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EFFECTS OF ENERGY CONSERVATION AND SATURATION IN MUELLER'S DIPOLE CASCADE FORMULATION*

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The effects of energy conservation and saturation are studied in the coordinate space dipole formulation for QCD cascades. Preliminary results for dipole–dipole and dipole–nucleus scattering are presented. Very large effects are obtained from energy conservation, corresponding to a factor ~ 10 in the cross section and ~ 3 in $\lambda_{\rm eff}$. Some results on the gluon fusion process $g+g \to g$ are also presented.

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1. Introduction

The leading order BFKL equation has solutions with asymptotic behaviour $F \sim 1/x^{\lambda}$, with $\lambda = 4\ln 2\,\bar{\alpha} \approx 0.5$, for $\bar{\alpha} \equiv 3\alpha_{\rm s}/\pi = 0.2$. There are various corrections which reduce the value of λ. First the NLO corrections are very large. It is well-known that a large fraction of these corrections are related to energy conservation [1]. Secondly the growth for small x-values is reduced by saturation effects, due to multiple Pomeron exchange. These effects are taken into account in the Balitsky–Kovchegov equation [2] by a non-linear component in Mueller's dipole formulation [3], which is a description in transverse coordinate space ($\boldsymbol{r}_\perp$) instead of transverse momentum space ($\boldsymbol{k}_\perp$). In this talk I want to discuss effects of energy conservation in the dipole formalism, and compare with the effects of non-linearity. Some preliminary results are presented for dipole–dipole scattering ($\gamma^*\gamma^*$-collisions) and dipole–nucleus scattering. At the end I also give some comments on effects of gluon–gluon fusion, $g + g \to g$, in the parton cascade. The results are obtained in collaboration with Emil Avsar and Leif Lönnblad, and some of the results are presented in Avsar's diploma thesis, Ref. [4].

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2. Formalism

Study the process $\gamma^* \to Q\bar{Q} \to Qg\bar{Q} \to Qgg\bar{Q} \to \ldots$. Here a virtual gluon is split into a $Q\bar{Q}$ colour dipole, which is first split into two dipoles by the emission of a gluon, then into three dipoles by a second gluon, *etc.* The process is illustrated in transverse coordinate space in Fig. 1. The probability for such a dipole splitting is given by the expression (for notation see Fig. 1)

$$\frac{dP}{dy} = \frac{\bar{\alpha}}{2\pi} d^2\boldsymbol{r}_2 \frac{r_{01}^2}{r_{02}^2\, r_{12}^2} \cdot S\,, \qquad S = \exp\left[-\frac{\bar{\alpha}}{2\pi}\int dy \int d^2\boldsymbol{r}_2 \frac{r_{01}^2}{r_{02}^2\, r_{12}^2}\right]. \quad (1)$$

Here S denotes a Sudakov form factor. We note that the integral over $d^2\boldsymbol{r}_2$ in the exponent diverges for small values of r_{02} and r_{12}. Therefore, Mueller introduced a cutoff ρ, such that the integration region satisfies $r_{02} > \rho$ and $r_{12} > \rho$. A small cutoff value ρ will here imply that we get very many dipoles with small r-values.

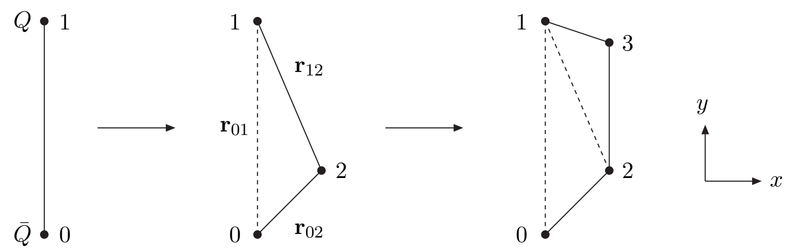

Fig. 1. A quark–antiquark dipole in transverse coordinate space is split into successively more dipoles via gluon emission.

If a dipole size, $\boldsymbol{r}$, is small, it means that the gluons are well localised, which must imply that transverse momenta are correspondingly large. This implies that not only the new gluon gets a large $k_\perp \sim 1/r$, also the original gluon, which is close in coordinate space, gets a corresponding recoil. Let us study the example in Fig. 2. For the emissions of the gluons marked 2, 3, and 4 the dipole sizes become smaller and smaller, $a \gg b \gg c \gg d$, in each step of the evolution. The corresponding $k_\perp$, therefore, become larger and larger in each step. After the minimum dipole, with size d, the subsequent emissions, 5 and 6, give again larger dipoles with correspondingly lower $k_\perp$ values. The probability for this chain is proportional to

$$\frac{d^2\boldsymbol{r}_2\, a^2}{a^2\, b^2} \cdot \frac{d^2\boldsymbol{r}_3\, b^2}{b^2\, c^2} \cdot \frac{d^2\boldsymbol{r}_4\, c^2}{c^2\, d^2} \cdot \frac{d^2\boldsymbol{r}_5\, d^2}{e^2\, e^2} \cdot \frac{d^2\boldsymbol{r}_6\, e^2}{f^2\, f^2} \quad (2)$$

For the first emissions, 2 and 3, we in this expression recognise the product of factors $d^2\boldsymbol{r}_i/r_i^2 \sim \prod d^2\boldsymbol{k}_i/k_i^2$, just as is expected from a “DGLAP evolution” of a chain with monotonically increasing $k_\perp$. Emission number 4 corresponds to the minimum dipole size, d, and we here note that the factors of d cancel in Eq. (2). We, therefore, get the weight $d^2\boldsymbol{r}_4 \sim d^2\boldsymbol{k}_{\max}/k_{\max}^4$, which corresponds to a hard gluon–gluon collision. When the dipole sizes get larger again, this gives factors corresponding to a "DGLAP chain" from the other end of the chain, up to the central hard subcollision.

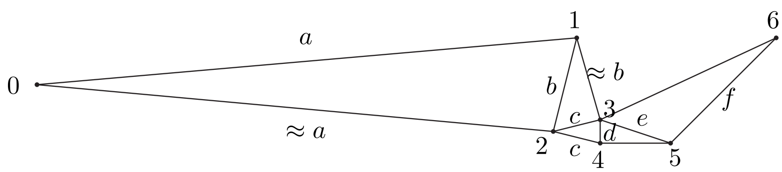

Fig. 2. A dipole cascade, where a chain of smaller and smaller dipoles is followed by a set of dipoles with increasing sizes. This is interpreted as one $k_\perp$-ordered cascade from the left and one from the right, up to a central hard subcollision, which is represented by the dipole with minimum size and, therefore, maximum $k_\perp$.

It is also easy to see that for a chain with increasing dipole sizes up to a maximum value, $r_{\max}$, which thus corresponds to a minimum transverse momentum, $k_{\perp\min}$, we get the weight $d^2\boldsymbol{r}_{\max}/r_{\max}^4 \sim d^2\boldsymbol{k}_{\min}$. Therefore, there is no singularity for the minimum $k_\perp$-value. This result agrees exactly with the result in the Linked Dipole Chain model, LDC [5], which is a reformulation of the CCFM model [6], interpolating between DGLAP and BFKL for non-$k_\perp$-ordered chains.

To study $\gamma^*\gamma^*$ scattering we imagine that the two virtual photons split up into quark–antiquark pairs, which develop into dipole cascades as schematically illustrated in Fig. 3. When the two central dipoles collide, it implies a recoupling, as indicated by the arrow, and the probability for this is given by the expression [7]

$$f = \frac{\alpha_{\mathrm{s}}^2}{2}\left\{\ln\left[\frac{|\boldsymbol{r}_1 - \boldsymbol{r}_3|\cdot|\boldsymbol{r}_2 - \boldsymbol{r}_4|}{|\boldsymbol{r}_1 - \boldsymbol{r}_4|\cdot|\boldsymbol{r}_2 - \boldsymbol{r}_3|}\right]\right\}^2. \tag{3}$$

As the dipole cascades from the two virtual photons branch out, it is also possible to have *multiple interactions* with dipoles from the left and from the right. The total cross section is then given by

$$\sigma \sim \int d^2b\left(1 - e^{-\sum f_{ij}}\right), \tag{4}$$

where b denotes the impact parameter.

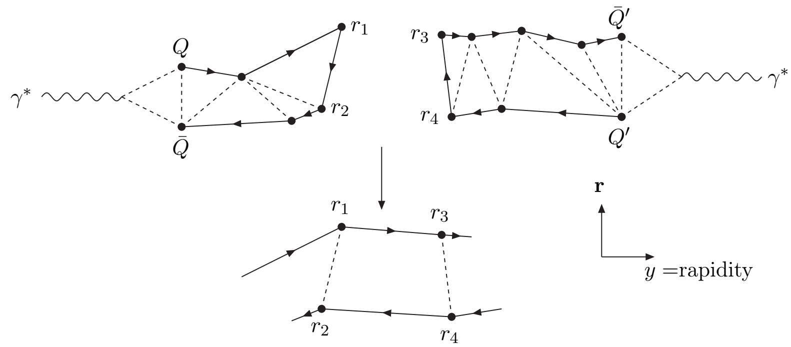

Fig. 3. A symbolic picture of a $\gamma^*\gamma^*$ collision in rapidity-$\boldsymbol{r}_\perp$-space. The two dipole chains interact and recouple with probability f given by Eq. (3).

With a small cutoff ρ ($r > \rho$) we get, as mentioned above, very many small dipoles. If these are interpreted as real emissions, it would imply a violation of energy-momentum conservation. The emission of these small dipoles must be compensated by virtual emissions. Thus the result in Eq. (4) will describe the inclusive cross section, but the many dipoles produced in all the branching chains will not correspond to the production of exclusive final states. To be able to describe final state properties we will make the following conjecture:

In the cascade an emission with a (transverse) dipole size r corresponds to transverse momentum $k_\perp = 1/r$, as discussed above. Emissions satisfying energy-momentum conservation correspond to real emissions. Keeping only these emissions corresponds to keeping what is called "primary gluons" in Ref. [5] and "backbone gluons" in Ref. [8]. These emissions determine both the total cross section and the structure of exclusive final states, and all softer emissions can be treated as final state emissions.

This conjecture implies that a chain of small dipoles is regarded as a single "effective" dipole. Keeping only the energy conserving emissions implies a *dynamical cutoff*, $\rho(\Delta y)$, which is large for small steps in rapidity, Δy, but gets smaller for larger Δy. (Alternatively it could be described as a cutoff for Δy which depends on ρ.) This is similar to the approach by Andersen–Stirling and Orr–Stirling [9] in transverse momentum space. Conserving also the negative lightcone momentum p_- implies that we in a similar way also get a maximum value for r in each emission.

This conjecture is very easy to implement in a MC simulation. The result is that the number of dipoles grows much more slowly with energy. It is straight forward to calculate cross sections and to study saturation effects, by comparing the unitarised expression $\int d^2b(1 - e^{-\sum f_{ij}})$ in Eq. (4) with

$\int d^2b \sum f_{ij}$ representing single $I\!P$ exchange. (The large numerical complications in MCs without energy conservation, discussed in Ref. [7], are not present.) The next section describes some preliminary results obtained with a fixed coupling $\bar{\alpha} = 0.2$.

3. Applications

Dipole–dipole scattering. Let us study the scattering of two dipoles with sizes r_1 and r_2. With a fixed coupling the scaled cross section, σ/r_2^2, depends only on the ratio r_1/r_2. We can imagine a target with size $r_2 \sim 1/M$, and a varying projectile size $r_1 \sim 1/\sqrt{Q^2}$. The results in Fig. 4 show that the cross section grows faster with the total rapidity range, $Y \sim \ln s$, for smaller r_1 (larger Q^2). This corresponds to a larger effective power λ_{eff} for larger virtuality Q^2 (see Fig. 4(b)), in a way qualitatively similar to the behaviour of the proton structure function. The effect of energy conservation is demonstrated in Fig. 4 by the results obtained for the case $r_1 = r_2$, with a constant cutoff, $\rho = 0.02\, r_i$. We see that energy conservation has a very strong effect, reducing σ by a factor 10 for $Y \sim 13$ and λ_{eff} by a factor of 3.

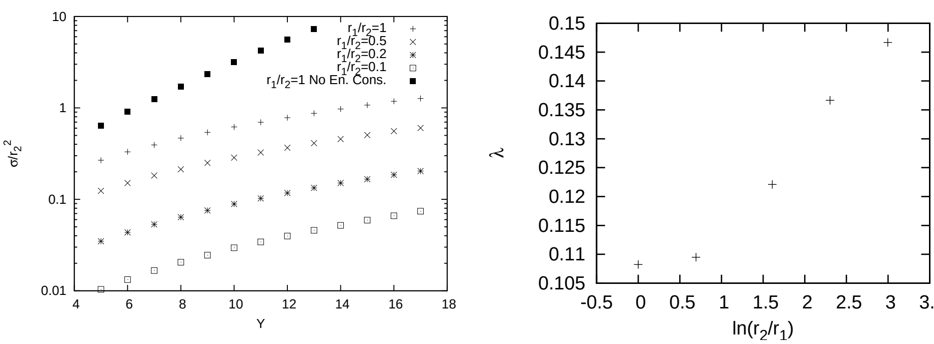

Fig. 4. Left: The scaled cross section, σ/r_2^2, for scattering of two dipoles with sizes r_1 and r_2, respectively, as a function of $Y = \ln s$ for different ratios r_1/r_2. The filled squares show results without energy conservation for $r_1 = r_2$. Right: The effective power λ as function of r_2/r_1.

For dipole–dipole scattering we find that the effect of multiple $I\!P$ exchange (saturation) is much smaller than the effect of energy conservation. The result is a reduction in the cross section which increases with energy up to about 20% for $Y \sim 15$.

Dipole–nucleus collisions. We study a toy model nucleus with a Gaussian distribution in dipole size r and impact parameter b. The dipole density is given by

$$dN = B \cdot d^2\boldsymbol{r}\, e^{-r^2/r_0^2} \cdot d^2\boldsymbol{b}\, e^{-b^2/b_0^2}\,. \tag{5}$$

The widths of the distributions are taken to be $r_0 = 1\text{fm}$ and $b_0 = A^{1/3} \cdot 1\,\text{fm}$ (where A is the mass number of the nucleus). The normalisation constant B is adjusted so that the transverse energy is given by $A \cdot 1\,\text{GeV}$.

Some results are shown in Fig. 5 for $A = 200$. We see here that multiple $I\!\!P$ effects are important for large projectile sizes and large target nuclei, being about 40% for $r_{\text{proj}} = 1/(1\,\text{GeV})$ and $A = 200$. We also see the effect of colour transparency for small projectile sizes, as multiple $I\!\!P$ exchange gives only a small correction for $r_{\text{proj}} = 1/(10\,\text{GeV})$, also for large A-values.

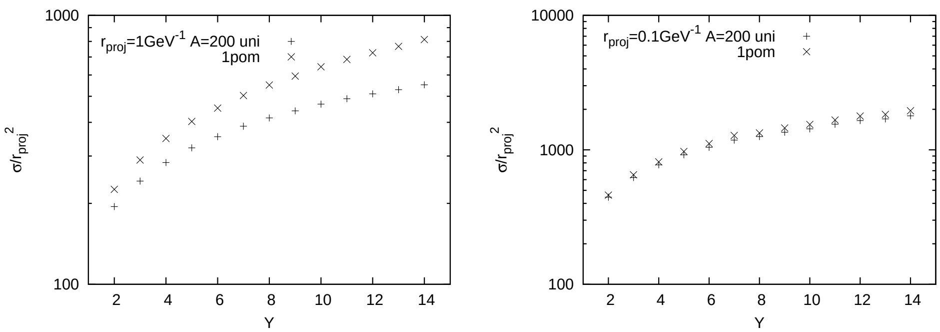

Fig. 5. Dipole–nucleus cross sections, σ/r^2_{proj}, for $A = 200$ and $r_{\text{proj}} = 1/(1\text{GeV})$ (left) and $r_{\text{proj}} = 1/(10\text{GeV})$ (right). The crosses show the one $I\!\!P$ contribution, and the plus signs the uniterised result.

4. Dipole fusion

At large rapidities and high gluon densities we can imagine that a process where two dipoles combine to a single dipole, becomes important. We note that this effect is not related to the multi-$I\!\!P$ exchange processes included in the Balitsky–Kovchegov equation, and in the results presented above. We now make three important assumptions or approximations:

1. $N_c = \infty$.
2. Local factorising approximation.
3. Forward–backward symmetry.

The first point implies that only neighbouring dipoles can fuse. The assumptions that the fusion can be described by a local factorising expression, independent of the rest of the cascade, and is such that the same result is obtained if the cascade was generated from right to left, instead of from left to right, implies that the probability that the dipoles $\boldsymbol{r}_{ij}$ and $\boldsymbol{r}_{jk}$ fuse to form the dipole $\boldsymbol{r}_{ik}$ must have the form:

$$P_{\text{fuse}} = C\, \frac{r_{ij}^2\, r_{jk}^2}{r_{ik}^2}\, dy\,. \tag{6}$$

Here C is a constant with dimension $(length)^{-2}$. The only relevant scale in this process is Λ_{QCD}, and we, therefore, use the notation $C = \xi\, (\bar{\alpha}/2\pi)\, \Lambda_{\text{QCD}}^2$, where ξ is a dimensionless phenomenological parameter, which defines the strength of the fusion process.

The expression in Eq. (6) gives, however, rise to an infrared problem. It implies that the fusion process becomes independent of the position, $\boldsymbol{r}_j$, of the common vertex for the fusing dipoles. Thus the total contribution $\propto \int d^2 r_j$ diverges. Obviously the confinement mechanism must suppress contributions from very large dipoles, where the point $\boldsymbol{r}_j$ is very far away. To describe this we introduce an infrared cutoff: when a dipole r_{01} is split into two dipoles r_{02} and r_{12}, we add a suppression factor $\exp[-D\,(r_{02}+r_{12})]$ to the expression in Eq. (1). Here D is a constant, which should be of order Λ_{QCD}, and we introduce the notation $D = \hat{\xi}\, \Lambda_{\text{QCD}}$.

Thus we conclude that in order to describe the dipole fusion process, we must introduce two fundamental parameters of order Λ_{QCD}, related to confinement. We have studied the effect of this fusion process for different parameter values. The result is that very large values are needed for a significant effect. For $\xi = 100$ and $\hat{\xi} = 1$ the cross section for dipole–dipole scattering is only reduced by 25% for $Y = 10$.

REFERENCES

[1] See *e.g.* G.P. Salam, *Acta Phys. Pol. B* **30**, 3679 (1999).

[2] I.I. Balitsky, *Nucl. Phys.* **B463**, 99 (1996); V. Kovchegov, *Phys. Rev.* **D60**, 034008 (1999).

[3] A.H. Mueller, *Nucl. Phys.* **B415**, 373 (1994); A.H. Mueller, B. Patel, *Nucl. Phys.* **B425**, 471 (1994).

[4] E. Avsar, `hep-ph/0406150`.

[5] B. Andersson, G. Gustafson, J. Samuelsson, *Nucl. Phys.* **B467**, 443 (1996); B. Andersson, G. Gustafson, H. Kharraziha, *Phys. Rev.* **D57**, 5543 (1998); H. Kharraziha, L. Lönnblad, *J. High Energy Phys.* **03**, 006 (1998).

[6] M. Ciafaloni, *Nucl. Phys.* **B269**, 49 (1988); S. Catani, F. Fiorani, G. Marchesini, *Nucl. Phys.* **B336**, 18 (1990).

[7] G.P. Salam, *Nucl. Phys.* **B461**, 512 (1996); A.H. Mueller, G.P. Salam, *Nucl. Phys.* **B475**, 293 (1996).

[8] G.P. Salam, *J. High Energy Phys.* **9903**, 009 (1999).

[9] J.R. Andersen, W.J. Stirling, *J. High Energy Phys.* **0302**, 018 (2003); L.H. Orr, W.J. Stirling, *Phys. Rev.* **D56**, 97 (1997).