FRAGMENTATION @ LEP*

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Recent results on fragmentation obtained by the LEP experiments are discussed centering on the comparison of gluon and quark fragmentation.

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1. Energy dependence of hadron production in e^+e^-

The LEP collaborations have completed the analysis of charged hadron production for the LEP II running [1–4]. The increase of the charged hadron multiplicity with energy is mainly due to gluon radiation off the initial quarks. It is well described by fragmentation models as well as by perturbative predictions. In both cases the absolute normalization is determined from a fit to the data. The comparison of the stable hadron fragmentation functions (FFs) at $\sqrt{s} \sim 200$ GeV and the Z pole shows clear evidence for scaling violations. Current parameterizations [5] of the FFs tend to underestimate the amount of scaling violations [1]. At small hadron momentum particle production is reduced due to soft gluon coherence. The results for the energy evolution of the maximum of the $\xi = -\ln x = -\ln 2E_h/E_{\rm CM}$ distribution is well consistent with the MLLA expectations.

2. Comparing gluon and quark fragmentation

Beside the study of quark fragmentation the presence of 3-jet ($q\bar{q}g$) events in e^+e^- annihilation allows to study gluon fragmentation. These analyses require to identify partons and jets at "tree level". Quarks and gluons are identified by energy ordering and/or heavy quark tagging. Limited experimental purities can be considered by unfolding methods.

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Assuming massless jets the kinematic is determined by the angles θ_i ($\theta_1 < \theta_2 < \theta_3$) between the jets and the CM energy of the event. The jet with index i is situated opposite to the corresponding angle θ_i and $E_1 > E_2 > E_3$. The study of dynamical dependencies requires the specification of evolution scales. In general jet evolution in 3-jet events is a two scale problem, however, for the low energy jets the transverse energy

$$\kappa = E_{\text{jet}} \sin \frac{\theta}{2} \tag{1}$$

with respect to the closest colour connected jet proves to be the relevant scale [6]. Here θ is the opening angle between both jets. In inclusive e^+e^- annihilation the beam energy corresponds to κ.

The assignment of particles to jets involves jet algorithms and in consequence implies ambiguities. In practice several strategies are used to mitigate these ambiguities: for multiplicities only the one of full events [7–9] or of event hemispheres are studied [10]. Alternatively fully symmetric topologies are employed [11] which eventually require boost algorithms [12]. In view of FFs fast hadrons are least effected by effects of jet selection.

2.1. Quark and gluon fragmentation functions

Recently a comprehensive study of FFs for inclusive, light and b-quarks as well as for gluons was presented [4]. Besides the Z data this analysis bases on the high energy (LEP II) data. Inclusive results and results obtained from jets in 3-jet events are compared. Thereby a large CM energy range spanning from 10 GeV to 200 GeV for quarks and from 10 GeV to almost 100 GeV for gluons is covered by the analysis. The inclusive quark results are similar to those sketched already in Sect. 1.

The results obtained for quarks in 3-jet events are widely consistent with measurements in low energy e^+e^- annihilation and available FF parameterizations [5] (*i.e.* also with the extrapolation of the high energy data). In the region of overlap there is also consistency with an older similar measurement of gluon and quark FFs [6] and previous results for gluons [10]. It turns out that the parameterizations of the gluon FF agree far less well with the data than in the quark case. This is especially so at large values of the scaled momentum $x = E_h/E_{\text{jet}} \geq 0.5$. Here the log energy slope in the data is stronger than in the parameterizations. This is troubling as the slope is a direct consequence of QCD evolution.

The validity of the 3-jet analysis is tested in the study by comparing to inclusive $q\bar{q}$ and gg results in the Monte Carlo. Also here a discrepancy is present which is strongest for gluons at low scales (κ). Consequently the slope in the 3-jet analysis is bigger as in the inclusive case. A previous

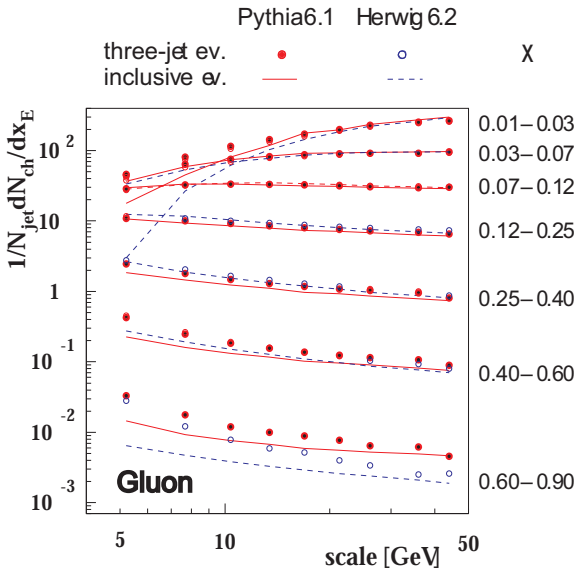


Fig. 1. Comparison of the inclusive gg FF to that obtained with the 3-jet analysis in the Pythia and Herwig model (by OPAL).

analysis [11] at small scales showed a similar discrepancy for the gluon FF, however, not for other quantities like the g multiplicity.

The observed discrepancy between data and parameterization as well as the inconsistency of the Monte Carlo study perturbs the faith in the present measurements of the gluon FF. This is especially so as the measurement using leading gluons [10] due to its small statistics provides little information at high x and the measurement obtained from the longitudinal FF are weakened by strong non-perturbative effects [13].

During the preparation of this talk it turned out that the observed discrepancy can be understood due to the limited experimental resolution of the jet energy. The definition of the FF uses $x = E_{\mathrm{hadron}}/E_{\mathrm{parton}}$ whereas in the measurement from 3-jets E_{parton} is replaced by E_{jet}. Although the analysis and the implied unfolding procedures warrant that in average $E_{\mathrm{parton}} = E_{\mathrm{jet}}$ (for any given topology) this is not so event by event. There is a (topology dependent) smearing of the jet energy which is *not* accounted for in *any* existing 3-jet analysis. This smearing implies biases of distributions strongly depending on E_{jet}, *i.e.* also of the FF which is implicitly E-dependent via x.

For symmetric events a study of the angular smearing between parton and jet axis showed an about constant resolution of $\sim 3^\circ$ [14]. This leads about to an inverse power law behaviour $\propto 1/E$ of the smearing of the jet energy with respect to the parton energy. Using simple error propagation and an analytical parameterization of the gluon and quark FFs [6] it was

then possible to qualitatively verify the deviations observed by the OPAL Monte Carlo study. The smaller effect for the quark FF is due to the smaller fall-off of the FF at high x compared to the gluon case.

A proper measurement of the FFs from 3-jet events requires the unfolding of the jet energy resolution effects. It is unlikely that the LEP experiments will be able to still perform this correction as it requires a reanalysis of the data. Alternatively the quoted effects can be considered in the FF parameterization fits by a smearing of the jet energy according to the resolution taken from Monte Carlo models.

2.2. Coherent particle production perpendicular to the 3-jet event plane

Soft gluons emitted at large angles due to their large "wave length" provide only little sensitivity to the colour structure of the underlying hard partons. Therefore soft hadrons emitted perpendicular to the event plane of 3-jet events are well suited to study effects due to colour coherence. A LO prediction for the ratio of gluon emission perpendicular to a $q\bar{q}g$ (3-jet) to a $q\bar{q}$ (2-jet) ensemble reads [15]:

$$\frac{N_\perp^{q\bar{q}g}}{N_\perp^{q\bar{q}}} = \frac{C_\mathrm{A}}{C_\mathrm{F}} \cdot r_\mathrm{t} = \frac{C_\mathrm{A}}{C_\mathrm{F}} \cdot \frac{1}{4}\left[\widehat{q\,g} + \widehat{\bar{q}\,g} - \frac{1}{N_C^2}\widehat{q\,\bar{q}}\right], \qquad \widehat{i\,j} = 2\sin^2\frac{\theta_{ij}}{2}. \quad (2)$$

The antenna terms $\widehat{i\,j}$ describe the emission from the individual quark–gluon dipoles. The contribution $\propto 1/N_C^2$ is due to destructive interference. The term r_t is sensitive to the topology of the underlying partons and — for fixed CM energy — represents the "scale" of soft gluon emission. Eq. (2) is remarkable as soft gluon emission in 3-jet events is directly proportional to the colour factor ratio $C_\mathrm{A}/C_\mathrm{F}$ and the topological scale r_t.

The prediction Eq. (2) has been compared to the charged hadron multiplicity observed in cones of 30° opening angle situated perpendicular to the event plane of 3-jet events [16]. Events were selected with the angular ordered Durham algorithm at fixed $y_\mathrm{cut} = 0.015$ in order to distinguish 2-, 3- and 4- or more jet events. The scale r_t can be determined from the inter-jet angles θ_i assuming massless kinematics. For general 3-jet events (at fixed CM energy) two angles suffice to describe the event topology. Mirror symmetric events (defined here by $\theta_3 - \theta_2 < 5°$) are specified by a single angle.

Fig. 2 (upper) shows the topology dependence of the cone multiplicity in 3-jet events compared with the expectation of Eq. (2). The normalisation is fixed by the corresponding multiplicity in cones perpendicular to the event axis in 2-jet events. The outer error bars beside the statistical error include systematic uncertainties of the data-to-theory comparison obtained from the variation of the jet algorithm, y_cut and the cone opening angle.

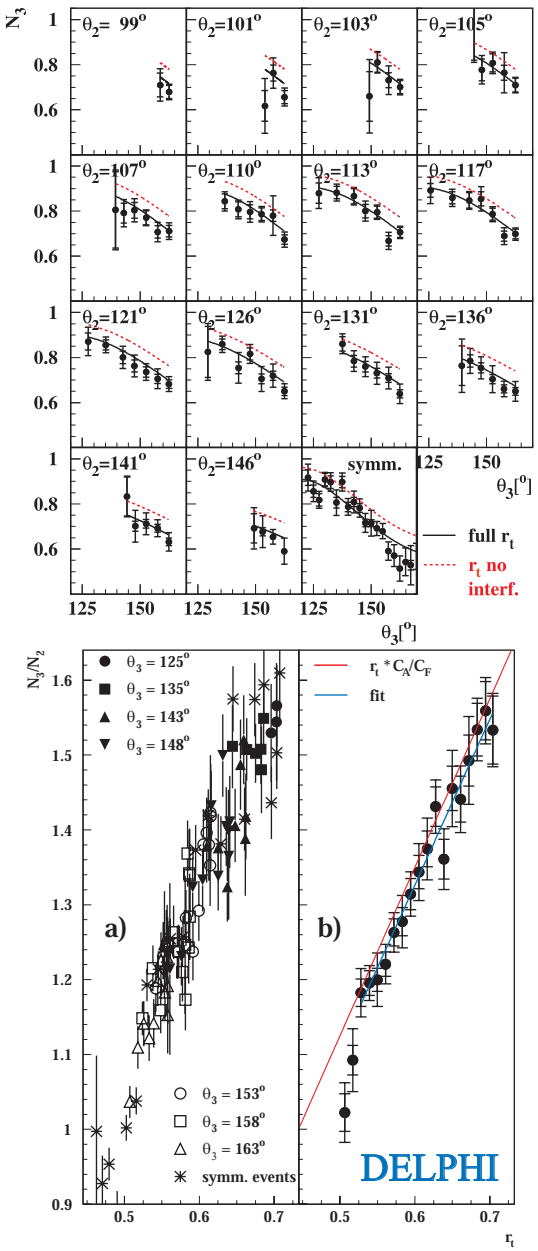


Fig. 2. Comparison of the multiplicity in cones $\perp$ to the 3-jet event plane with the expectation Eq. (2). Upper: as function of the angles θ_2 and θ_3. Lower: (a) as a function of r_t and θ_3, (b) as function of the averaged r_t.

The absolute LO prediction of Eq. (2) describes the data very well. Omission of the interference term leads to an overestimate of the multiplicity. Fitting an amplitude factor k multiplied to the interference term to the data yields for general 3-jet topologies:

$$k = 1.37 \pm 0.05(\text{stat.}) \pm 0.33(\text{syst.}) \qquad \frac{\chi^2}{Ndf} = 1.2\,. \tag{3}$$

The result is in agreement with the expectation $k = 1$ but differs significantly from 0. This implies a direct verification of destructive gluon interference representing a purely quantum mechanical effect beyond the classical probabilistic parton-shower picture.

The sensitivity of Eq. (2) to $C_\mathrm{A}/C_\mathrm{F}$ is best verified when binning the data as function of r_t (see Fig. 2 (lower)). At first this representation is verified in Fig. 2 (a) where the data is additionally binned as function of θ_3. The full line in Fig. 2 (b) representing Eq. (2) is in perfect agreement with the data. A homogenous straight line fit in the indicated r_t-range yields for the slope:

$$2.182 \pm 0.009(\text{stat.}) \pm 0.055(\text{syst.}) \qquad \text{for general 3-jet topologies.}$$

This value agrees amazingly well with the LO prediction $C_\mathrm{A}/C_\mathrm{F} = 2.25$.

2.3. Topology dependence of the multiplicity of 3-jet events

In the limit of high energy the ratio $r = N_{gg}/N_{q\bar{q}}$ of the hadron multiplicity of colour singlet gg and $q\bar{q}$ systems is predicted to resemble the QCD colour factor ratio $C_\mathrm{A}/C_\mathrm{F}$ as the colour factors can be interpreted as the colour charge of the gluon and quark, respectively. The multiplicity is severely affected by non-perturbative effects, however. Therefore, it has been suggested to measure the ratio of the derivatives of the gluon and quark multiplicity with respect to the energy, $r^{(1)} = N'_{gg}/N'_{q\bar{q}}$ [17]. A MLLA calculation relates these derivatives [12]. Consequently the integration of this relation leads to a constant of integration which can be fixed by a direct measurement of N_{gg} at $\sqrt{s} \simeq 10$ GeV [18]. The hadron multiplicity of 3-jet events is predicted [19] to be:

$$N_{q\bar{q}g} \;=\; N_{q\bar{q}}(L_{q\bar{q}}, \kappa_\mathrm{Lu}) + \frac{1}{2} N_{gg}(\kappa_\mathrm{Le}) \tag{4}$$

with

$$\kappa_\mathrm{Lu} \;=\; \ln \frac{s_{qg} s_{\bar{q}g}}{s\Lambda^2} = \kappa_\mathrm{Le} + \ln \frac{s_{q\bar{q}}}{s}\,, \qquad L_{q\bar{q}} = \ln \frac{s_{q\bar{q}}}{\Lambda^2}\,. \tag{5}$$

Here coherence effects are accounted for by the proper choice of scales. The second argument of the quark contribution reflects the phase-space restriction of the $q\bar{q}$-system due to the resolution of the gluon jet [12].

The measurement strategy [9] is similar to that described in Sect. 2.2. The prediction describes the data well except for almost 2-jet like events where resolution problems are expected. A fit of the colour factor ratio $C_\mathrm{A}/C_\mathrm{F}$ and an additional free offset term N_0 to the data yields:

$$\frac{C_\mathrm{A}}{C_\mathrm{F}} = 2.261 \pm 0.14(\mathrm{stat.}) \pm 0.36(\mathrm{exp.}) \pm 0.052(\mathrm{theo.}) \pm 0.041(\mathrm{cluster})\,. \quad (6)$$

This result is the most precise measurement of the colour factor ratio so far. The presence of the offset term assures that the information about $C_\mathrm{A}/C_\mathrm{F}$ is determined from the multiplicity slope only. Moreover the fit of N_0 implies a measurement of the multiplicity difference δ_{bl} between b and light quark events at $\sqrt{s'} \sim 60$ GeV.

Instead of determining $C_\mathrm{A}/C_\mathrm{F}$ Eq. (4) can be solved for N_{gg} by subtracting the known quark contribution. This result is shown in Fig. 3 (left) compared with other measurements [10,11,18,20] and $N_{\bar{q}q}$. The about twice as strong increase of the multiplicity with energy for gluons compared to quarks clearly demonstrates the higher colour charge of the gluon. A good mutual agreement between the different measurements of N_{gg} and the prediction is observed. The measurement covers a wide energy range allowing to study the ratio r of the gluon to quark multiplicity as well as the ratio of the energy slopes $r^{(1)}$. The results are shown in Fig. 3 (right). The measurement of r (upper plot) is compared with the LO [21], NLO [22] and 3NLO [23]

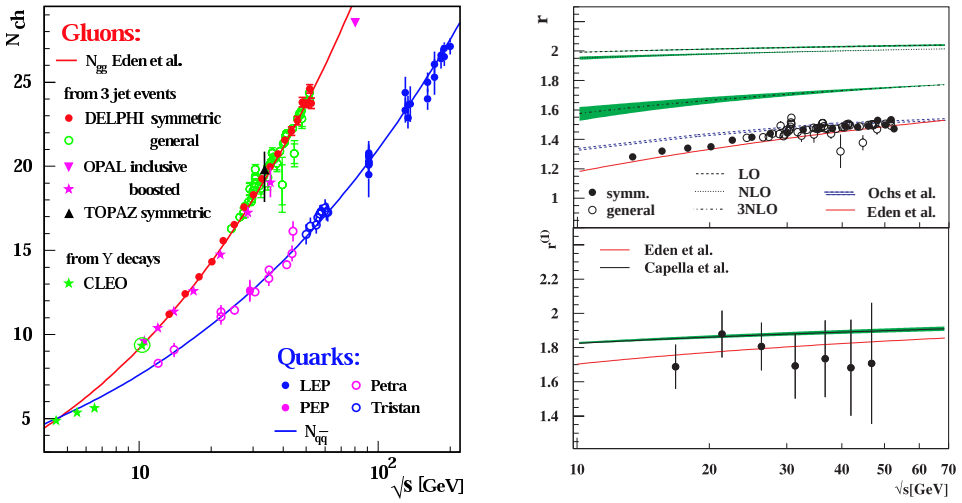


Fig. 3. The energy dependence of the multiplicity of gg compared to $\bar{q}q$ colour singlet systems (left). The ratio r of the gg to the $\bar{q}q$ multiplicity (upper right) and the ratio of the multiplicity slopes $r^{(1)}$ (lower right).

predictions as well as with a numerical calculation [24]. The perturbative calculations significantly overestimate r, however the predictions [12, 23] agree reasonably with the measured slope ratio $r^{(1)}$ (lower plot). This indicates the presence of non-perturbative contributions leading to offset terms in the multiplicities which are unimportant for the determination of $r^{(1)}$.

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