ON THE DEBYE SCREENING IN THE $q\bar{q}$ PLASMA*

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The existence of a *Hückel–Debye* color screening is often proposed as a signal of the presence of a QCD plasma. Here the particular case is examined where the quark density is largely dominant over the gluon density. In this case the two body correlation shows a spatial decay like $\exp[-ar^{2/3}]/r$ to be compared with the usual electric case $\exp[-ar]/r$. The usual electrodynamical case is rapidly re-examined in order to compare it with the chromodynamical case.

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1. The commutative case

One starts from the definition of canonical partition function [1]:

$$Z = \frac{Z_0}{V^N} \int e^{-\beta U(\boldsymbol{r})} d^{3N} r \,.$$

The interaction is given by the Coulomb potential

$$U = \sum_{i<j} u_{ij} \,, \quad u_{ij} = \alpha z_i z_j / r_{ij} \,, \quad r_{ij} = |\boldsymbol{r}_i - \boldsymbol{r}_j| \,,$$

where the behavior at $r_{ij} \to 0$ must be suitably regularized in order to prevent a divergence of the partition function.

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The integrand of Z can be expanded in multiple correlations as:

$$\begin{aligned} e^{-\beta U(\boldsymbol{r})} &= \prod_l \mathcal{D}(\boldsymbol{r}_l) + \sum_{i<j} \mathcal{C}^{(2)}(\boldsymbol{r}_i, \boldsymbol{r}_j) \prod_{l\neq i,j} \mathcal{D}(\boldsymbol{r}_l) \\ &\quad + \sum_{i<j<k} \mathcal{C}^{(3)}(\boldsymbol{r}_i, \boldsymbol{r}_j, \boldsymbol{r}_k) \prod_{l\neq i,j,k} \mathcal{D}(\boldsymbol{r}_l) + \cdots . \end{aligned}$$

For a uniform plasma the one-body distribution is a constant:

$$\mathcal{D}(\boldsymbol{r}_l) = (Z/Z_0)^{1/N} .$$

The many-body functions $\mathcal{C}^{(2)}, \mathcal{C}^{(3)}, \ldots,$ are defined to be pure correlations, *i.e.*:

$$\int \mathcal{C}^{(J)} d^3\boldsymbol{r}_J = 0 .$$

It is useful to renormalize the functions $\mathcal{C}^{(J)} = (Z/Z_0)^{J/N} C^{(J)}$ so that the expressions of the many-body distributions become:

$$W_2(\boldsymbol{r}_i, \boldsymbol{r}_j) = \frac{Z_0}{ZV^{N-2}} \int e^{-\beta U(\boldsymbol{r})} \prod_{l\neq i,j} dr_l = \left[1 + C^{(2)}(\boldsymbol{r}_i, \boldsymbol{r}_j)\right] ,$$

$$\begin{aligned} W_3(\boldsymbol{r}_i, \boldsymbol{r}_j, \boldsymbol{r}_k) &= 1 + \left[C^{(2)}(\boldsymbol{r}_i, \boldsymbol{r}_j) + C^{(2)}(\boldsymbol{r}_i, \boldsymbol{r}_k) + C^{(2)}(\boldsymbol{r}_j, \boldsymbol{r}_k)\right] \\ &\quad + C^{(3)}(\boldsymbol{r}_i, \boldsymbol{r}_j, \boldsymbol{r}_k) . \end{aligned}$$

And so on.

Using then the expression of W_2, W_3 one gets for $C \equiv C^{(2)}$ the following equation

$$\frac{\partial C(\boldsymbol{r}_1, \boldsymbol{r}_2)}{\partial r_{1,v}} = -\beta\left(\frac{\partial u_{12}}{\partial r_{1,v}} + \frac{1}{V}\sum_{l\neq 1,2} \int d^3 r_l \left[\frac{\partial u_{12}}{\partial r_{1,v}} C(\boldsymbol{r}_l, \boldsymbol{r}_2)\right]\right) \quad v = x, y, z .$$

Then one follows the standard procedure: by taking a second derivative of C and using the geometrical symmetries of the problem the known result is obtained:

$$C(r) \propto \frac{1}{r}\exp[-ar] , \qquad a = \sqrt{\beta\alpha n} , \qquad n = \frac{N}{V} .$$

The length $1/a$ is the Debye radius of the system, in order for the whole treatment to be consistent the radius at which the potential has been regularized must be much smaller than $1/a$.

2. The noncommutative case

The formal definition of the canonical partition function is the same [1], but since there is a matrix structure in color space a matrix multiplication is understood and, finally, a trace must be taken. The integrand of the partition function is expanded into multiple correlations as in the commutative case and the plasma is assumed uniform in space and isotropic in the color so the one particle distribution is a constant diagonal in the color indices $\mathcal{D}(q_i) = R$. The functions $\mathcal{C}^{(J)}$ are again defined as the pure correlations of order J and we redefine them as $\mathcal{C}^{(J)} = R^J C^{(J)}$. So for the two-body distribution we still have an expansion as before, with W and C which are matrices in color space. Now we would like to find an equation for the two-body distribution $W(q_i, q_j)$. In general the derivative of U will not commute with U, because they are matrices, so we use the representation:

$$\frac{d}{dt}e^{A} = \int_{0}^{1} e^{xA}\frac{dA}{dt}e^{(1-x)A}dx$$

which may be verified by comparing the series expansion of both sides.

Identifying now A with $-\beta U$ and defining $\tau = x\beta$ distributions at different temperatures enter into the game [2], so the variable τ appear, as an index, in the color matrices C_τ. The equation for the two-body correlation is now:

$$\frac{\partial C_\beta(\boldsymbol{r}_1, \boldsymbol{r}_2)}{\partial r_{1,v}} =$$
$$-\int_{0}^{\beta} d\tau \left(\frac{\partial u_{12}}{\partial r_{1,v}} + \frac{1}{3V}\sum_{l\neq 1,2}\int d^3 r_l \left[\frac{\partial u_{12}}{\partial r_{1,v}} C_{\beta-\tau}(\boldsymbol{r}_l, \boldsymbol{r}_2) + C_\tau(\boldsymbol{r}_l, \boldsymbol{r}_2)\frac{\partial u_{12}}{\partial r_{1,v}} \right] \right).$$

Here also one takes the second derivative, owing to the presence of an integration in the inverse temperature τ, one performs the *Laplace*-transform with respect to β [conjugate variable s] and the *Fourier*-transform with respect to space [conjugate variable $\boldsymbol{k}$]. With these transformations the result takes the form:

$$-k^2\check{C}(s;k) = \frac{\alpha T}{s^2} + \frac{\alpha}{3Vs}\sum_{l\neq 1,2}\left[T\check{C}(s;k) + \check{C}(s;k)T\right].$$

[1] The literature on QCD is immense, the few references here given [2–4] leads either to general reviews or to papers which contain arguments or points of view related to the present attempt.

[2] The presence of integrations over the inverse temperature is a well known feature of Quantum Statistics, see standard textbooks [5].

Both $\check{C}(s;k)$ and T are color matrices: in order to proceed on, the explicit color structure is needed, one can start from the definitions:

$$\check{C} = M \quad \text{for the pair } q\bar{q}\,, \quad M = \text{“1”, “8”},$$

$$\check{C} = Q \quad \text{for the pair } qq\,, \quad Q = \text{“}\bar{3}\text{”, “6”},$$

$$\check{C} = \bar{Q} \quad \text{for the pair } \bar{q}\bar{q}\,, \quad \bar{Q} = \text{“3”, “}\bar{6}\text{”}\,.$$

The term T come from the potential u (gluon exchange): it is a pure octet in *t-channel*. For two incoming quarks (with colors a, c) and two outgoing quarks (with colors b, d) the interaction is $I^{b,d}_{a,c} = [\delta^d_a \delta^b_c - \delta^b_a \delta^d_c/3]/2$.

For a quark–antiquark pair with incoming colors a, d and outgoing colors b, c the interaction is $-I^{b,d}_{a,c}$.

The unit tensor in color space: $U^{b,d}_{a,c} = \delta^b_a \delta^d_c$ is introduced, it has the property that $U^{b,d}_{a,c} I^{a,c}_{b,d} = 0$, The color projectors are explicitly written out in terms of U and I

$$^1\Pi = 2\frac{I}{3} + \frac{U}{9}\,, \quad ^8\Pi = -2\frac{I}{3} + 8\frac{U}{9}\,, \quad ^3\Pi = -I + \frac{U}{3}\,, \quad ^6\Pi = I + 2\frac{U}{3}$$

and they are normalized to state multiplicity

$$^1\Pi^{f,g}_{f,g} = 1\,, \quad ^8\Pi^{f,g}_{f,g} = 8\,, \quad ^3\Pi^{f,g}_{f,g} = 3\,, \quad ^6\Pi^{f,g}_{f,g} = 6\,,$$

(one has the same projectors for “3” and “$\bar{3}$” and the same projectors for “6” and “$\bar{6}$”). The projector expressions are then introduced into the different forms of $\check{C}$.

$$M = {}^1\Pi F_1 + {}^8\Pi F_8 \quad Q = {}^3\Pi F_3 + {}^6\Pi F_6 \quad \bar{Q} = {}^3\Pi \bar{F}_3 + {}^6\Pi \bar{F}_6$$

and the final result is an inhomogeneous system of linear equations, where the inhomogeneous terms come from T. Using the orthogonality between I and U the identities

$$F_8 = -\frac{F_1}{8}\,, \quad F_6 = -\frac{F_3}{2}\,, \quad \bar{F}_6 = -\frac{\bar{F}_3}{2}$$

are obtained and the system of equations is reduced to:

$$\begin{aligned}
k^2 F_1 - \frac{4\alpha}{3s^2} + \frac{\alpha}{2s}[2\rho F_3 + 2\bar{\rho}\bar{F}_3 + (\rho + \bar{\rho})F_1] &= 0\,,\\
k^2 F_3 - \frac{2\alpha}{3s^2} + \frac{\alpha}{2s}[2\rho F_3 + \bar{\rho} F_1] &= 0\,,\\
k^2 \bar{F}_3 - \frac{2\alpha}{3s^2} + \frac{\alpha}{2s}[2\bar{\rho}\bar{F}_3 + \rho F_1] &= 0\,.
\end{aligned}$$

Here ρ and $\bar{\rho}$ are the densities of quarks and antiquarks and α is the chromodynamical fine-structure constant.

This system yields $F_1 = F_3 + \bar{F}_3$ so it is furthermore reduced to a

$$\begin{aligned} k^2 F_3 - \frac{2\alpha}{3s^2} + \frac{\alpha}{2s}[(2\rho + \bar{\rho})F_3 + \bar{\rho}\bar{F}_3] \ &= \ 0\,, \\ k^2 \bar{F}_3 - \frac{2\alpha}{3s^2} + \frac{\alpha}{2s}[(2\bar{\rho} + \rho)\bar{F}_3 + \rho F_3] \ &= \ 0\,. \end{aligned}$$

The solution depends only on the total fermionic density $n = \rho + \bar{\rho}$, it is:

$$F_3 = \bar{F}_3 = \frac{2\alpha}{3}\frac{1}{k^2 s^2 + n\alpha s} = \check{G}(k^2, s)\,.$$

From

$$\check{G}(k^2, s) = \frac{2}{3n}\left[\frac{1}{s} - \frac{1}{s + \alpha n/k^2}\right]$$

we get its Laplace-anti-transform:

$$\hat{G}_\beta(k^2) = \frac{2}{3n}\Big[1 - \exp\left[-\beta\alpha n/k^2\right]\Big]\,.$$

From this expression it is possible to calculate the correlation energy, but in order to understand how the correlations behave in space the Fourier transform is needed:

$$G_\beta(r^2) = \frac{1}{(2\pi)^3}\int \exp[i\boldsymbol{k}\cdot r]\hat{G}_\beta(k^2) d^3k\,.$$

After performing the angular integration the resulting expression is estimated by means of the saddle point method [6], for large values of r:

$$G_\beta(r^2) = \frac{1}{2\pi^2 r}\Im\int_0^\infty \exp[ikr]\hat{G}_\beta(k^2) k dk$$

with the result:

$$G_\beta(r^2) \propto \frac{1}{r}\exp\left[-3(ar/2)^{2/3}/2\right]\cos\left[3\sqrt{3}(ar/2)^{2/3}/2 - \pi/3\right] + \cdots\,.$$

The subsequent terms contain higher negative powers of r but the same exponential behavior.

It is simple to read out the main result: the Debye screening is still present, but there are two differences: the screening is not a simple exponential, the decay is in fact slower since at the exponent one finds a power of r smaller than 1, moreover there in an oscillating behavior, controlled by the same parameter a. We note also that at this approximation the behavior of the $(q\bar{q})$-pair is the same as the behavior of the (qq)-pair.

3. Concluding observations and outlook

There are treatments of the same physical system which are from the beginning very different from the present one, they are strong coupling methods [4,7,8]; the attitude of the present investigation has been to keep the procedure as close as possible to the electric case in order to make the comparison easier[3]. The conclusion is that the $q\bar{q}$-plasma behaves in a way similar but not strictly equal to the Coulomb plasma. At this level of approximation only the total number of fundamental charges is relevant, not the number of quarks and antiquarks separately. The result for the $q\bar{q}$ pair is the same as for the qq, so the screening effect on meson and on baryon formation is the same[4].

The main neglected effect is the presence of gluons:

> Virtual gluons reflect in the running coupling constant, the correction is possible, the final calculation (*i.e.* the inversion of the Laplace and Fourier transforms) become very complicated, but no very significant result may be foreseen.
>
> Two (or more) gluons in *t-channel* make the color structure more complicated.

More important is the presence of real gluons. This involves a situation which is not realized in the electric case. Since all color channels are coupled also the gluon–gluon case should be considered; an investigation is now being performed looking for the influence of gluons over quark correlations *i.e.* systems where the quarks still dominate over the gluons but the effects of these latter are not negligible.

Added in proofs: In all formulae where it appears, except in the first one, the expression α must be substituted by $4\pi\alpha$.

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