^{\rm out}| \,, \qquad Q_{\perp,\mathcal{C}} = \sum_{i\in\mathcal{C}} |\vec{q}_{\perp i}| \tag{6}$$

and adds to it a recoil term, *e.g.*

$$\mathcal{R}_{\perp,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}} \left| \sum_{i\in\mathcal{C}} \vec{q}_{\perp i} \right| . \tag{7}$$

The recoil enhanced thrust minor is then given by

$$T_{m,\mathcal{R}} \equiv T_{m,\mathcal{C}} + \mathcal{R}_{\perp,\mathcal{C}} \,. \tag{8}$$

These observables are often named indirectly global observables, since they are indirectly sensitive of emissions in the unobserved region through recoil effects. The drawback of these observables is that in the region of extremely small V the NLL function $\mathcal{F}$, parameterising multiple emission effects [6], has a divergence. The presence of this divergence is well understood, and is not specific of indirectly global observables. A divergence occurs whenever, due to cancellation between contributions from different emissions, an observable can be very small in the presence of radiation. The origin of the divergence can be understood with simple arguments: the master resummation formula assumes that the mechanism responsible for keeping the value of the observable small is a LL Sudakov effect. However, in the phase space region of very small V, it is more likely that V is small because of bi-dimensional cancellations in $\mathcal{R}_{\perp,\mathcal{C}}$. The NLL function $\mathcal{F}$ cannot compensate a wrong LL behaviour and manifests this with a divergence. Despite the presence of the divergence, explicit results show that most of the distribution is in a region which is under control of NLL resummations [10]. What makes these observables challenging experimentally, is the accuracy with which the recoil term can be measured, indeed $\mathcal{R}_{\perp,\mathcal{C}}$ is subject to big cancellations between almost equal and opposite large transverse momenta.

4.3. Observables with exponentially suppressed forward terms

A variant of directly global observables is to define a class of observables with exponentially suppressed forward terms.

One introduces the mean transverse-energy weighted rapidity $\eta_{\mathcal{C}}$ and total transverse momentum of a given central region $\mathcal{C}$

$$\eta_{\mathcal{C}} = \frac{1}{Q_{\perp,\mathcal{C}}} \sum_{i\in\mathcal{C}} \eta_i \, q_{\perp i} \,, \qquad Q_{\perp,\mathcal{C}} = \sum_{i\in\mathcal{C}} q_{\perp i} \tag{9}$$

and with particle in the forward region one defines an exponentially suppressed forward term

$$\mathcal{E}_{\bar{\mathcal{C}}} = \frac{1}{Q_{\perp,\mathcal{C}}} \sum_{i\notin\mathcal{C}} q_{\perp i} \, e^{-|\eta_i - \eta_{\mathcal{C}}|} \,, \tag{10}$$

then *e.g.* the thrust minor with exponential forward suppression is

$$T_{m,\mathcal{E}} = T_{m,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}} \,. \tag{11}$$

These observables are intended to have some of the better features of both purely global and indirectly global observables. Theoretical predictions are not affected by divergences and from an experimental side these observables are optimal in that there is no need for a fine resolution in rapidity and azimuth in the forward region, and calorimeter information should be enough to determine the forward term $\mathcal{E}_{\bar{\mathcal{C}}}$.

5. Final considerations

Despite the fact that observables with exponentially suppressed forward terms seem to be more suitable in many respects, we point out that observables in different classes have complementary sensitivities to perturbative and non-perturbative (NP) radiation, so that the *simultaneous study* of a number of observables within each class is a powerful tool to investigate properties of QCD radiation, *e.g.* for:

- *studies of underlying event*, since the forward sensitivity (to beam-fragmentation) can be specifically tuned, suppressing it for purely PT studies, or deliberately enhancing it when studies NP effects. This allows one to test quantitative, as well as qualitative features of existing models, *e.g.* the sensitivity of the underlying event to the partonic channel (*i.e.* $qq \to qq$, $qg \to qg$ or $gg \to gg$);
- *studies of hadronization corrections in multi-jet events*, in particular for tests of power-corrections beyond the "Feynman Tube model" [11, 12];
- *studies of the non-trivial quantum evolution of colour*, indeed the novel perturbative QCD colour evolution structure that arises in events with 4-jet topology [13] has never been investigated experimentally before.

Given the much more challenging environment in hadronic collisions, compared to e^+e^- and DIS, we believe that only an automated approach can make such a programme feasible. A number of other possible measurements are given in [10], while results for the automated analysis and resummation of a large number of observables are available from `http://qcd-caesar.org`.

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