SATURATION IN DEEP INELASTIC SCATTERING*

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We review the evidence for saturation seen at HERA, and we discuss a few theoretical aspects of saturation in deep inelastic electron proton scattering.

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1. Introduction

Measurements of deep inelastic structure functions at small x at HERA have stimulated novel ideas on parton dynamics in QCD, in particular the possible existence of states with high gluon density in electron proton scattering ("saturation") [1]. More recently, these ideas have been extended to heavy ion collisions [2], and arguments have been given that high gluon densities of the incoming heavy ions may initiate the formation of the searched-for quark gluon plasma. This talk summarizes the present evidence for saturation collected in deep inelastic electron proton scattering (DIS) at HERA; I also address a few theoretical aspects of saturation in DIS.

In a reference frame where the proton has a large longitudinal momentum and the photon carries only a transverse momentum, the leading twist DGLAP description of deep inelastic scattering can be visualized in a cascade picture: the interaction of the virtual photon with the fast proton is through a single parton cascade which has its beginning long before the interaction with the photon. At small x, according to the linear DGLAP evolution equations, the cascade is mainly gluonic, and the cross section for this process becomes large, *i.e.* the probability for the photon of "finding a small-x" gluon grows. Consequently, for sufficiently small values of x, also the probability of seeing a second, third, … cascade (Fig. 1) starts to become non-negligible. The density of participating gluons grows, and interactions

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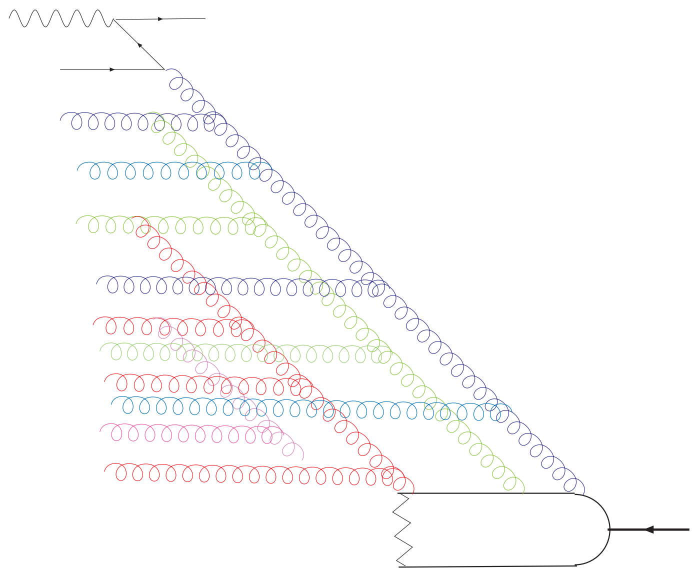

Fig. 1. multi-cascade configurations at small x.

among these gluons come into play. The net result of these interactions is a weakening of the growth of the gluon density at small x. Once the density of gluons is high, the concept of "partons" is no longer appropriate, and the language of "strong classical fields" becomes more suitable. This picture of the photon as "seeing" a field of high gluon density of the proton has also been named "color glass condensate" [2]. Here the term "condensate" hints at the high density, whereas "glass" refers to the life-time of the gluonic field which is much longer than the interaction time of the photon: the gluon field appears as a viscid medium.

An important feature of this saturation phenomenon is the appearance of a x-dependent momentum scale, $Q_{\rm s}^2(x)$. The onset of saturation, as we have described, depends on the momentum scale Q^2: the multi-cascade interactions start to become important at a certain x-value which decreases with increasing Q^2. This dependence can be inverted to define the saturation momentum scale $Q_{\rm s}^2(x)$; calculations based upon the BFKL Pomeron [3, 4] lead to the functional form

$$Q_{\rm s}^2(x) = c\left(\frac{1}{x}\right)^{\lambda}, \qquad (1)$$

where typically $\lambda \approx 0.3$. The constant scale parameter c, so far, cannot be calculated, but has to be determined from a (model-dependent) comparison with data. For Q^2 values larger than the saturation scale (1) the QCD parton model with the linear DGLAP evolution equations holds, whereas for smaller Q^2 values the saturation effects become visible. More elaborate calculations show that (1) is a too crude approximation: there are logarithmic corrections in front of the exponential, and also the exponent is a slightly more complicated function [5].

The experimental verification of saturation in DIS is an important task. If true, it means that partons with very small x originate from regions of high density and are probing subtle QCD dynamics. This would represent a step beyond the QCD-based parton picture, which deals with dilute partons only. As to practical applications, saturation has implications both for the analysis of heavy ion collisions at RHIC and of proton–proton scattering at the LHC. For the latter, saturation in DIS is expected to begin with the presence of multiple parton cascades; their measurement could be used to estimate, in proton–proton collisions, the effect of multiple partonic interactions and to understand the general structure of events. This clearly will help to control the background of new physics.

The HERA kinematic domain is not large enough, and the determination of the gluon density not precise enough to observe, as a signal of saturation, a flattening of the gluon density at fixed Q^2 and small x. So we have look for other signals of saturation. The x-dependence of the saturation scale $Q_{\mathrm{s}}^2(x)$ allows to trace saturation not only at fixed Q^2 as a function of x, but also at fixed x as a function of Q^2: this suggests to look, in DIS, for saturation effects also in the region of smaller Q^2 values where we expect to see the transition from the QCD parton picture to nonperturbative strong interactions. Clearly, in this region the use of perturbative arguments is less reliable. Nevertheless, this is the region where at HERA, so far, the strongest evidence for saturation comes from.

2. Evidence for saturation at HERA

In the following I review three different observations which, in my opinion, are indicative of saturation being present in the small-x and low-Q^2 region:

(i) models based upon saturation ideas are most successful in describing the deep inelastic proton structure function F_2 in the small-x region at low and at moderate Q^2;

(ii) the observed geometric scaling of F_2 is a fundamental feature of saturation;

(iii) the observed constant (with energy) ratio of DIS diffractive and DIS total cross sections has a natural explanation in saturation models.

Let me briefly comment on these observations.

The classical simple dipole saturation model (GBW) is due to Golec-Biernat and Wüsthoff [6]: with 3 parameters it successfully describes HERA data of F_2 in the low and intermediate Q^2 region; with a fourth parameter the description extends down to $Q^2 = 0$. Because of its simple analytic form it is straightforward to determine a saturation scale: the dipole cross section depends upon $r/R_{\rm s}(x)$, where $R_{\rm s}^2(x) = 1/Q_0^2(x/x_0)^{\lambda}$. Inserting this into the dipole formula one arrives at the structure function F_2 which depends on the ratio $Q^2/Q_{\rm s}^2(x)$ where $Q_{\rm s}^2(x) = 1/R_{\rm s}^2(x)$, and hence, predicts the observed property of geometric scaling [7]. In the x–Q^2 plane (Fig. 2)[1], the lower line is defined by $Q^2 = Q_{\rm s}^2$, and it marks the transition region from

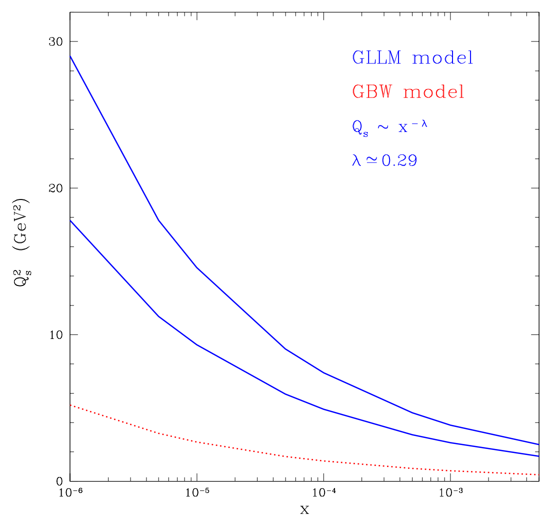

Fig. 2. Estimates of the saturation scales in two different models.

pQCD to nonperturbative physics. Above this line the QCD parton model holds, and F_2 can be expanded in inverse powers of Q^2 (twist expansion [8]). Below, in the small-Q^2 region, a power series expansion in Q^2 applies. The transition between the two regions is not sharp, and it is not clear how far above the line corrections to DGLAP (higher twist corrections) could be significant. An improved version [9] of the GBW saturation model includes logarithmic scaling violations: this not only improves the quality of the fit to HERA data, but also extends the validity towards larger Q^2 values. An alternative saturation model [10], based upon an approximate solution of the nonlinear Balitsky–Kovchegov equation, also describes the HERA data. As to the transition from pQCD to nonperturbative strong interactions, it leads to a somewhat different conclusion. The the upper two lines in Fig. 2 present, in the model of [10], two different definitions of the transition line,

[1] I thank M. Lublinsky for Fig. 2.

and the region between these lines can be interpreted as a "transition strip". Compared with the GBW model, the limit of the linear evolution equations lies considerably higher, *i.e.* it is shifted towards larger Q^2 values. The discrepancy between the two models helps to illustrate the present uncertainty of where, in the x–Q^2 plane, the applicability of the linear DGLAP evolution ends.

As it has been said already, saturation leads to scaling properties of the dipole cross section and of the structure function F_2. In particular

$$F_2(x, Q^2) = F_2 \, \frac{Q^2}{Q_{\mathrm{s}}^2(x)} \, . \tag{2}$$

This feature has clearly been seen in the data. Scaling has also been derived within the vector dominance model [11]; however, the energy dependence of the scaling momentum is different from the one of the saturation models.

DIS diffraction, most likely, provides the most sensitive test of saturation. One of the striking experimental results is the energy dependence of the diffractive cross section σ_{diff}: the ratio $\sigma_{\mathrm{diff}}/\sigma_{\mathrm{tot}}^{\gamma^* p}$ is nearly constant with energy, and the saturation models, by a subtle interplay of the scales, reproduce this distinctive feature in a much more convincing manner than other models. It should, however, be noted that the saturation models for F_2, as far as diffraction is concerned, are not completely satisfactory. Neither of them fully contains the diffractive $q\bar{q}$ and $q\bar{q}g$ final states (see below) which at HERA have been shown to contribute.

The presented features provide evidence that saturation may be present in the small-x region at low/intermediate Q^2-values of F_2 and in DIS diffraction. Clearly, each feature by itself may allow for a different interpretation; on the other hand, it is remarkable that the simple idea of the high gluon density allows to explain different phenomena which, at first sight, look quite uncorrelated.

To collect further evidence we need to look for other — if possible: more direct — signals. This will be the task of the next few years. One direction of future research is the impact parameter (b) dependence of the dipole cross section. So far (*i.e.* in $\sigma_{\mathrm{tot}}^{\gamma^* p}$ and in the diffractive cross section at zero momentum transfer t) we have been dealing with b-integrated cross sections; but HERA data also include the dependence on t: studies of b-dependent dipole cross sections have been started [12] and need further attention. Another route of looking for signals of saturation is the investigation of multi-parton chains. As we have mentioned before, large gluon densities start with the formation and the interaction of multiple chains of partons. One should therefore look for signals of these multiple interactions. Direct evidence for the presence of double chains follows from the presence of DIS diffraction:

the hard part of the diffractive final states cannot be counted as being part of the initial conditions to the (leading twist) parton densities. A recent analysis [13] has been based upon this fact, and it shows that the proper account of this fraction of diffractive data may lead to changes in the global fit of parton densities in the low Q^2 region. The presence of multi-chain configurations also affects the cross sections for multijet final states in DIS. Such jet configurations can originate from both single chains or from multi-chain configurations. The conventional hard scattering formalism takes into account only single chains; a deviation from its predictions, therefore, might be indicative for the presence of multi-chains. Work in this direction is in progress.

3. A remark on the use of the Balitski–Kovchegov equation in DIS

An attractive theoretical tool for studying saturation in QCD is given by the Balitsky–Kovchegov (BK) equation [14], which represents a nonlinear generalization of the LO BFKL equation and, when written in configuration space variables, has a particularly simple mathematical form. Since it seems natural to use this equation as a model in DIS, it is important to understand the content of this equation and to be able to compute necessary corrections. In the context of using the BK equation also for DIS diffraction, there is particular interest in the question of which part of the DIS diffractive cross section is included in the BK equation.

A good starting point of such an analysis is QCD reggeon field theory, derived from momentum space Feynman diagrams [15]. This approach allows one to analyze s-channel unitarity cuts, to compute NLO corrections and to keep a connection with hard scattering processes in QCD. In this field theory reggeized gluons play the role of the elementary fields, and the BFKL Pomeron represents the bound states of two gluons. The $2 \to 4$ gluon vertex [16] describes the splitting of one Pomeron into two Pomerons, and invariance under Möbius transformations has been proven for both the BFKL kernel and for this $2 \to 4$ vertex. In order to obtain a scattering amplitude, the Green's function couple to external impact factors; because of gauge invariance they have special properties which define the space of functions in which the reggeon field theory operators are acting.

In this language, a simple nonlinear generalization of the BFKL equation is the fan diagram equation [17] which sums all fan-like diagrams, with all BFKL Pomerons at the lower end of the fan structure coupling to a single common dipole (eikonal approximation): this equation can be used as a model for the scattering of a single small dipole (upper end) on another larger dipole (lower end). Making use of the Möbius properties of the reggeon field theory and taking the limit $N_c \to \infty$ it as been shown [18], for this simple model, that the Fourier-transform of the $2 \to 4$ vertex coincides with the BK kernel. From the point of view of the QCD reggeon field theory, this model represents a handy approximation; steps beyond it contain, for example, closed Pomeron loops [19] and higher order vertices.

Addressing the question, within this simple model, of how much diffraction of the upper dipole is included in this fan diagram equation, we have to compute the energy discontinuity. At first sight it might seem as if, since there is always a rapidity gap between the upper dipole and the first triple Pomeron vertex underneath, there is no contribution from the elastic rescattering of the upper dipole. A closer inspection of the triple Pomeron vertex [16], however, shows that this is not correct: in momentum space, the triple Pomeron vertex contains "virtual pieces" which do not originate from real s-channel gluon production (quite in analogy to the BFKL vertex, which also consists of a "real" and a "virtual" contribution). These pieces can be traced back to diagrams of the type shown in Fig. 3, *i.e.* to parts of the elastic rescattering of the quark–antiquark dipole. A triple Pomeron vertex without these virtual contributions has been derived in [20]: it leads to an integral operator which — when Fourier-transformed to coordinate space — in [18] has been shown to differ from the BK-kernel. Conversely, however, this does not mean that the BK kernel contains all of the elastic scattering: the computation of the closed quark loop shows that there are other pieces which belong to the reggeization of the gluon. In summary, the momentum space triple Pomeron vertex which — when Fourier-transformed in the large-N_c limit — has been shown to agree with the BK kernel, contains already a part of the elastic scattering (but not all of it). Therefore, elastic scattering cannot simply be added to the BK equation.

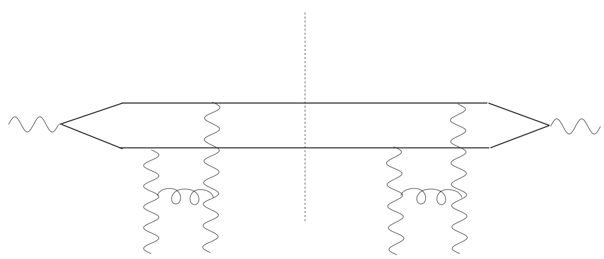

Fig. 3. Elastic scattering of a quark pair.

4. Concluding remarks

We have reviewed a few phenomena in DIS at HERA which have a natural interpretation if saturation is assumed to be present at small-x and moderate Q^2. Each them, when considered separately, could possibly be explained by another and different model or mechanism; nevertheless it is remarkable that the simple idea of saturation provides a natural explanation of this variety of seemingly independent phenomena. An attractive theoretical framework for saturation in DIS is given by the nonlinear Balitsky–Kovchegov equation. Recent investigations indicate that this equation does not fully include the diffractive production of $q\bar{q}$ and $q\bar{q}g$ states: hence further theoretical work is needed to provide a realistic theory for HERA data.

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