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STATISTICAL DESCRIPTION OF THE THERMAL SHAPE FLUCTUATIONS USING REALISTIC MICROSCOPIC AND MACROSCOPIC MODELS*

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We calculate the probability distribution describing the thermal fluctuations of the nuclear shape, using Lublin–Strasbourg Drop (LSD) model for the macroscopic nuclear energies and deformed Woods-Saxon model for the single-nucleonic level densities. Examples of applications are presented in the form of the GDR spectra of hot ^{46}Ti and ^{216}Rn nuclei.

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1. Introduction

In the following we are going to present the new calculation results of the statistical shape-fluctuations in hot, spinning nuclei. The total nuclear energy at temperatures $T \sim 1$ MeV will be approximated using the recent macroscopic approach, the Lublin–Strasbourg Drop (LSD) model [1, 2]. We will also calculate the single-nucleonic level-densities using the microscopic deformed Woods–Saxon mean-field with the universal parameters of Ref. [3]. As an example of this approach, we will then present the modeling of the Giant Dipole Resonance (GDR) spectra in hot, rotating nuclei, using a rather well known formalism based on the cranked Harmonic Oscillator [4].

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There have been several studies in the past employing the concept of the thermal fluctuations in the hot, rotating nuclei, while aiming at the description of the Giant Dipole Resonances (*e.g.* Ref. [5–8] and references therein). In the present approach some refinements were undertaken. Firstly, the temperature is calculated locally at each deformation point assuming the fixed total excitation energy of the nucleus. This is done by using the realistic single-particle spectra as well as the newest, realistic model for the macroscopic nuclear energy. The nuclear rotation-coupling to the GDR-vibrations is modeled with the help of the (simple but microscopic) approach involving the rotating harmonic oscillator.

2. Thermal fluctuations of the nuclear shapes

Let us consider a nucleus at an excitation energy E^*. In a simplified approach followed here, a part of this energy can be associated with the deformation of the system while the other part with the thermal excitation of the individual-nucleonic degrees of freedom. This thermal excitation allows to introduce the notion of the entropy and free energy in accordance with the standards of statistical physics. At fixed excitation energy the latter quantities, in general, depend on the deformation. Since the probability of a given configuration, and thus of an associated shape, in turn depends on the free-energy, there should be a whole ensemble of such configurations (thus shapes) in the description. Each shape will appear with a certain statistical probability, therefore one may speak of the shape probability distributions and, less precisely, of shape fluctuations.

Let us introduce the probability for the nucleus to be in a given state of deformation 'def', $P(\mathrm{def}, E^*)$, defined as usual in statistical physics by

$$P(\mathrm{def}, E^*) \sim \exp\left\{-\frac{F(\mathrm{def}, E^*)}{kT(\mathrm{def}, E^*)}\right\}, \tag{1}$$

where E^* is the excitation energy, $F(\mathrm{def}, E^*)$ is the nuclear free energy, $T(\mathrm{def}, E^*)$ is the nuclear temperature, and k is the Boltzmann constant. One can show that the nuclear partition function that will be needed below has the following expression (see Ref. [9], p. 281–289, for details):

$$\begin{aligned}\ln Z(\alpha_n, \alpha_p, \beta) \; = & \int\limits_{-\infty}^{\infty} g_n(\epsilon) \ln\left[1 + \exp(\alpha_n - \beta\epsilon)\right] d\epsilon \\ & + \int\limits_{-\infty}^{\infty} g_p(\epsilon) \ln\left[1 + \exp(\alpha_p - \beta\epsilon)\right] d\epsilon \,. \end{aligned} \tag{2}$$

Above, the level densities $g_p(\epsilon)$ and $g_n(\epsilon)$ that depend explicitly on the single-nucleonic spectra will be computed using a realistic-cranking deformed Woods–Saxon single-particle potential. The values of α_p, α_n and β originate from the Fermi gas model:

$$\beta = T^{-1} \quad \text{and} \quad \alpha_\kappa = \lambda_\kappa^{\mathrm{F}}\, T \quad \text{with} \quad \kappa \in \{p, n\}, \tag{3}$$

where $\lambda_\kappa^{\mathrm{F}}$ are the Fermi energies for neutrons ($\kappa = n$) and for protons ($\kappa = p$). One can demonstrate that the nuclear entropy and the free energy have the following expressions[1]:

$$S(\alpha_n, \alpha_p, \beta) \;=\; -\alpha_n N - \alpha_p Z + \beta E + \ln Z(\alpha_n, \alpha_p, \beta)\,, \tag{4}$$

$$F(\alpha_n, \alpha_p, \beta) \;=\; E - TS(\alpha_n, \alpha_p, \beta)\,, \tag{5}$$

where E is the actual deformation energy of the nucleus, here approximated by the LSD expression. The nuclear temperature T is obtained from the energy conservation:

$$E^* = E_p(T) + E_n(T) - E_p(T=0) - E_n(T=0)\,, \tag{6}$$

where

$$E_\kappa(T) \;=\; \int\limits_{-\infty}^{\infty} g_\kappa(\epsilon)\Big[1 + \exp\left[(\epsilon - \lambda_\kappa^{\mathrm{F}})/T\right]\Big]^{-1} \epsilon d\epsilon \tag{7}$$

and

$$E_\kappa(T=0) \;=\; \int\limits_{-\infty}^{\lambda_\kappa^{\mathrm{F}}} g_\kappa(\epsilon)\epsilon d\epsilon\,. \tag{8}$$

Fermi energies $\lambda_\kappa^{\mathrm{F}}$ are calculated from the particle-number conservation.

Below, the nuclear quadrupole deformation will be represented using the usual (β, γ)-plane; for convenience we will use the Cartesian coordinates

$$x \;=\; \beta\cos(\gamma + 30^\circ)$$

and

$$y \;=\; \beta\sin(\gamma + 30^\circ)\,. \tag{9}$$

In order to calculate the deformation-averaged value of a nuclear observable, say $f(\mathrm{def})$, we integrate it with the normalized probabilities after replacing ‘def’ $\leftrightarrow \{x, y\}$:

[1] These can be found in [9], p. 288.

$$\langle f(E^*)\rangle_{\rm def} = \iint\limits_{x,y} P(x,y;E^*) f(x,y)\, dV\,. \tag{10}$$

The volume element dV can be represented by $dxdy$ or $\beta^4|\sin(3\gamma)|d\beta d\gamma$, depending on the metric used (see discussion in [8,9]).

The upper-left panel of Fig. 1 shows the macroscopic nuclear energy calculated with the help of the LSD model [1,2] in the (β_2,γ) plane. Using this energy and the Woods–Saxon single-particle spectra, the individual level densities and then the free energy, deformation-dependence of the temperature, and the probabilities to find the nucleus in any (x,y)-deformation point can be calculated. The results are shown also in Fig. 1.

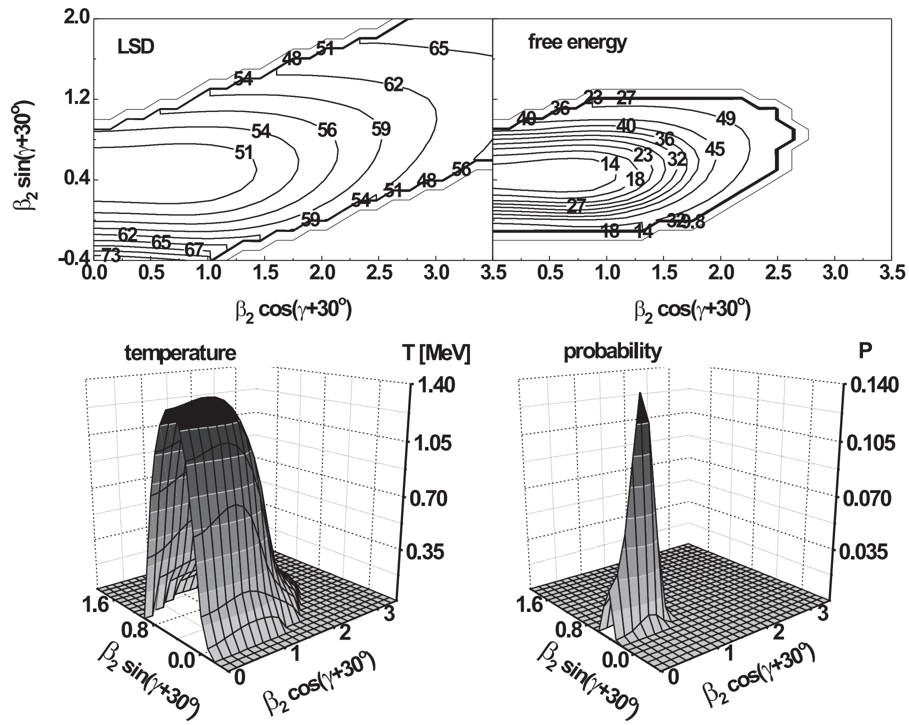

Fig. 1. The macroscopic LSD potential energy and free energy maps (top). The macroscopic energy is normalized in such a way that the corresponding energy of the *spherical* nucleus is, by definition, zero. The free energy corresponds then to Eq. (5). The deformation-dependent temperature and the deformation probability distributions in the (β_2,γ) plane for the ^{46}Ti nucleus at spin $I = 30\hbar$ and $E^* = 60$ MeV are given in the bottom.

3. Application for the GDR spectrum of a hot nucleus

In the following, we summarize an approach of Ref. [4], based on the deformed harmonic oscillator (HO) Hamiltonian, $\hat{H}^\omega$, with a cranking term. It allows to model a GDR spectrum for a nucleus in a given state of deformation. In this approach, the rotation gives rise to a Coriolis splitting of the two GDR frequencies for the oscillation perpendicular to the axis of rotation and thus the GDR strength function consists, in general, of 5 components. In the case of rotation about the '1'-axis, the cranking-model harmonic-oscillator Hamiltonian reads:

$$\hat{H}^\omega = \sum_{\alpha=1}^{3} \Big(\frac{\hat{p}_\alpha^{\,2}}{2m} + \frac{m}{2}\,\omega_\alpha^2\,\hat{x}_\alpha^2 \Big) - \omega\,(\hat{x}_2\,\hat{p}_3 - \hat{x}_3\,\hat{p}_2)\,. \tag{11}$$

This Hamiltonian can be diagonalized analytically, Ref. [4], and we obtain:

$$\hat{H}^\omega = \sum_{\alpha=1}^{3} \Omega_\alpha(\boldsymbol{a}_\alpha^+ \boldsymbol{a}_\alpha + \tfrac{1}{2})\,, \tag{12}$$

where $\boldsymbol{a}_\alpha^+$ and $\boldsymbol{a}_\alpha$ are the usual creation and annihilation operators, and

$$\Omega_1 = \omega_1; \quad \Omega_2^2 = \frac{1}{2}(\omega_2^2+\omega_3^2)+\omega^2+\Delta; \quad \Omega_3^2 = \frac{1}{2}(\omega_2^2+\omega_3^2)+\omega^2-\Delta \tag{13}$$

with

$$\Delta = \sqrt{\tfrac{1}{4}(\omega_2^2-\omega_3^2)^2 + 2\omega^2(\omega_2^2+\omega_3^2)}\,. \tag{14}$$

It turns out that the final radiation probability, $\Gamma(E_\gamma)$, corresponding to the radiation energy E_γ from a nucleus turning about the '1'-axis is:

$$\begin{aligned}\Gamma(\mathrm{def};E_\gamma) \;\sim\; & \frac{1}{2m_n\Omega_1}\left[1+e^{-\Omega_1/T}\right]^{-1} L(E_\gamma,\Omega_1)\\ & +\frac{1}{2}(\alpha+\beta)^2\left[1+e^{-\Omega_2/T}\right]^{-1} L(E_\gamma,\Omega_2-\omega)\\ & +\frac{1}{2}(\gamma+\delta)^2\left[1+e^{-\Omega_3/T}\right]^{-1} L(E_\gamma,\Omega_3-\omega)\\ & +\frac{1}{2}(\alpha-\beta)^2\left[1+e^{-\Omega_2/T}\right]^{-1} L(E_\gamma,\Omega_2+\omega)\\ & +\frac{1}{2}(\gamma-\delta)^2\left[1+e^{-\Omega_3/T}\right]^{-1} L(E_\gamma,\Omega_3+\omega)\,,\end{aligned} \tag{15}$$

where m_n is the nucleonic mass and the symbol L represents the Lorentz function

$$L(E_\gamma,E_i) = \frac{W_i E_\gamma^2}{(E_\gamma^2-E_i^2)^2+(E_iW_i)^2}\,. \tag{16}$$

Above, $W_i = W_0(E_i/E_0)^\rho$, contains a phenomenological scaling factor (here we use $\rho = 1.9$), W_0 and E_0 are the GDR intrinsic width and centroid for the ground state of spherical nucleus, while W_i and E_i correspond to the width and centroid of the given GDR component, respectively (see *e.g.* [10,11] for more details). The quantities α, β, γ and δ are functions of the three HO frequencies ω_i and the cranking frequency ω:

$$\alpha^2 = \frac{1}{2m_n\Omega_2}\frac{\Omega_2^2 - \omega_3^2 + \omega^2}{2\Delta}\,, \tag{17}$$

$$\beta^2 = \frac{1}{2m_n\Omega_2}\frac{\Omega_2^2 - \omega_2^2 + \omega^2}{2\Delta}\,, \tag{18}$$

$$\gamma^2 = \frac{1}{2m_n\Omega_3}\frac{\Omega_3^2 - \omega_2^2 + \omega^2}{2\Delta}\,, \tag{19}$$

and

$$\delta^2 = \frac{1}{2m_n\Omega_3}\frac{\Omega_3^2 - \omega_3^2 + \omega^2}{2\Delta}\,, \tag{20}$$

(the reader is referred to Refs. [4,13] for details). According to Eq. (10), the average total radiation probability corresponding to GDR built on a nucleus with excitation energy E^* is:

$$\Gamma_{\rm total}(E_\gamma; E^*) = \iint\limits_{x,y} P(x,y;E^*)\Gamma(x,y;E_\gamma)dV\,. \tag{21}$$

4. Example of computed GDR spectra

Figure 2 shows the calculated GDR spectra obtained by applying the technique presented in the preceding Sections for two cases: ^{46}Ti and ^{216}Rn. To visualize the role of the rotation (*i.e.* of the Coriolis splitting of the GDR components), for the ^{46}Ti case, the spectra with $\omega = 0$ value are also computed (for the Rn case, in the experimental range of interest here the ω remains anyway small). For comparison, the experimentally extracted GDR strength functions for ^{46}Ti [12,14] and ^{216}Rn [15] are displayed in the same figure. The results show clearly that introducing the effect of rotation one improves the comparison with experiment considerably in the case of light nuclei. It should also be emphasized that the use of the presented here thermal shape fluctuation approach based on LSD model, results in an overall good quality of the reproduction of the GDR spectra both for ^{46}Ti and ^{216}Rn nuclei.

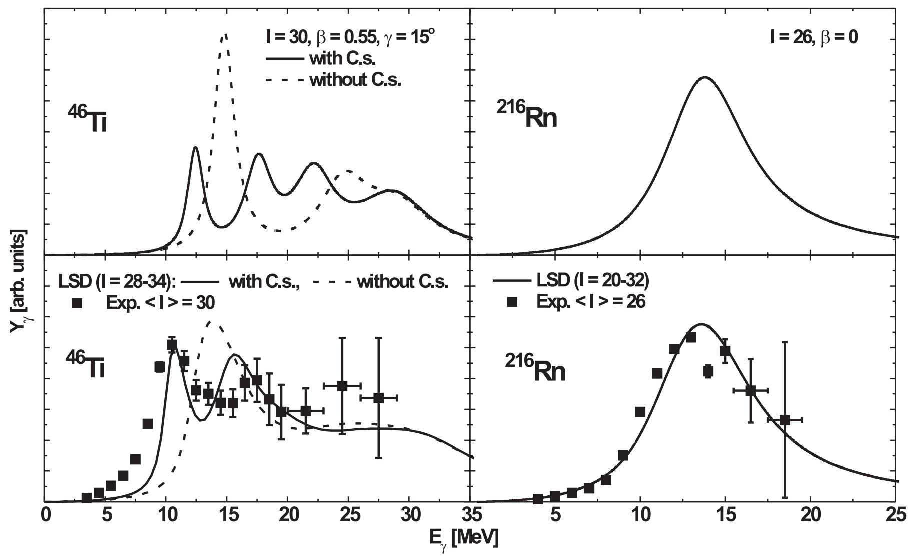

Fig. 2. Computed spectra (lines) and the experimental data (points) for the ^{46}Ti and the ^{216}Rn nuclei. The dashed curves correspond to $\omega = 0$ condition. The computed spectra in the top panels are for the equilibrium deformation predicted by LSD model at a given spin, while those in the bottom ones are the results of thermal shape fluctuations for a spin range corresponding to the experimental one. In all calculations the intrinsic GDR width W_0 equals to 6 MeV.

5. Conclusion

The statistical approach presented here uses the recent LSD macroscopic model as well as the single particle spectra from the deformed Woods–Saxon mean-field approximation with the universal parameters. It seems that the approach provides a powerful tool permitting to extend the use of models designed for cold nuclei to nuclei with thermal excitations. As shown in the example, the statistical description of GDR spectra gives a very good agreement with experimental data, both for light and heavy nuclei, despite of the simplicity of the model used for the GDR strength function (cranked Harmonic Oscillator). This makes us confident that statistical approach of the kind described here for the theoretical description of the properties of hot and rotating nuclei, especially in function of spin and very large deformations, where the LSD approximation works very satisfactorily, will be very useful also in the future.

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