Vol. 36 (2005) ACTA PHYSICA POLONICA B No 2

A SUM RULE FOR ELASTIC SCATTERING*

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(Received October 25, 2004)

A sum rule is derived for elastic scattering of hadrons at high energies which is in good agreement with experimental data on $p\bar{p}$ available upto the maximum energy $\sqrt{s} = 2$ TeV. Physically, our sum rule reflects the way unitarity correlates and limits how large the elastic amplitude can be as a function of energy to how fast it decreases as a function of the momentum transfer. The universality of our result is justified through our earlier result on equipartition of quark and glue momenta obtained from the virial theorem for massless quarks and the Wilson conjecture.

PACS numbers: 12.38.–t, 13.60.Hb, 13.85.Lg

1. Introduction

Consider the elastic scattering of two hadrons (A and B) with the following kinematics

$$A(p_a) + B(p_b) \;\rightarrow\; A(p_c) + B(p_d)$$

with

$$\begin{aligned} s &= (p_a + p_b)^2 = (p_c + p_d)^2\,; \\ t &= (p_a - p_c)^2 = (-p_b + p_d)^2 = -\vec{q}^{\,2}\,; \\ u &= (p_a - p_d)^2 = (p_c - p_b)^2\,, \end{aligned} \tag{1.1}$$

* Presented at the XXXIV International Symposium on Multiparticle Dynamics, Sonoma County, California, USA, July 26–August 1, 2004.

and let us normalize the elastic amplitude $F(s,t)$ so that the elastic differential cross-section and the total cross-section (for high energies) read as

$$\left(\frac{d\sigma}{dt}\right) = \pi|F(s,t)|^2 ; \quad \sigma_{\rm tot}(s) = 4\pi {\rm Im}\ F(s,t=0)\,. \tag{1.2}$$

In the impact parameter representation

$$F(s,t) = i\int\limits_0^\infty (bdb) J_0(b\sqrt{-t})\tilde{F}(s,b)\,, \tag{1.3}$$

and the partial b-wave amplitude is given by

$$\tilde{F}(s,b) = 1 - \eta(s,b)e^{2i\delta(s,b)}\,, \tag{1.4}$$

where the inelasticity factor η lies between $(0 \le \eta(s,b) \le 1)$ and $\delta(s,b)$ is the real part of the phase shift. Directly measureable quantities are (a) $|F(s,t)|$ through $(\frac{d\sigma}{dt})$ and (b) Im $mF(s,0)$ through $\sigma_{\rm tot}$. In Section 2, we shall obtain lower and upper bounds for a dimensionless quantity $I_0(s)$ constructed by integrating $|F(s,t)|$ over all momentum transfers t. Under rather mild assumptions, at high energies $(s \to \infty)$ it is sharpened into a sum rule

$$I_0(s) = (1/2)\int\limits_0^\infty (dt)\sqrt{\frac{d\sigma}{\pi dt}} = \int\limits_0^\infty (qdq)|F(s,q)| \ \to \ 1\,. \tag{1.5}$$

In Section 3, we compare these predictions with the experimental data and find that already at the Tevatron $\sqrt{s} = 2$ TeV, the integral has the value 0.98 ± 0.03 very close to its asymptotic limit 1. Our extrapolation for LHC gives 0.99 ± 0.03. Also, a brief discussion of the assumptions and an estimate of the elastic cross-section is presented here. In Section 4, we present arguments based on an equipartition of energy between quark and glue derived earlier, for the universality of the above result for all hadrons made of light quarks. In the concluding section, we consider future prospects and possible applications.

2. Lower and upper bounds and the elastic sum rule

The dimensionless b-wave cross sections are

$$\frac{d^2\sigma_{\rm el}}{d^2b} \ = \ 1 - 2\eta(s,b)\cos 2\delta(s,b) + \eta^2(s,b)\,, \tag{2.1a}$$

$$\frac{d^2\sigma_{\rm inel}}{d^2b} \ = \ 1 - \eta^2(s,b)\,, \tag{2.1b}$$

$$\frac{d^2\sigma_{\rm tot}}{d^2b} \ = \ 2[1 - \eta(s,b)\cos 2\delta(s,b)]\,. \tag{2.1c}$$

The maximum permissible rise for the different cross sections allowed by unitarity [1–4] is when there is total absorption of "low" partial waves, *i.e.*, when

$$\eta(s,b) \to\ 0,\ \text{as } b \to\ 0 \text{ and } s \to\ \infty\,, \tag{2.2}$$

and the "geometric" limit is reached

$$\frac{d^2\sigma_{\rm el}}{d^2b} = \frac{d^2\sigma_{\rm inel}}{d^2b} = (1/2)\frac{d^2\sigma_{\rm tot}}{d^2b}\ \to\ 1\ (b \to\ 0; s \to\ \infty)\,. \tag{2.3}$$

Most models with rising total cross-sections satisfy the above [5–10]. Often times, one defines $\eta(s,b)\ = e^{-n(s,b)/2}$ and $n(s,b)$ is interpreted as the number of collisions at a given impact parameter b and energy $\sqrt{s}$.

Now let us consider bounds for the dimensionless integral $I_0(s)$ defined in Eq. (1.5). The lower bound is easily obtained

$$I_0(s) \geq \int_0^\infty (qdq)|\mathrm{Im}\ F(s,q)| \geq \int_0^\infty (qdq)\mathrm{Im}\ F(s,q), \tag{2.4a}$$

which upon using Eq. (1.3) leads to

$$I_0(s) \geq \int (qdq) \int (bdb) J_0(qb)[1-\eta(s,b)\cos\delta(s,b)]\,, \tag{2.4b}$$

so that we have finally

$$I_0(s) \geq 1 - \eta(s,0)\cos 2\delta(s,0) \geq 1-\eta(s,0)\,. \tag{2.4c}$$

The upper bound requires more input [23]. If we assume (an ugly technical assumption) that $\sin 2\delta(s,b)$ does not change sign (to leading order in s), then one has the following upper and lower bounds

$$\left(1 + \frac{K}{\ln(s/s_0)}\right)\ \geq\ I_0(s)\ \geq\ 1-\eta(s,0)\,, \quad (K\ >\ 0)\,. \tag{2.5}$$

These bounds have been obtained incorporating *(i)* unitarity, *(ii)* positivity, *(iii)* correct behavior near $b\ =\ 0$ and *(iv)* the asymptotic behavior for $b \to\ \infty$.

Some useful remarks: (1) For hadrons (*not* quarks and glue), the lowest hadronic state has a finite mass ($m_\pi\ >\ 0$), hence there is a finite range of interaction. Thus, in the limit of both b and s going to ∞, we have

$$1-\eta(s,b)\cos 2\delta(s,b)\ \to\ 0; \eta(s,b)\sin 2\delta(s,b)\ \to\ 0\,, \tag{2.6}$$

faster than an exponential in b. (2) The higher moments

$$I_n(s) = \int (dt)(-t)^n |F(s,t)|\,,\ (n \ = \ 1, 2, ...)\,, \tag{2.7}$$

are dimensional and go to zero in the asymptotic limit. Thus, they are less useful than the zeroeth moment.

From Eq. (2.5), we obtain the sum rule as $s \to \ \infty$

$$I_0(s) \to 1\,,\ \text{as}\ s \ \to \ \infty\,. \tag{2.8}$$

3. Comparison of the sum rule with experimental data

The integral $I_0(s)$ should rise from its threshold value $2|a_0|k \ \ \to \ \ 0$, where a_0 is the S-wave scattering length (complex for $p\bar{p}$) and k is the CM 3-momentum, to its asymptotic value 1 as s goes to infinity. In Fig. 1, we show a plot of this integral for available data [11–18] on pp and $p\bar{p}$ elastic scattering for high energies [19]. Highest energy data at $\sqrt{s} = 1.8$ TeV for $p\bar{p}$ from the Fermilab Tevatron [11], give an encouraging value of $0.98 \ \pm \ 0.03$ demonstrating that indeed the integral is close to its asymptotic value of 1. We expect it to be even closer to 1 at the LHC (our extrapolation gives the value 0.99 ± 0.03 for LHC).

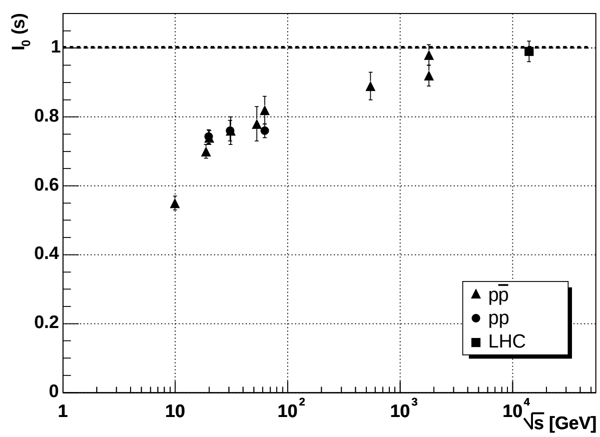

Fig. 1. A plot of $I_0(s)$ vs. $\sqrt{s}$ using experimental data [11–18]. The last point is our extrapolation for LHC.

4. Universality of the sum rule

It can be shown that the central value of the inelasticity $\eta(s,0) \to \ 0$ at asymptotic energies $s \ \to \ \infty$ for *all* hadrons made of light quarks. Hence, we have the universal result [23] that $I_{AB}(s) \to 1$ as $s \to \infty$, where A, B are either nucleons or mesons made of light quarks. The reasons are as follows:

(i) For nucleons as well as light mesons, half the hadronic energy is carried by glue. In QCD such an equipartition of energy is rigorously true [20, 24] for hadrons made of massless quarks if the Wilson area law holds.

(ii) If we couple *(i)* to the notion that the rise of the cross-section is through the gluonic channel, which is flavour independent, the asymptotic equality of the rise in all hadronic cross-section automatically emerges.

5. Conclusions

Our (dimensionless) sum rule reflects the fact that unitarity strongly correlates the fall off in the momentum transfer to the magnitude of the scattering amplitude at high energies. Its satisfaction by experimental data at the highest energy confirms our initial hypothesis that the rise in the total cross-section as a function of the energy is indeed proportional to the fall off in the momentum transfer. As a by product, we find that the ratio $\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} \to (1/4)$, which is again in very good agreement with data at the highest Tevatron energy $\sqrt{s} = 2$ TeV.

We also find universality. That is, asymptotically, $I_{AB} \to 1$ for any hadrons A, B made of light quarks. These may be testable at future LHC and RHIC measurements with heavy ions (or by other means [21].

Currently, we are extending similar considerations for one particle inclusive cross-sections.

It is a pleasure to thank Paolo Giromini and Allan Widom for fruitful discussions. We also take the opportunity to thank Bill Gary and other organizers of this meeting for their kind hospitality and for making this conference stimulating and enjoyable.

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