HARMONIC MIXING IN A BISTABLE DEVICE*

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(Received January 21, 2005)

Dedicated to Professor Andrzej Fuliński on the occasion of his 70th birthday

A Brownian particle hopping in a symmetric double-well potential can be statistically confined into either well by the action of two periodic input signals that rock the potential simultaneously. The underlying harmonic mixing mechanism exhibits resonant behavior responsible for asymmetry inversion. Asymmetric confinement through harmonic mixing can be conveniently controlled by tuning the input signal parameters (frequencies, relative phase, and amplitudes).

PACS numbers: 05.40.–a, 02.50.Ey, 82.20.–w

1. Introduction

A charged particle spatially confined on a periodic substrate is capable of mixing two alternating input electric fields of angular frequencies Ω_1 and Ω_2, its response containing all possible higher harmonics of Ω_1 and Ω_2. For commensurate input frequencies, *i.e.*, $m\Omega_1 = n\Omega_2$, the output contains a dc component, too [1,2]; harmonic mixing (HM) thus induces a rectification effect of the $(n+m)$-th order in the dynamical parameters of the system [3]. At variance with common ratchet devices [4], such an effect takes place also on a reflection-symmetric substrate, as it is an intrinsically nonlinear effect.

* Presented at the XVII Marian Smoluchowski Symposium on Statistical Physics, Zakopane, Poland, September 4–9, 2004.

Then the question rises naturally as how HM affects the Brownian dynamics in a symmetric bistable potential.

A Brownian particle bound by a bistable potential diffuses symmetrically between two potential minima; this is the case of Kramers' dynamics [5], where the particle is activated solely by thermal fluctuations, as well as of stochastic resonance (SR) [6], where the particle escape over the potential barrier is controlled by the interplay of noise and external periodic drive(s). In both cases the time-averaged particle distribution densities peak symmetrically in correspondence with the potential minima; particle localization into one well is customarily achieved by applying an external static bias that breaches the symmetry of the system [7].

For practical purposes experimenters are interested in confining the diffusing particle around one stable configuration and then manipulating it by means of various techniques from magnetic flux microscopy [8] to chemical reaction control [9]. However, in most circumstances adding an external bias to the system under study is inconvenient; hence, the need for an alternate approach to the confinement problem.

In a recent paper we proved that confinement in a noisy bistable device may be achieved without apparent symmetry breaking [10]. A Brownian particle driven by a white, zero-mean Gaussian noise (mimicking thermal fluctuations) and, possibly, by an additive sinusoidal force with angular frequency Ω_1 [11, 12], can be localized into one state by modulating the potential barrier separating the two degenerate states. To this purpose one can either input a sinusoidal control signal with frequency Ω_2 or recycle the additive noise back through a noisy transmission line with time delay τ_d and residual correlation λ. In both schemes the corresponding steady distribution densities develop one prominent peak, whose relative magnitude hits a *resonance* maximum (of over 95%) for optimal values of the input parameters (Ω_1 and Ω_2, or τ_d and λ, respectively) which degenerate state the particle gets trapped into, depends on the switch-on phase of the modulating signal.

In the present article we prove that HM in symmetrically confined systems causes a static effect in the form of a dynamical symmetry breaking of the relevant average probability distributions.

The present article is organized as follows. In Sec. 2 we introduce a simple model of bi-harmonically rocked double-well. In Sec. 3 we show that the overdamped stochastic dynamics which takes place over the rocked barrier tends to be confined into one preferred well, depending on the frequency of the applied drive components. Note that this phenomenon has been overlooked even in the earlier SR literature [6, 13, 14]. In Sec. 4 we interpret the resonant nature of asymmetric confinement as a synchronization phenomenon; our analysis is based on the notion of residence time distribution developed in Refs. [15–18].

2. Model

The key mechanism underlying the phenomenon of asymmetric confinement is apparent in the study model, illustrated in Fig. 1, of an overdamped Brownian particle of coordinate $x(t)$ diffusing in a quartic double-well potential $V(x) = -ax^2/2 + bx^4/4$, with $a, b > 0$, subjected to a zero-mean Gaussian noise $\xi(t)$ and a bi-harmonic, zero-mean valued drive $F(t)$ with harmonic components of period $T_i = 2\pi/\Omega_i$, $i = 1, 2$, *i.e.*

$$\dot{x} = ax - bx^3 + ax_0 F(t) + \xi(t)\,, \tag{1}$$

where

$$F(t) = A_1 \cos(\Omega_1 t + \phi_1) + A_2 \cos(\Omega_2 t + \phi_2) \tag{2}$$

and $\langle \xi(t)\xi(0) \rangle = 2D\delta(t)$. Here, the additive signal $F(t)$ tilts the potential sidewise between the two configurations sketched in figure 1(lower). In the absence of a drive, the barrier separating the degenerate minima

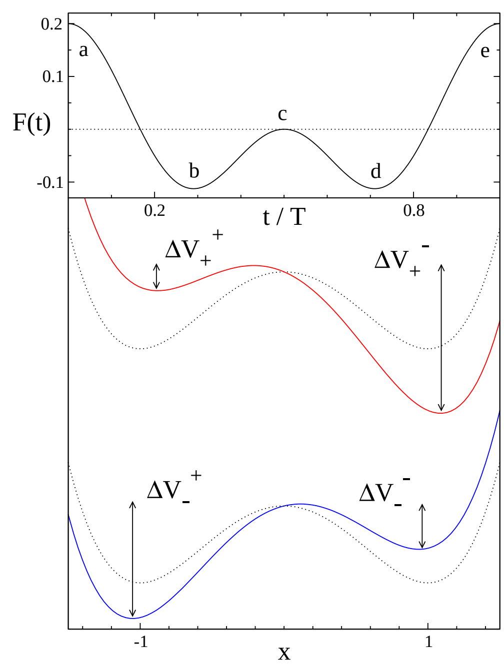

Fig. 1. Top panel: Bi-harmonic drive (2) with $\phi_1 = \phi_2 = 0$ and $\Omega_2 = 2\Omega_1$. Bottom panel: Rocked quartic double-well potential (1): $a = b = 1$, $A_1 = A_2 = 0.1$. The two configurations for tilts a (upper) and b, d (lower) are contrasted with the unperturbed potential $V(x)$ (pointed curve). The relevant barrier heights are: $\Delta V_-^- = 0.147$, $\Delta V_+^+ = 0.08$, $\Delta V_-^+ = 0.37$, and $\Delta V_+^- = 0.48$ (see text).

$\pm x_0 = \pm\sqrt{a/b}$ is symmetric with $\Delta V_0 = a^2/4b$; being the waveform (2) plotted in Fig. 1(upper) asymmetric, the barrier to overcome during a left-to-right (right-to-left) jump, oscillates between the two different extremal values $\Delta V_{\pm}^{+}$ ($\Delta V_{\pm}^{-}$).

The steady (time-averaged) distribution densities $P(x)$ of the stochastic process (1) have been computed by standard numerical simulation. In order to quantify the asymmetry of the dynamics (1), we introduced [10] the subtracted asymmetry factor $\sigma \equiv P_-/P_+ - 1$, with $P_\pm \equiv \left\langle \int_0^\infty P(\pm x, t)dx \right\rangle_t$ and $\langle \ldots \rangle_t$ denoting the stationary time average

$$\langle \ldots \rangle_t \equiv \lim_{\tau\to\infty} \int\limits_0^\tau (\ldots)dt\,.$$

For periodic drives in the stationary regime τ can coincide with the forcing cycle.

3. Simulation results

An asymmetry in the time averaged probabilities $P(x)$ of a modulated bistable system can always be traced back to some inherent asymmetry of the driving mechanism. By inspecting Fig. 1(upper), it is apparent that $F(t)$ is not perfectly symmetric. Here, "perfectly symmetric" means that all force moments are invariant under sign reversal $F \to -F$; a vanishing dc component, $\lim_{t\to\infty} \frac{1}{t}\int_0^t F(s)ds = 0$, would not be sufficient! The two-frequency signal (2), although zero-mean valued, is not perfectly symmetric for commensurate frequencies (with the exception of special values of $\phi_1 - \phi_2$); therefore, the dynamics (1) is symmetric under signal reversal $F \to -F$ only for irrational Ω_1/Ω_2, see Fig. 2(b).

It is not surprising that the time averaged probability densities of the process (1) develop a certain degree of asymmetry for commensurate drive frequencies — signaled by a non-zero factor σ (Fig. 2). However, determining the sign of σ is not a straightforward task.

In the adiabatic limit of Fig. 2, $\Omega_1, \Omega_2 \to 0$ with $\Omega_2/\Omega_1 = 2$ and $\phi_1 = \phi_2$, the zero-mean periodic drive $F(t)$ is negative during a larger fraction of its period T_1 than it is positive. As a consequence, one predicts an accumulation of the particle distributions into the negative well, that is $\sigma > 0$. This is confirmed by the adiabatic estimate

$$P(x) = \langle P(x,t) \rangle_t\,, \tag{3}$$

with $P(x,t) = N(t)\exp[-V(x,t)/D]$, $V(x,t) = V(x) - ax_0xF(t)$, and $\int_{-\infty}^{\infty} P(x,t)dx \equiv 1$; the resulting asymmetry factor attains a positive maximum for vanishingly low commensurate frequencies, in agreement with our numerical simulation, see Fig. 2(b).

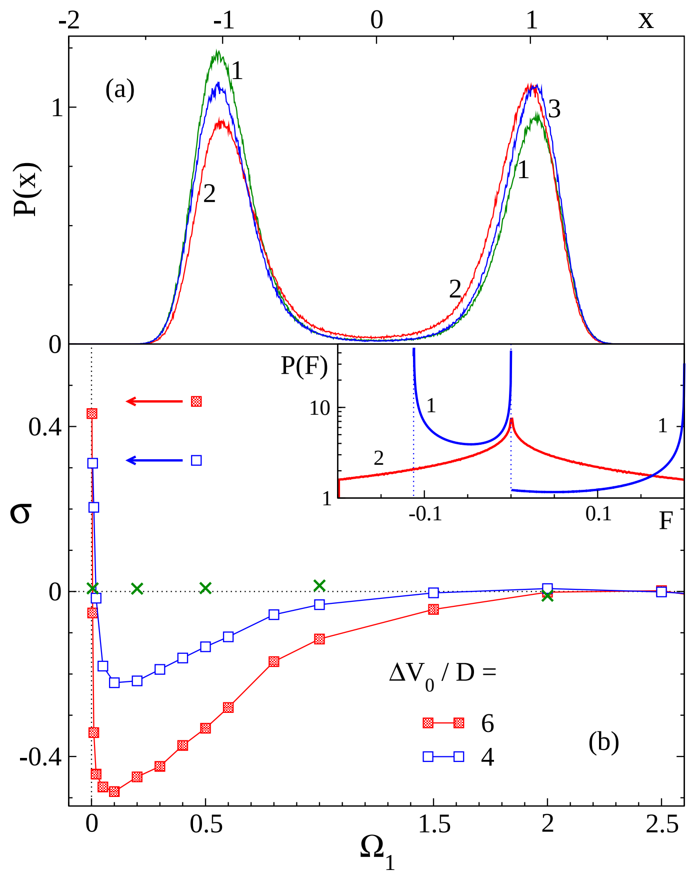

Fig. 2. Bi-harmonically rocked double-well potential (1)–(2): (a) Time-averaged probability densities $P(x)$ of the process (1) for $\Delta V_0/D = 4$, and $\Omega_1 = 0.005$ and $\Omega_2 = 0.01$ (curve 1), $\Omega_1 = 0.1$ and $\Omega_2 = 0.2$ (curve 2), and $\Omega_1 = 0.1$ and $\Omega_2 = 0.1 \times \sqrt{5}$ (curve 3). (b) Subtracted asymmetry σ *versus* Ω_1; empty squares: $\Omega_2 = 2\Omega_1$ and $\Delta V_0/D = 4$; solid squares: $\Omega_2 = 2\Omega_1$ and $\Delta V_0/D = 6$; crosses: $\Omega_2 = \sqrt{5}\Omega_1$ and $\Delta V_0/D = 4$. Horizontal arrows point to the analytic estimate (3) for $\Omega_2 = 2\Omega_1 \to 0$. Inset: Distribution of the simulation forcing signal (2) sampled with time step 0.001: curve 1: $\Omega_2 = 2\Omega_1$ (asymmetric, commensurate case); curve 2: $\Omega_2 = \sqrt{5}\Omega_1$ (symmetric, incommensurate case). Other simulation parameters: $a = b = 1$, $A_2 = A_1 = 0.1$, $\phi_1 = \phi_2$ and integration time step 0.001.

Furthermore, numerical simulation shows that on increasing Ω_1 with $\Omega_2/\Omega_1 = 2$, σ decreases and eventually changes sign. This is an instance of the phenomenon known as resonant activation [17, 19]. Due to the tilting action of $F(t)$, the bistable potential oscillates between the two configurations sketched in Fig. 1(lower), that is, when jumping to the right (left), the particle overcomes different tilted barriers with heights $\Delta V_{\pm}^{+}$ ($\Delta V_{\pm}^{-}$), respectively; here, these four barrier heights obey the inequalities $\Delta V_{+}^{+} < \Delta V_{-}^{-} < \Delta V_{-}^{+} < \Delta V_{+}^{-}$. Consequently, for low noise intensities the direct left-to-right escape time T_{+}^{+} is much shorter than the reverse escape time T_{-}^{-}. For $\phi_1 = \phi_2$ the forcing waveform (2) develops large amplitude crests of relatively short time duration: as long as the forcing period T_1 is larger than T_{+}^{+}, but shorter than T_{-}^{-}, the particle flow from left to right is favored and σ stays negative.

However, when the forcing period T_1 grows much longer than T_{-}^{-}, probability leakage through direct escape over both barrier configurations starts degrading asymmetric confining, so that P_{-}/P_{+} decays back *close* to unity. For extremely fast oscillations of $F(t)$ the Brownian particle sees an average potential $\langle V(x,t)\rangle_t = V(x)$, *i.e.* no asymmetry effects are detectable. This is the behavior displayed by the curves σ versus Ω_1 in Fig. 2(b).

The process (1) exhibits asymmetric confinement as a result of the nonlinear combination of the two harmonic components of $F(t)$. Such a mechanism, experimentally established as *harmonic mixing*, was not much explored in the context of transport theory until very recently [11, 20]. Note that the HM induced probability asymmetry changes sign with Ω_1 (for Ω_1/Ω_2 a constant rational number), thus making this effect less robust and predictable than the gating effect of Refs. [10, 21].

4. Discussion

The dynamics of confinement under the conditions of Fig. 2 is further illustrated in Fig. 3, where the distributions of the escape times T from left to right, $N_{+}(T)$, and vice versa, $N_{-}(T)$, are plotted for three choices of the modulation frequency Ω_1 corresponding to σ close to its maximum (zero-frequency limit), minimum (resonant confinement), and zero asymptotic value (high frequency limit). As expected [15, 16], all plotted escape time distributions can be regarded as the superposition of an exponential background accounting for totally random switches, and a regular structure of relatively sharp peaks clocked by the external signal $F(t)$. In panel 3(c) the peak structure merges into the background as Ω_1 is too large to synchronize the dynamics (1) appreciably; $N_{\pm}(T)$ tend to overlap and their decay time approaches the Kramers rate over the unperturbed activation barrier ΔV_0. This is consistent with a vanishingly small asymmetry factor, $\sigma \to 0$.

In panel 3(b) the peak structure is dominant and spans over dozens of forcing cycles T_1; this implies that the dynamical locking of the hopping particle to the external drive is not very efficient [15, 16]. The envelope of both peak structures decays exponentially, but faster for $N_{+}(T)$ than for

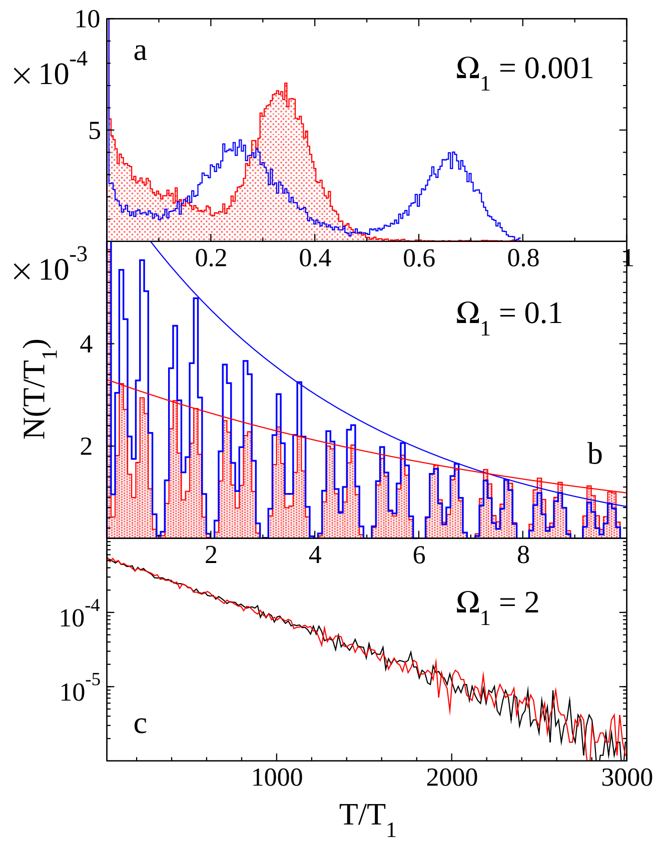

Fig. 3. Distributions of the left-to-right, $N_+(T)$, and the right-to-left escape times, $N_-(T)$, for the process (1)-(2) for $\Delta V_0/D = 6$, $A_1 = A_2 = 0.1$, $\phi_1 = \phi_2$ and $\Omega_2 = 2\Omega_1$. In panels (a) and (b) the curves $N_-(T)$ are shaded; in panel (c) $N_+(T)$ and $N_-(T)$ are statistically indistinguishable. The exponential envelopes of the peak structures in panel (b) are drawn as a guide to the eye.

$N_-(T)$; as a consequence, the particle distribution tends to accumulate into the positive potential well rather than into the negative one and σ turns negative.

Finally, in panel 3(a), as we enter the adiabatic regime, T_1 is larger than, or comparable with all escape times $T_\pm^\pm$, so that the random switch background grows noticeable, again; moreover, the hopping dynamics gets synchronized more closely to the drive swings, as proved by the few detectable $N_\pm(T)$ peaks shown. As anticipated in Sec. 3, the particle tends to relax into the tilted potential configurations of Fig. 1(lower) and then spends more time in the negative well than in the positive one, *i.e.* $\sigma > 0$.

As a clearcut evidence of the strong input-output synchronization in the adiabatic regime, we remark that in panel 3(a) the peak structure of $N_-(T)$ exhibits only one visible peak per forcing period T_1, at variance with $N_+(T)$ which shows two peaks per cycle. The interpretation of this feature is simple: The left-tilted configuration of the potential $V(x)$ (Fig. 1(lower), top) occurs only once per period, while the right-tilted configuration (Fig. 1, bottom) repeats itself twice in correspondence with the symmetric negative minima b, d of $F(t)$ in Fig. 1(upper); it follows that the right-to-left jumps are activated along the $\mathrm{a} \rightarrow \mathrm{b}$ branches of the drive waveform, while the left-to-right jumps take place preferably along either the shorter $\mathrm{d} \rightarrow \mathrm{e}$ or the longer $\mathrm{b} \rightarrow \mathrm{e}$ branches of $F(t)$, as at the midpoints c the tilt F vanishes. On the contrary, in the intermediate frequency regime of panel 3(b) both pairs of external tilt extrema a, c and b, d are capable of inducing left-to-right and right-to-left jumps, respectively, with comparable likelihood.

5. Conclusions

In conclusion, the nonlinear mixing of two periodic additive zero-mean signals is capable of localizing a Brownian particle into one well of a symmetric bistable potential through a resonant mechanism of stochastic symmetry breaking, a mechanism that went unnoticed in previous work. Correspondingly, preliminary evidence suggests that the Brownian motion on a symmetric substrate under appropriate modulation conditions may exhibit resonant rectification [11, 20]. Direct applications of the confinement techniques proposed in this article are within the reach of existing experimental technologies [22].

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