BOOTSTRAPPING GENERALIZED PARTON DISTRIBUTIONS*

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A non-perturbative approach to the calculation of generalized parton distributions (GPD) through their relation to deeply virtual Compton scattering (DVCS) and vector meson photoproduction is suggested. In the first approximation, GPDs are proportional to the imaginary part of the DVCS amplitude. A model for DVCS, developed earlier in the framework of the analytic S-matrix and incorporating duality between resonances and Regge behavior, is used for this purpose. Furthermore, a bootstrap procedure is suggested in which this GPD is used as an input in the handbag diagram, whose convolution with the perturbative kernel (loop diagram) results in the DVCS amplitude to be reconstructed from the analytic S-matrix theory and/or the experiment.

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Generalized parton distributions (GPD) [1] unify the concept of ordinary parton distributions and form factors. Apart from the momentum fraction variable x and skewedness $\xi \sim x_{\rm B}/2 \sim 1/s$ (see below), GPDs depend on the invariant momentum transfer t.

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Contrary to ordinary parton distributions, related to inclusive cross sections or, equivalently to the imaginary part of the forward Compton scattering amplitude, GPDs are supposed to be extracted from exclusive reactions, such as $ep \to ep\gamma$, where the outgoing photon may be replaced by a vector meson or by a virtual photon, decaying into a lepton pair. The configuration $ep \to ep\gamma$ implies three processes, one being the genuine deeply virtual Compton scattering (DVCS) (Fig. 1(a)), and the other two corresponding to the accompanying en electromagnetic Bethe–Heitler process, in which the photon is radiated by the incoming or outgoing electron (Fig. 1(b) and 1(c), respectively). The interference of the two enables the experimental determination of the amplitude phase and opens the way to nuclear holography (spatial picture of the nucleon or the nucleus).

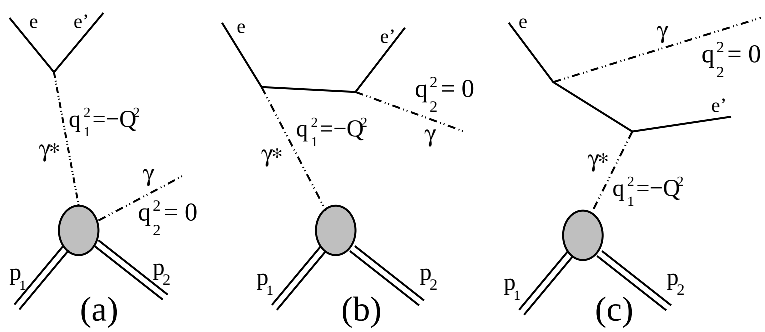

Fig. 1. (a) DVCS and (b), (c) accompanying Bethe–Heitler processes.

The usual notations for the DVCS kinematics are $P = p_1 + p_2$, $\Delta = p_2 - p_1$, $q = (q_1 + q_2)/2$, generalized Bjorken variable $\xi = -q^2/(P\,q)$ (see Fig. 1). They are related to the variables used in deeply inelastic scattering (DIS): $\Delta^2 = t$, $s = (p_1 + q_1)^2)$, $Q^2 = q_1^2$, and the Bjorken variable $x_{\rm B} = -q_1^2/(2p_1\,q_1)$. If the virtuality of the outgoing photon is different from zero, we have an extra variable, called skewedness, and defined as $\eta = (\Delta\,q)/(P\,q)$. For $q_2^2 = 0$ it is related to ξ, $\eta = \xi(1+t/(2Q^2))^{-1}$ and ξ is related to $x_{\rm B}$ by $\xi = x_{\rm B}\,(1+t/(2Q^2))/(2-x_{\rm B}+(x_{\rm B}t)/Q^2)$.

GPDs cannot be measured directly experimentally, instead they appear in convolution integrals of the form (so-called "handbag" diagram, Fig. 2)

$$A_{\rm DVCS}(\xi,\eta,t,x_{\rm B},Q^2) = \int_{-1}^{1} dx \frac{{\rm GPD}(x,\xi,\eta,t,x_{\rm B},Q^2)}{x-\xi+i\varepsilon}, \tag{1}$$

where x is a loop variable. A is the complex DVCS scattering amplitude and GPD is an unknown real function.

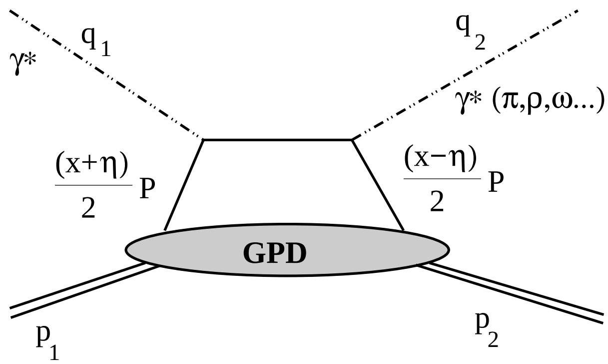

Fig. 2. "Handbag" diagram, corresponding to the convolution integral, Eq. (1).

Since these convolution integrals cannot be easily converted, GPD are often "extracted" from the data by writing [2,3] a model ansatz for the GPD with various free parameters. For example, a plausible ansatz at low t is GPD $\sim x^{-\alpha(t)}$, where α is a Regge trajectory.

If one is interested only in the imaginary part of the amplitude, $\mathcal{I}\mathrm{m}\, A_{\mathrm{DVCS}}$, then it can be immediately related to the GPD: from Eq. (1) one obtains

$$\mathcal{I}\mathrm{m}\, A_{\mathrm{DVCS}}\left(\xi,\eta,t,x_{\mathrm{B}},Q^2\right) = i\pi\ \mathrm{GPD}\left(x=\xi,\xi,\eta,t,x_{\mathrm{B}},Q^2\right) . \tag{2}$$

Our main idea is to consider $A_{\mathrm{DVCS}}(\xi,\eta,t,x_{\mathrm{B}},Q^2)$ as a generalization of the $A_{\mathrm{DIS}}(x_{\mathrm{B}},Q^2)$, see Fig. 3. Let us consider a symmetric case when $q_1^2 = q_2^2 = -Q^2$. Then $\eta = 0$, $\xi = x_{\mathrm{B}}\,(1+t/(4Q^2))/(1+x_{\mathrm{B}}\,t/(2Q^2))$ is not an independent variable anymore, but t may differ from zero, $t = -\vec{\Delta}_t^2 \neq 0$. In this limit

$$A_{\mathrm{DVCS}}(\eta=0,t,x_{\mathrm{B}},Q^2) = A_{\mathrm{DIS}}(t,x_{\mathrm{B}},Q^2)\,. \tag{3}$$

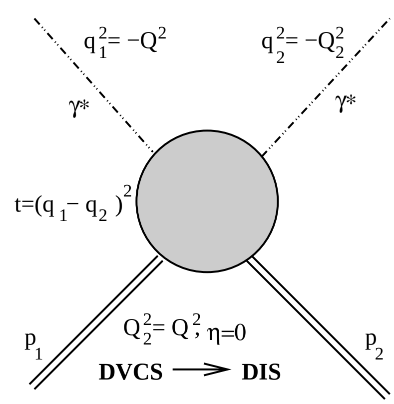

Fig. 3. Relating DVCS and DIS diagrams.

The authors of Refs. [4, 5] presented a model for DIS amplitude as an off-mass-shell continuation of an ordinary scattering amplitude $A(s,t)$, *e.g.* a Dual Model with Mandelstam Analyticity (DAMA) (see Fig. 4). The next step should be to develop a model for the scattering amplitude of an asymmetric $\gamma * p \to \gamma * p$ scattering with $\eta \neq 0$.

$$\begin{array}{ccccc} A(s,t) & \Rightarrow & A_{\mathrm{DIS}}(t, x_{\mathrm{B}}, Q^2) & \Rightarrow & A_{\mathrm{DVCS}}(\xi, \eta, t, x_{\mathrm{B}}, Q^2) \\ \text{DAMA} & \Rightarrow & \text{Modified DAMA [4,5]} & \Rightarrow & \text{DAMA for DVCS } \mathbf{?} \end{array}$$

Furthermore, for $\eta = 0$ and $t = 0$ we can relate our GPD to the nucleon structure function:

$$\begin{aligned} &\mathrm{GPD}(x = \xi = x_{\mathrm{B}}, \xi = x_{\mathrm{B}}, \eta = 0, t = 0, x_{\mathrm{B}}, Q^2) \\ &\quad \sim \mathcal{I}\mathrm{m}\; A_{\mathrm{DVCS}}(\xi = x_{\mathrm{B}}, \eta = 0, t = 0, x_{\mathrm{B}}, Q^2) \\ &\quad = \mathcal{I}\mathrm{m}\; A_{\mathrm{DIS}}(x_{\mathrm{B}}, Q^2) \;\sim\; F_2(x_{\mathrm{B}}, Q^2) = x_{\mathrm{B}}\; q(x_{\mathrm{B}}, Q^2)\,, \end{aligned} \tag{4}$$

where $q(x_{\mathrm{B}}, Q^2)$ is the ordinary parton distribution (PD) function[1]. Then, for example, in the small x_{B} limit (small ξ in our case) we have Bjorken scaling

$$\mathrm{GPD}\left(x = \xi = x_{\mathrm{B}} \to 0, \eta = 0, t = 0, x_{\mathrm{B}} \to 0, Q^2\right) \sim x_{\mathrm{B}}^{-\alpha(t=0)+1}\,. \tag{5}$$

Now the main question is how to generalize the scattering amplitude $A(t, x_{\mathrm{B}}, Q^2)$ to $A(\xi, \eta, t, x_{\mathrm{B}}, Q^2)$?

Let us consider the case of the real outgoing photon and small t. Then $-\eta = \xi$ and also $\xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}})$, the independent variables are only x_{B}, Q^2 ($t \ll s, Q^2$). If also $Q^2 \to 0$ we restore the symmetric situation and can write

$$\begin{aligned} &\mathrm{GPD}\left(x = \xi, \xi = x_{\mathrm{B}}/(2 - x_{\mathrm{B}}) \to 0, \eta = -\xi \to 0, t \to 0, x_{\mathrm{B}} \to 0, Q^2 \to 0\right) \\ &\quad = \mathcal{I}\mathrm{m}\; A_{\mathrm{DVCS}} \\ &\quad = \mathcal{I}\mathrm{m}\; A_{\mathrm{DIS}}(t \to 0, x_{\mathrm{B}} \to 0, Q^2 \to 0)\,. \end{aligned} \tag{6}$$

Our idea is to assume as a first approximation that the above expression is valid in all the range of x_{B} and Q^2, *i.e.* take

$$\begin{aligned} &\mathrm{GPD}_0(x, \xi = x, \eta = -x, t, x_{\mathrm{B}} = 2x/(1+x), Q^2) \\ &\quad = \mathrm{GPD}_0(x, t, Q^2) \\ &\quad = c\mathcal{I}\mathrm{m}\; A_{\mathrm{DIS}}(t, x_{\mathrm{B}} = 2x/(1+x), Q^2)\,, \end{aligned} \tag{7}$$

[1] The relation $F_2(x_{\mathrm{B}}, Q^2) = x_{\mathrm{B}}\; q(x_{\mathrm{B}}, Q^2)$ modifies if one takes into account higher order QCD corrections.

where c is a normalization coefficient. Then we use this GPD_0 as an input into the handbag diagram, whose convolution with the perturbative kernel (loop diagram) results in the complete DVCS amplitude. The resulting amplitude will have the good analytic and asymptotic properties s, t and Q^2, known from dual models.

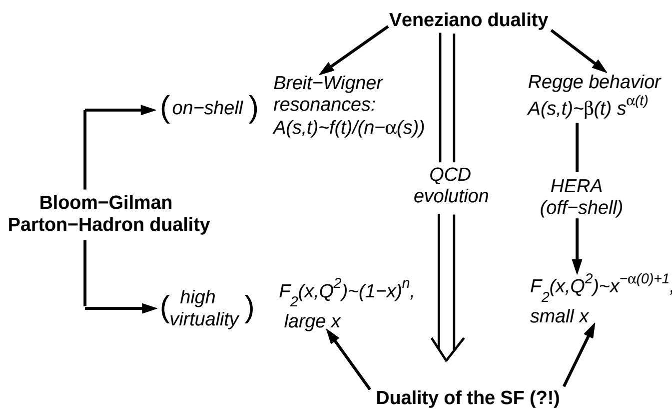

Fig. 4. Off-mass-shell continuation of the scattering amplitude. Veneziano, or resonance–reggeon duality [6] and Bloom–Gilman, or hadron–parton duality [7] in strong interactions. From [4].

As a candidate model for the DVCS we utilize the off-mass extrapolation of DAMA, so called modified DAMA (MDAMA) developed in [5]:

$$D(s,t,Q^2) = \int_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_{\mathrm{s}}(s')-\beta(Q^{2\prime\prime})-1} \left(\frac{1-z}{g}\right)^{-\alpha_t(t'')-\beta(Q^{2\prime})-1}, \tag{8}$$

where $a' = a(1-z)$, $a'' = az$, $\alpha_{\mathrm{s},t}$ are complex, nonlinear Regge trajectories in the corresponding channel, function $\beta(Q^2)$ is given by (see Ref. [5])

$$\beta(Q^2) = \begin{cases} -1 - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q^2+Q_0^2}{Q_0^2}\right) & \text{for } Q^2 \geq 0\,, \\ -1 - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q_0^2-Q^2}{Q_0^2}\right) & \text{for } Q^2 < 0\,, \end{cases} \tag{9}$$

and $g > 1$. Note that quarks and gluons here are manifest indirectly through the scaling behavior of the amplitude, due to the logarithmic asymptotic behavior of the trajectories. The complete amplitude is a sum of a singlet (pomeron) and nonsinglet (reggeon) terms (for more details see [4, 5]). Further calculations, and fits to the experimental data will be presented elsewhere [8].

To summarize, we have presented a fairly general model for GPDs. Its remaining flexibility can be used in solving the bootstrap program, to match with QCD evolution, and to fit the data on DVCS, measure now at JLab, HERA (HERMES, H1 and ZEUS) and at CERN (COMPASS). The virtue of the present approach is that we gain a complex DVCS amplitude, related to GPD. The knowledge of its explicit s-, t- and Q^2-dependence as well as of its complex phase (ratio of the real to the imaginary part) for all vales of the variables may provide us with a practical tool to be used in nuclear tomography and holography.

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